HOW TO CHOOSE THE FRICTION ANGLE FOR 3D CALCULATIONS

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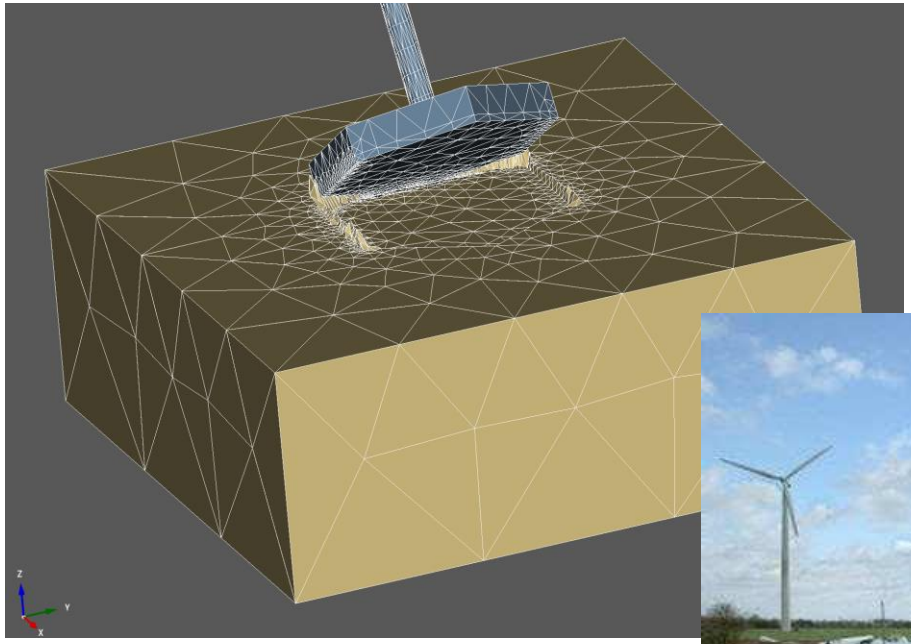
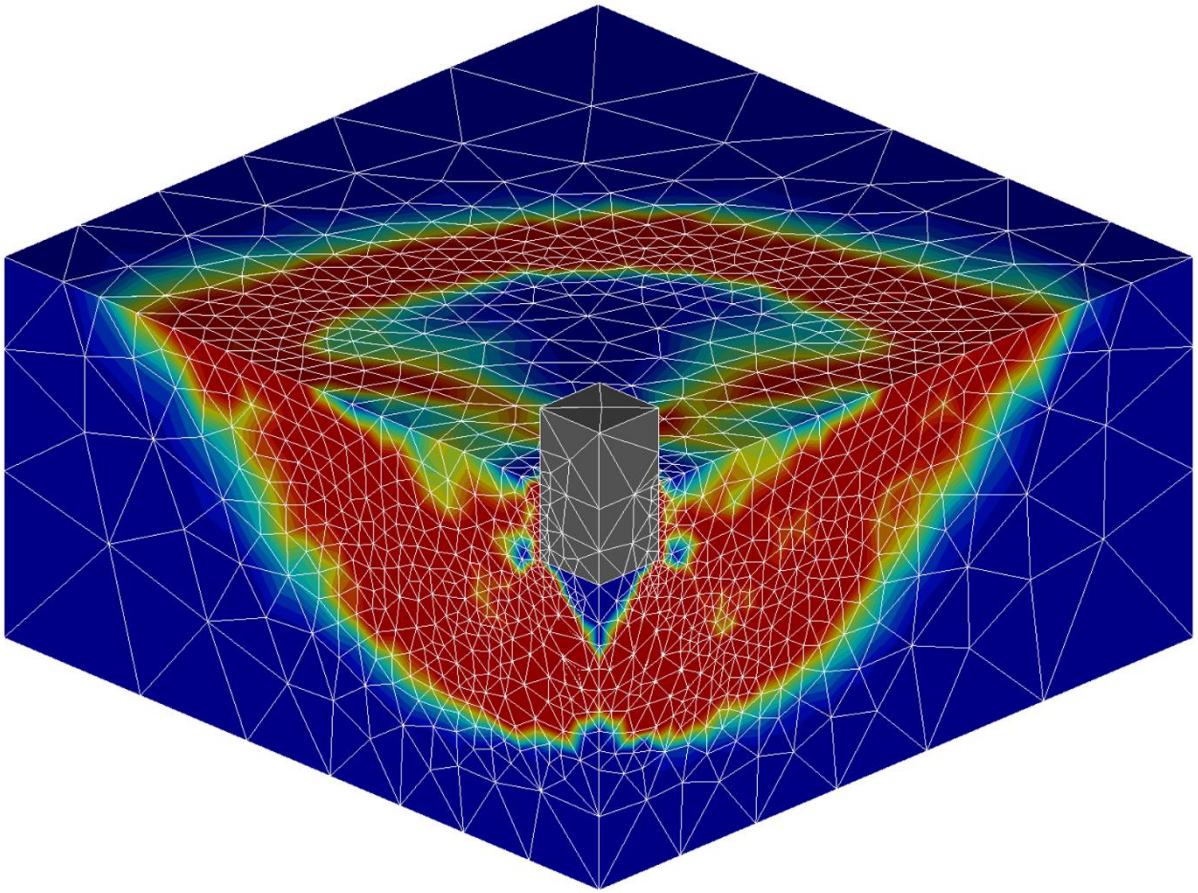
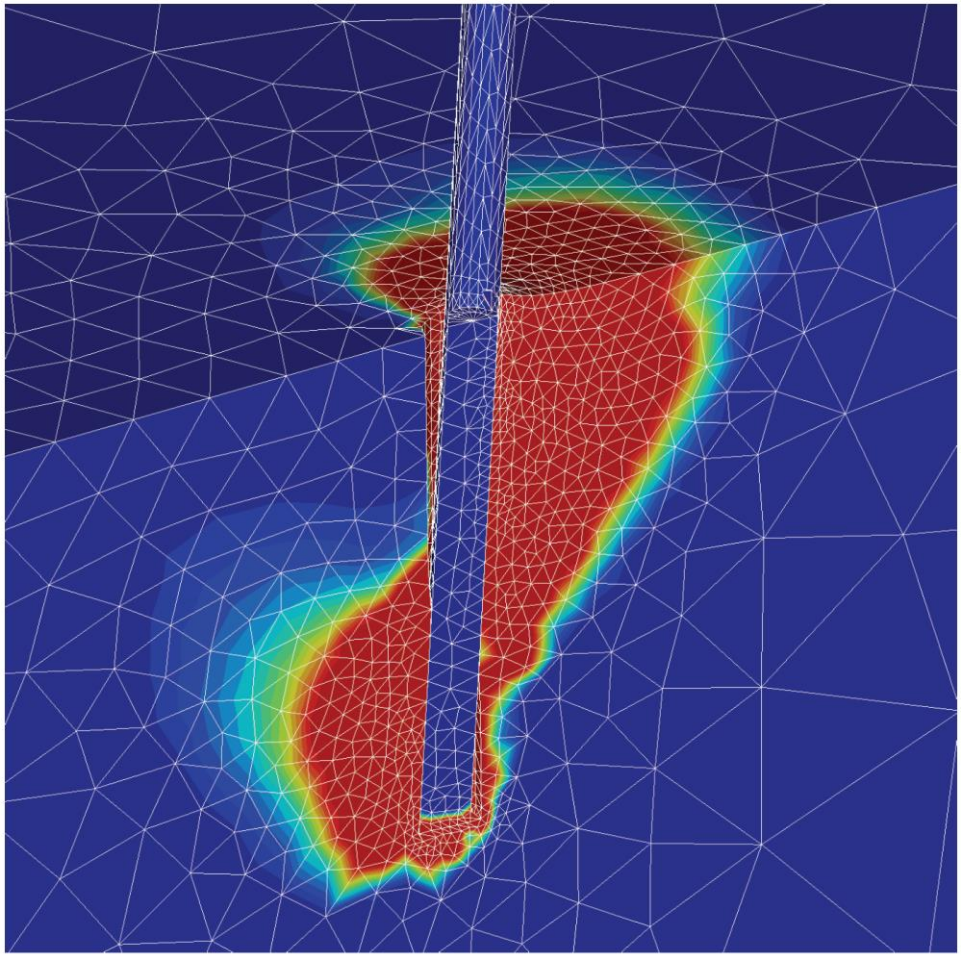
3D analyses



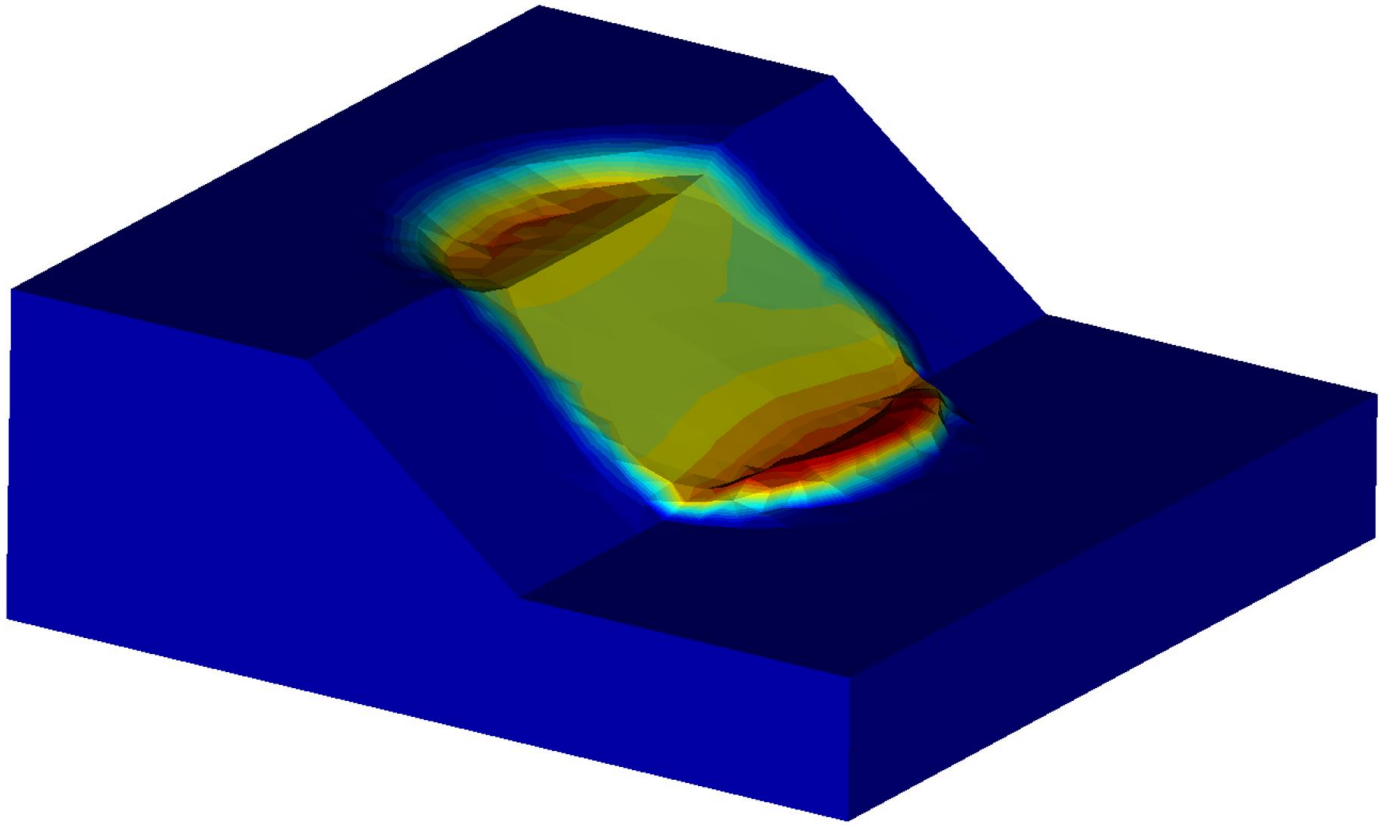
3D analyses



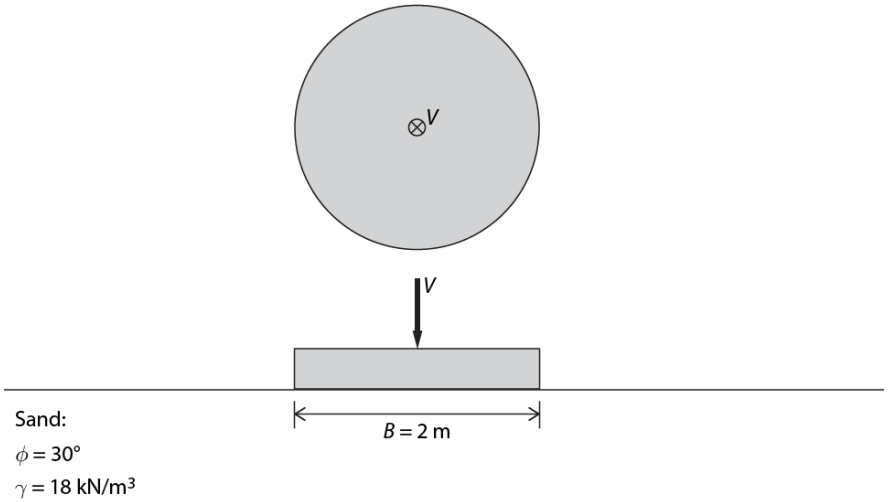
3D analyses



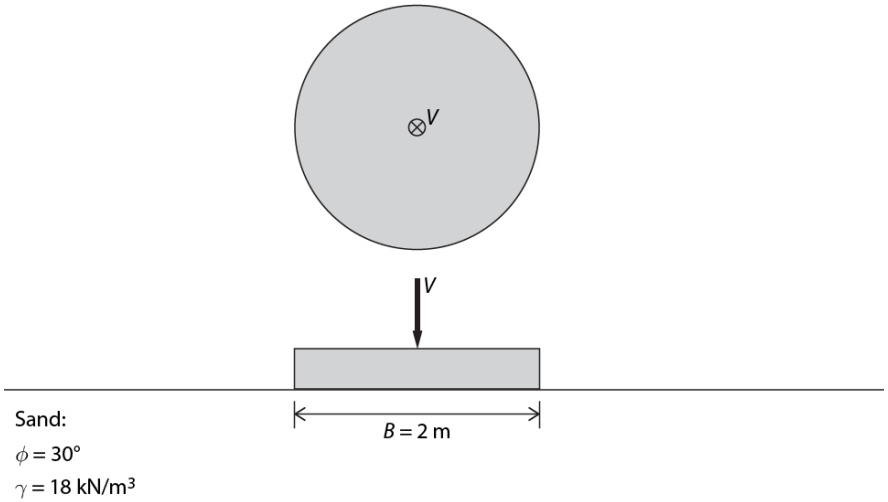
3D analyses



Circular foundation on sand (N_γ type problem)



Circular foundation on sand (N_γ type problem)



Bearing capacity equation (EC7 version):

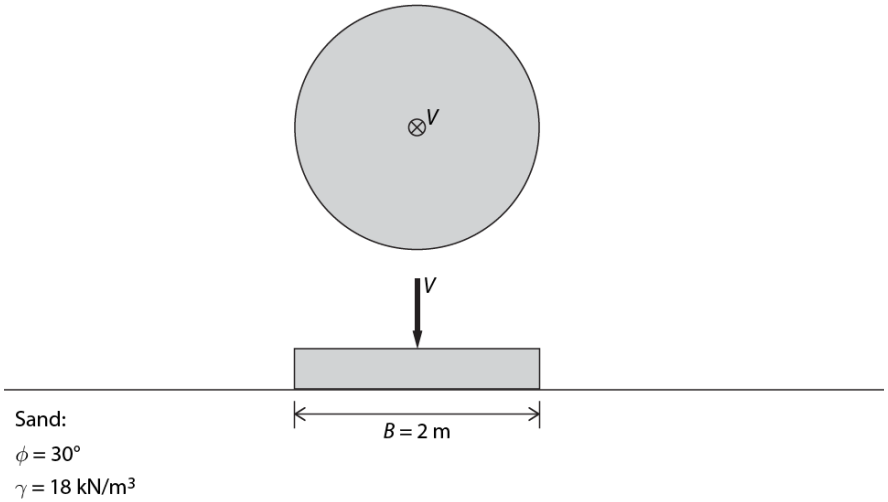
$$V_u/A = \tfrac{1}{2} B \gamma N_\gamma s_\gamma$$

where:

$$N_\gamma = 2(N_q - 1)\tan\phi = 20.1$$

$$s_\gamma = 0.7$$

Circular foundation on sand (N_γ type problem)



Bearing capacity equation (EC7 version):

$$V_u/A = \tfrac{1}{2} B \gamma N_\gamma s_\gamma = 253\,\mathrm{kPa}$$

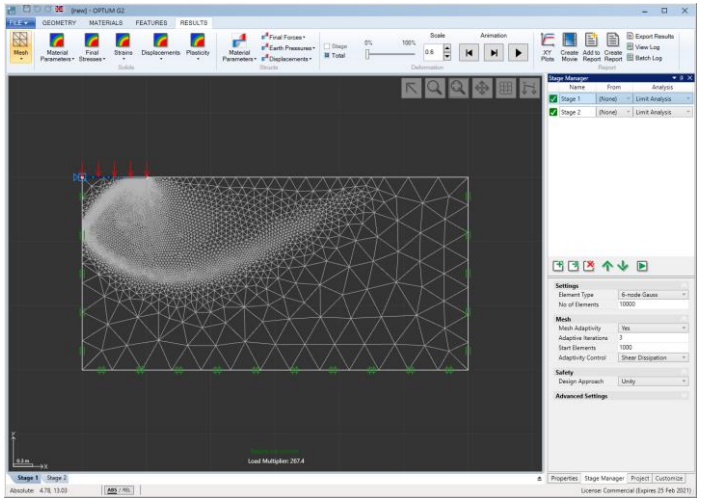
where:

$$N_\gamma = 2(N_q - 1)\tan\phi = 20.1$$

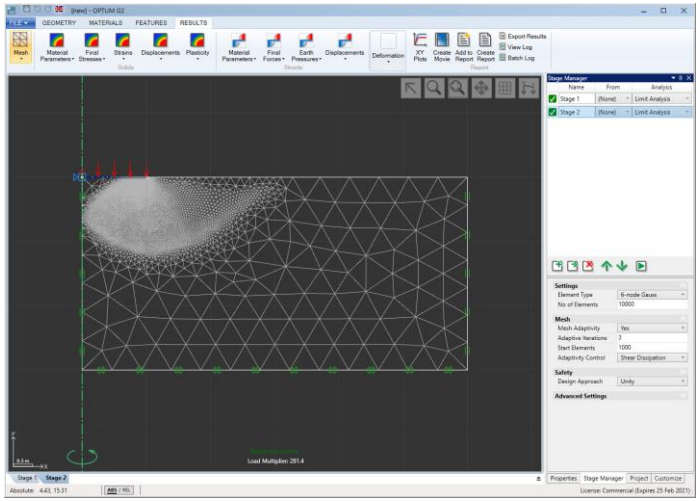
$$s_\gamma = 0.7$$

Circular foundation on sand (N_γ type problem)

OPTUM G2:



Strip: $N_\gamma = 14.8\ (20.1)$



Circular: $s_\gamma = 1.05\ (0.7)$

Circular foundation on sand (N_γ type problem)

Summary:

	N_γ	S_γ	V_u/A (kPa)
EC7	20.1	0.7	253
FE	14.8	1.05	280

Circular foundation on sand (N_γ type problem)

Summary:

	N_γ	S_γ	V_u/A (kPa)
EC7	20.1	0.7	253
FE	14.8	1.05	280

Hold on: N_γ should be calculated on the basis of the *plane strain angle*

Circular foundation on sand (N_γ type problem)

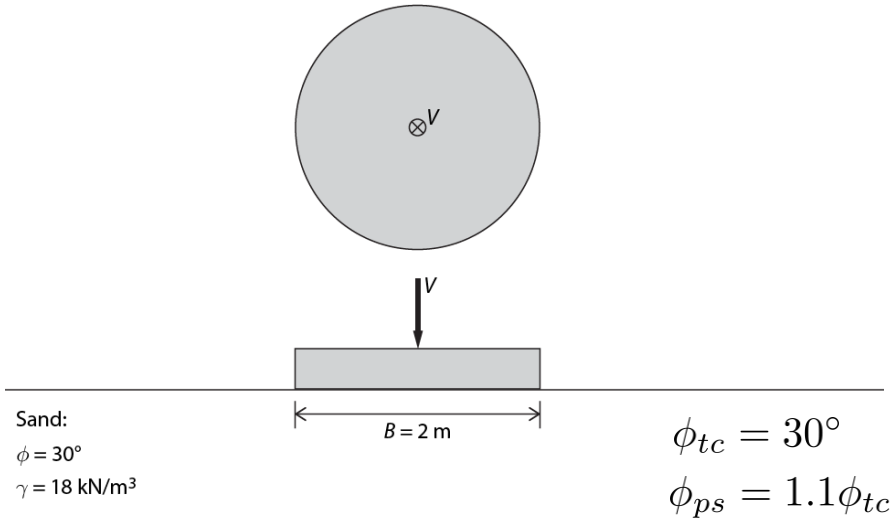
Summary:

	N_γ	S_γ	V_u/A (kPa)
EC7	20.1	0.7	253
FE	14.8	1.05	280

Hold on: N_γ should be calculated on the basis of the *plane strain angle*

That is why s_γ = 0.7 – both shape *and* stress states not corresponding to plane strain

Circular foundation on sand (N_γ type problem)



$$\phi_{tc} = 30°$$

$$\phi_{ps} = 1.1\phi_{tc} = 33°$$

Bearing capacity equation (EC7 version):

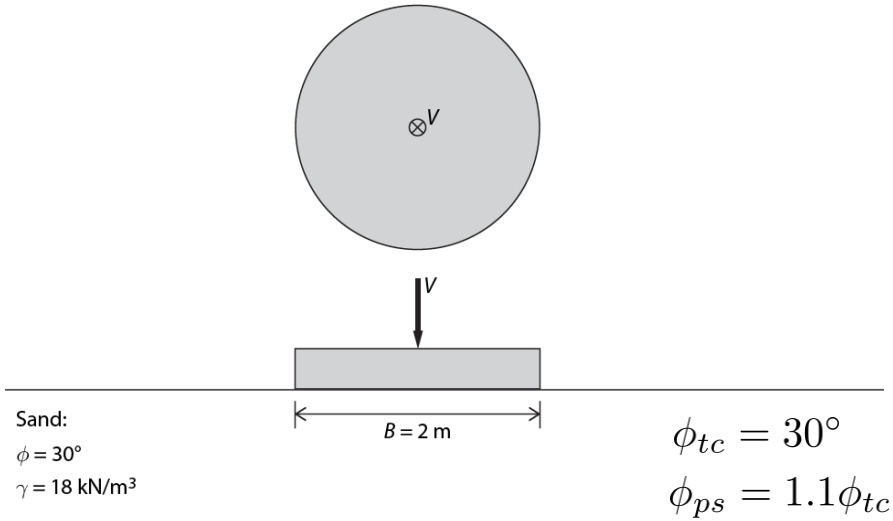
$$V_u/A = \tfrac{1}{2} B \gamma N_\gamma s_\gamma$$

where:

$$N_\gamma = 2(N_q - 1)\tan\phi = 32.6$$

$$s_\gamma = 0.7$$

Circular foundation on sand (N_γ type problem)



$$\phi_{tc} = 30°$$

$$\phi_{ps} = 1.1\phi_{tc} = 33°$$

Bearing capacity equation (EC7 version):

$$V_u/A = \tfrac{1}{2} B \gamma N_\gamma s_\gamma = 411\,\mathrm{kPa}$$

where:

$$N_\gamma = 2(N_q - 1)\tan\phi = 32.6$$

$$s_\gamma = 0.7$$

Circular foundation on sand (N_γ type problem)

Summary:

	N_γ	S_γ	V_u/A (kPa)
EC7 (30°)	20.1	0.7	253
FE (30°)	14.8	1.05	280
EC7 (33°)	32.6	0.7	411

Hold on: N_γ should be calculated on the basis of the *plane strain angle*

That is why s_γ = 0.7 – both shape *and* stress states <u>not</u> corresponding to plane strain

Circular foundation on sand (N_γ type problem)

Summary:

	N_γ	S_γ	V_u/A (kPa)
EC7 (30°)	20.1	0.7	253
FE (30°)	14.8	1.05	279
EC7 (33°)	32.6	0.7	411

Hold on: N_γ should be calculated on the basis of the *plane strain angle*

That is why s_γ = 0.7 – both shape *and* stress states not corresponding to plane strain

How about FE – f_{tc} or f_{ps}?

Circular foundation on sand (N_γ type problem)

Summary:

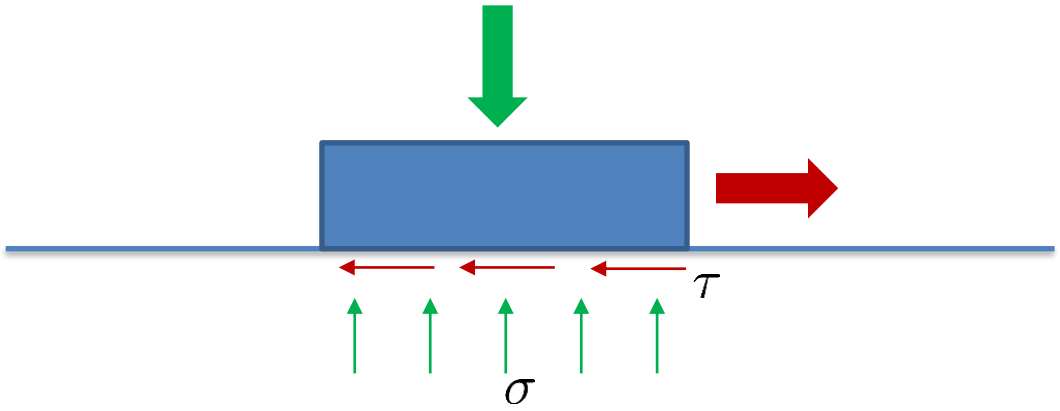
	N_γ	S_γ	V_u/A (kPa)
EC7 (30°)	20.1	0.7	253
FE (30°)	14.8	1.05	280
EC7 (33°)	32.6	0.7	411
FE (33°)	24.4	1.14	501

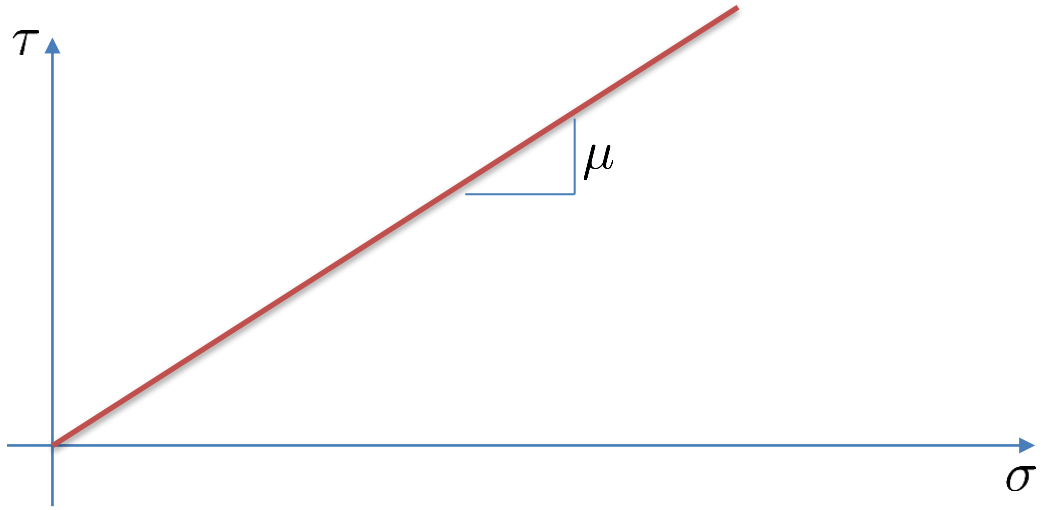
Hold on: N_γ should be calculated on the basis of the *plane angle*

That is why s_γ = 0.7 – both shape *and* stress states not corresponding to plane strain

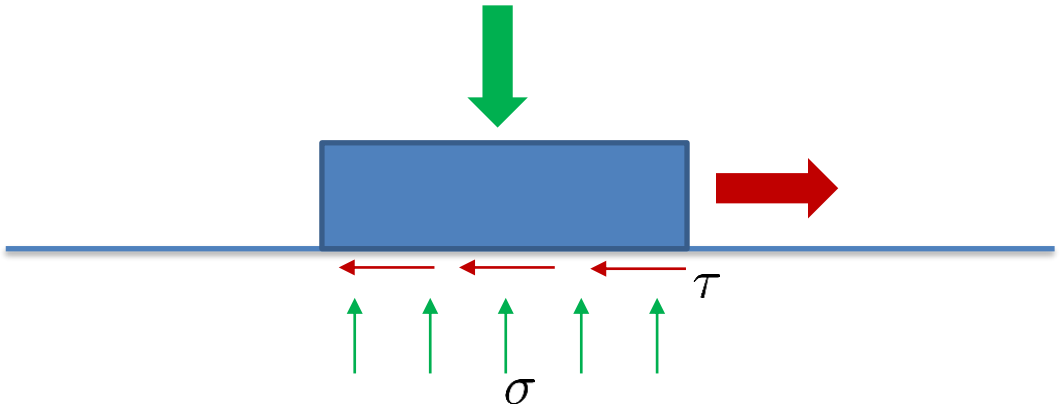
How about FE – f_{tc} or f_{ps}?

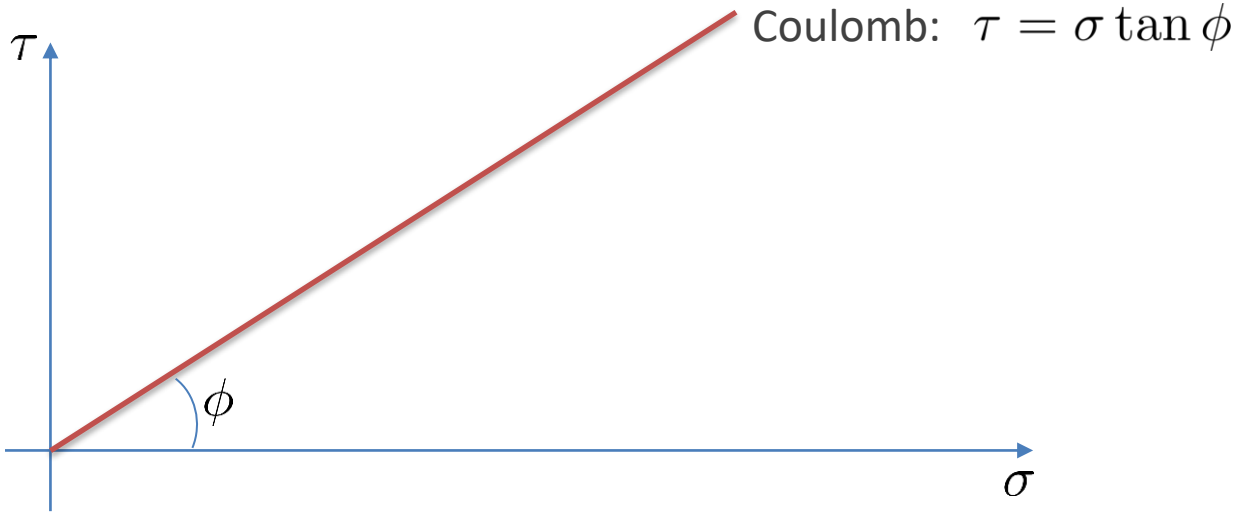
Friction angle





Friction angle





Coulomb: $\tau = \sigma \tan \phi$

Friction angle

General continuum setting:

Failure when

$$\tau = \sigma \tan \phi$$

on the *critical section*

Friction angle

General continuum setting:

Failure when

$$\tau = \sigma \tan \phi$$

on the *critical section*

In terms of principal stresses:

$$\sigma_1 - \sigma_3 + (\sigma_1 + \sigma_3) \sin \phi = 0$$
$$\sigma_3 - \sigma_1 + (\sigma_1 + \sigma_3) \sin \phi = 0$$
$$\sigma_1 - \sigma_2 + (\sigma_1 + \sigma_2) \sin \phi = 0$$
$$\sigma_2 - \sigma_1 + (\sigma_1 + \sigma_2) \sin \phi = 0$$
$$\sigma_2 - \sigma_3 + (\sigma_2 + \sigma_3) \sin \phi = 0$$
$$\sigma_3 - \sigma_2 + (\sigma_2 + \sigma_3) \sin \phi = 0$$

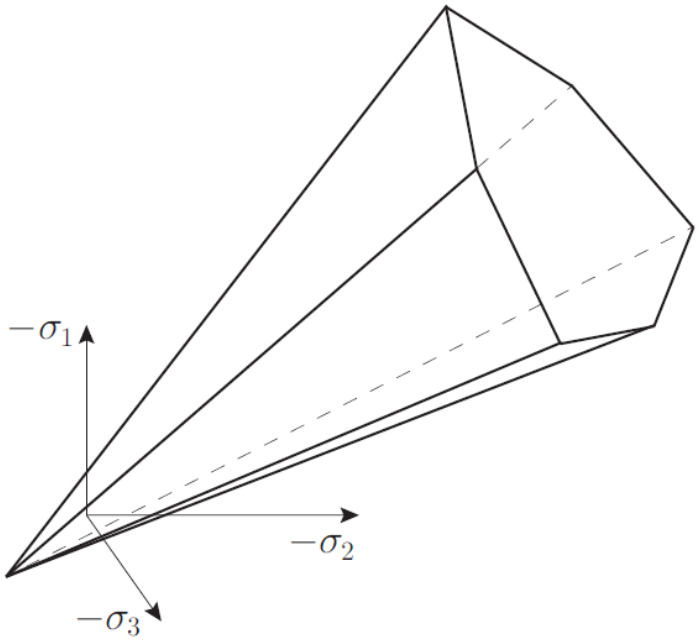
Friction angle

General continuum setting:

Failure when

$$\tau = \sigma \tan \phi$$

on the *critical section*



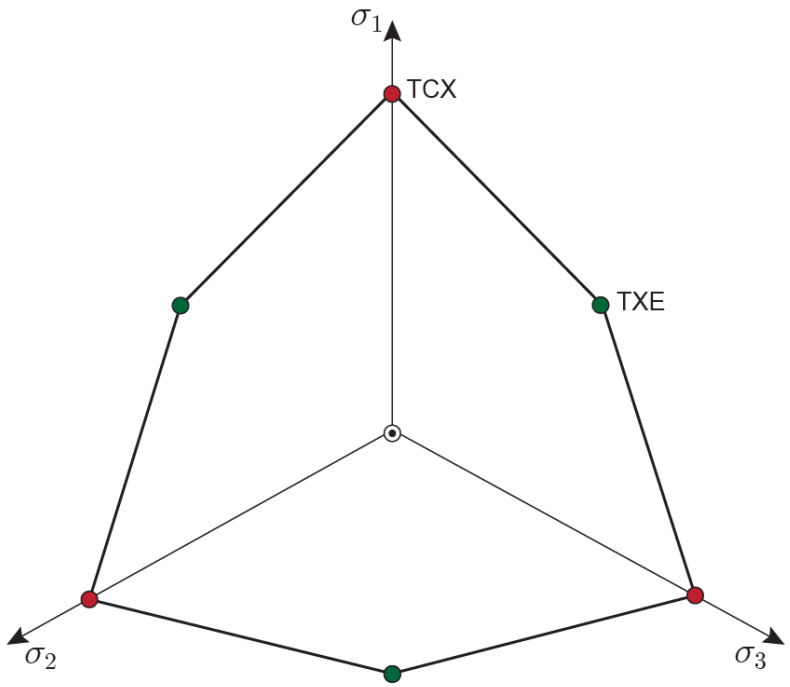
Friction angle

General continuum setting:

Failure when

$$\tau = \sigma \tan \phi$$

on the *critical section*



Friction angle

Experiments

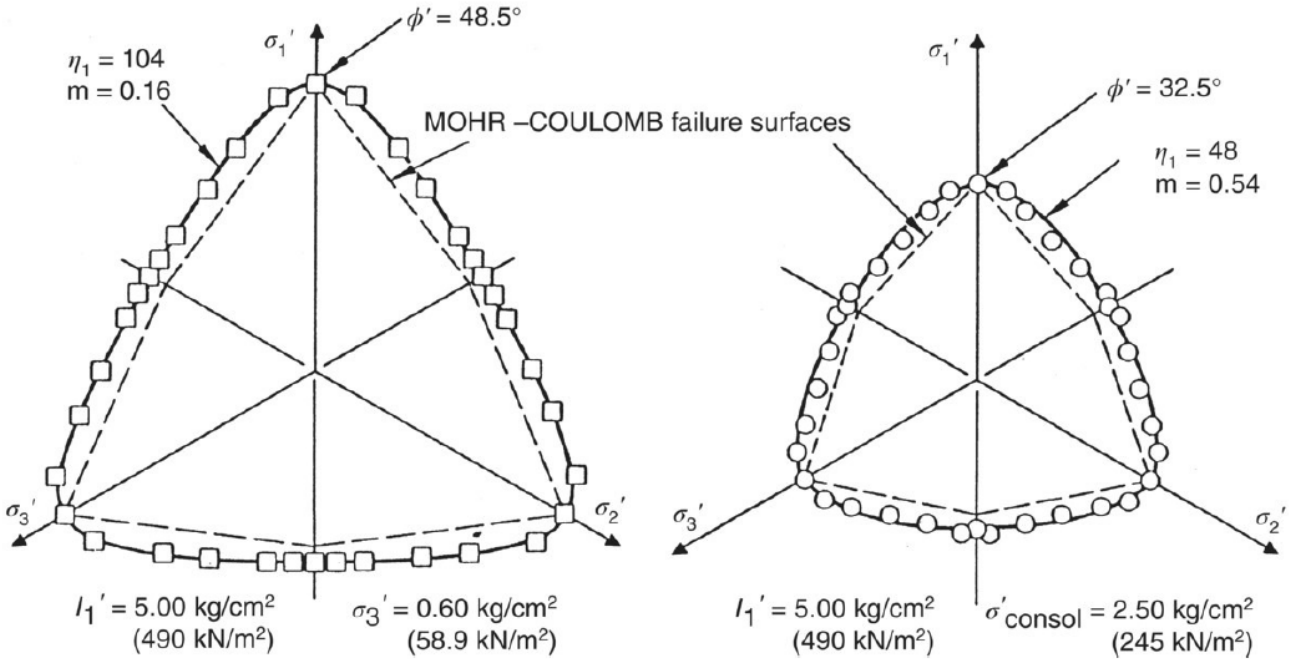
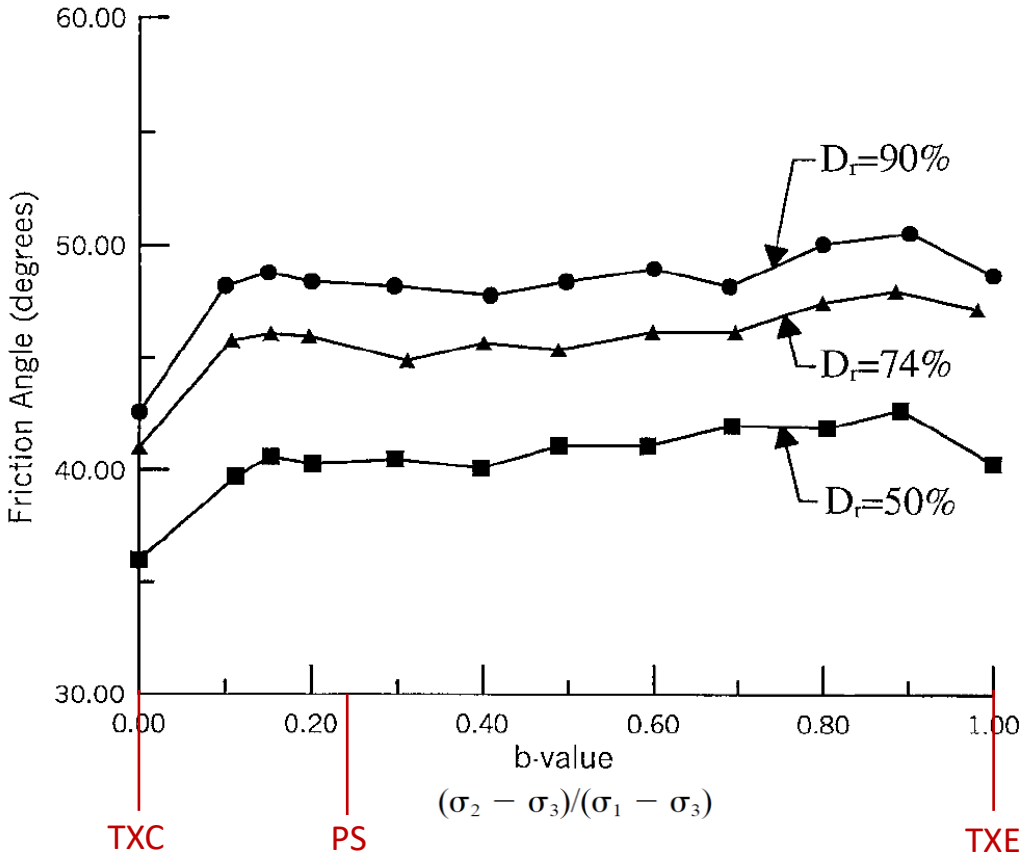


Figure 11.20 Comparisons between failure criterion and test data in terms of effective stresses for (a) dense Monterey No. 0 sand and (b) normally consolidated, remolded Edgar Plastic Kaolinite.

Reproduced from Lade 1984 by permission of John Wiley & Sons.

Friction angle

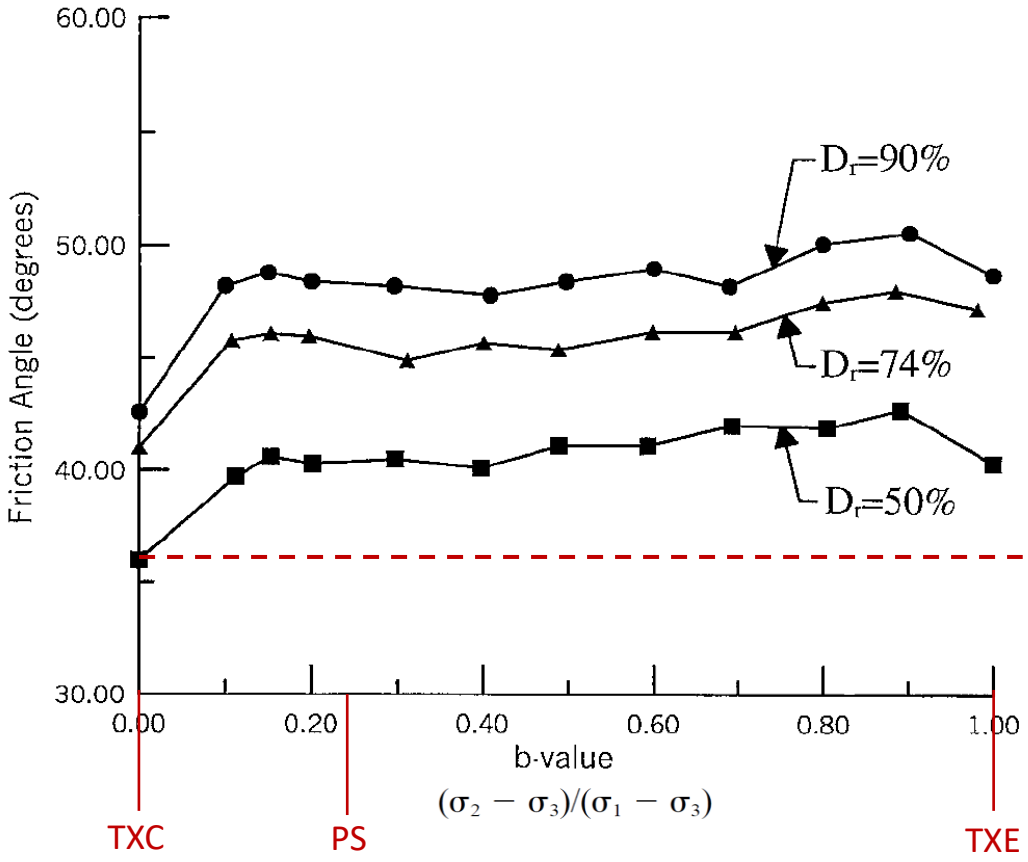
Experiments



Lade & Wang (2001)

Friction angle

Experiments



Lade & Wang (2001)

Friction angle

Experiments

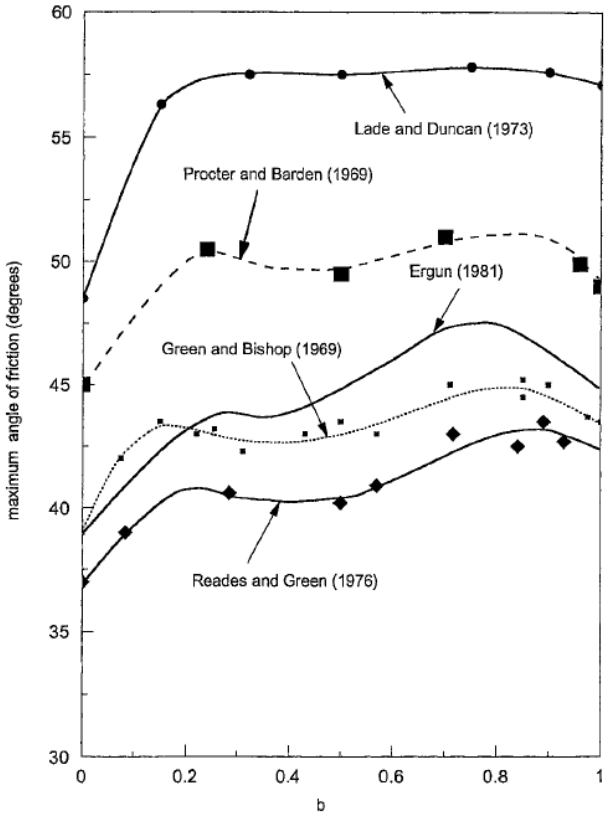
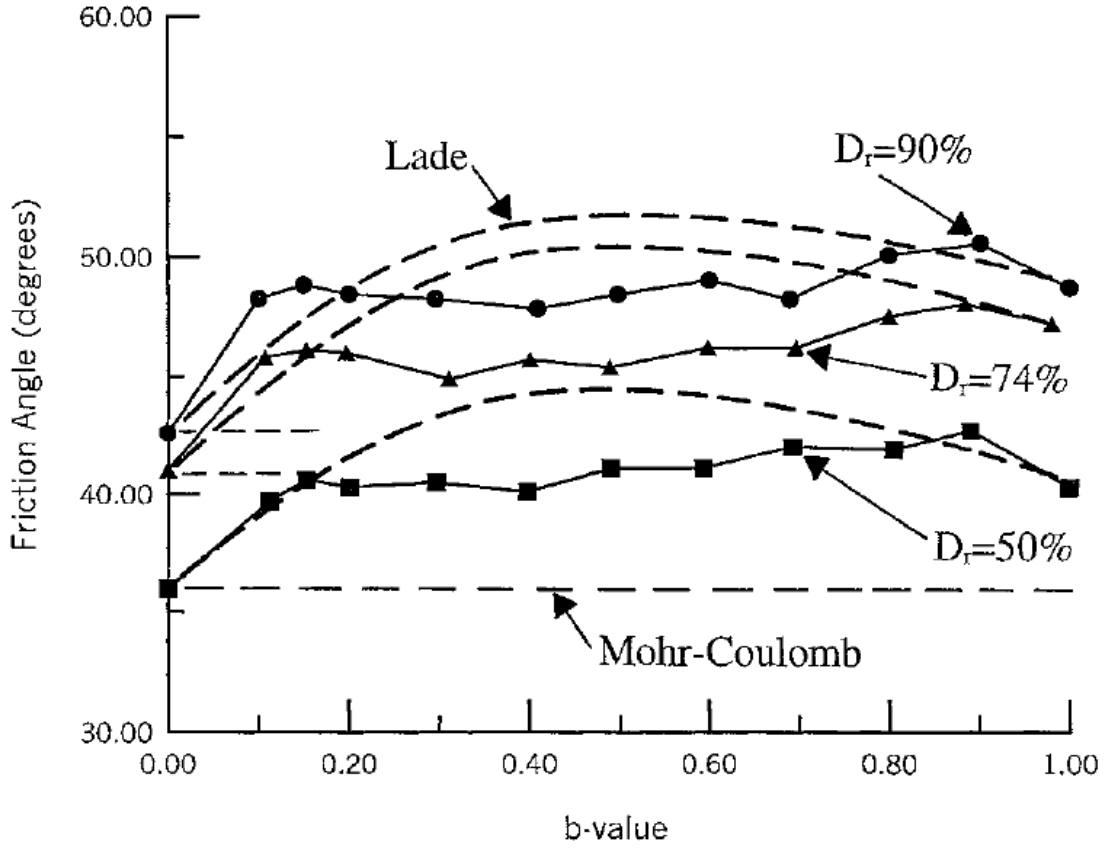


FIG. 1. Results of True Triaxial Tests on Sands Obtained by Various Authors, as Indicated on Diagram

Lade & Wang (2001)

Friction angle

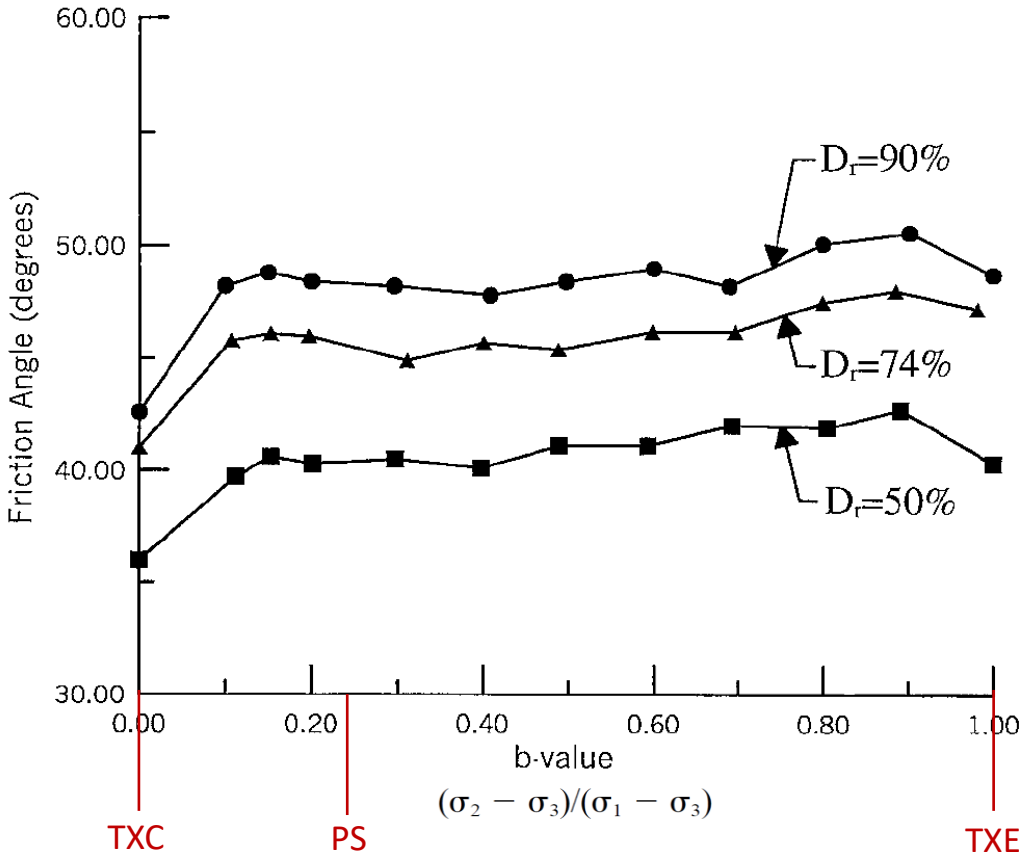
Experiments



Lade & Wang (2001)

Friction angle

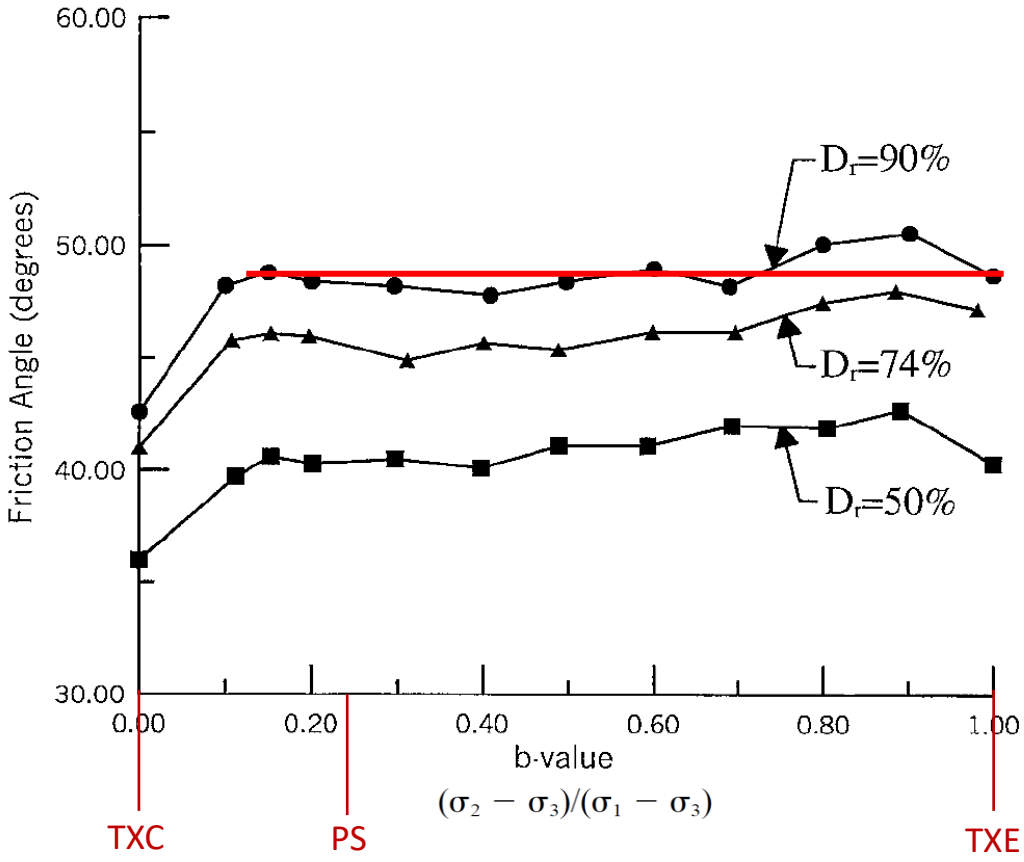
Experiments



Lade & Wang (2001)

Friction angle

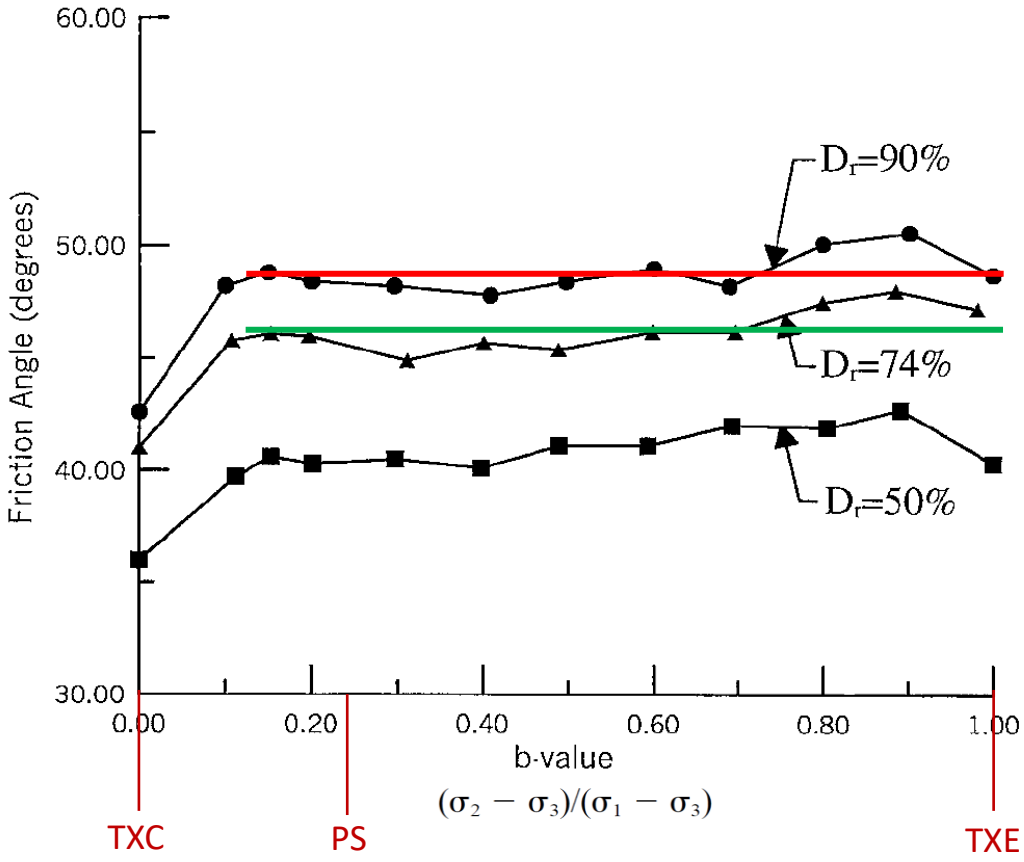
Experiments



Lade & Wang (2001)

Friction angle

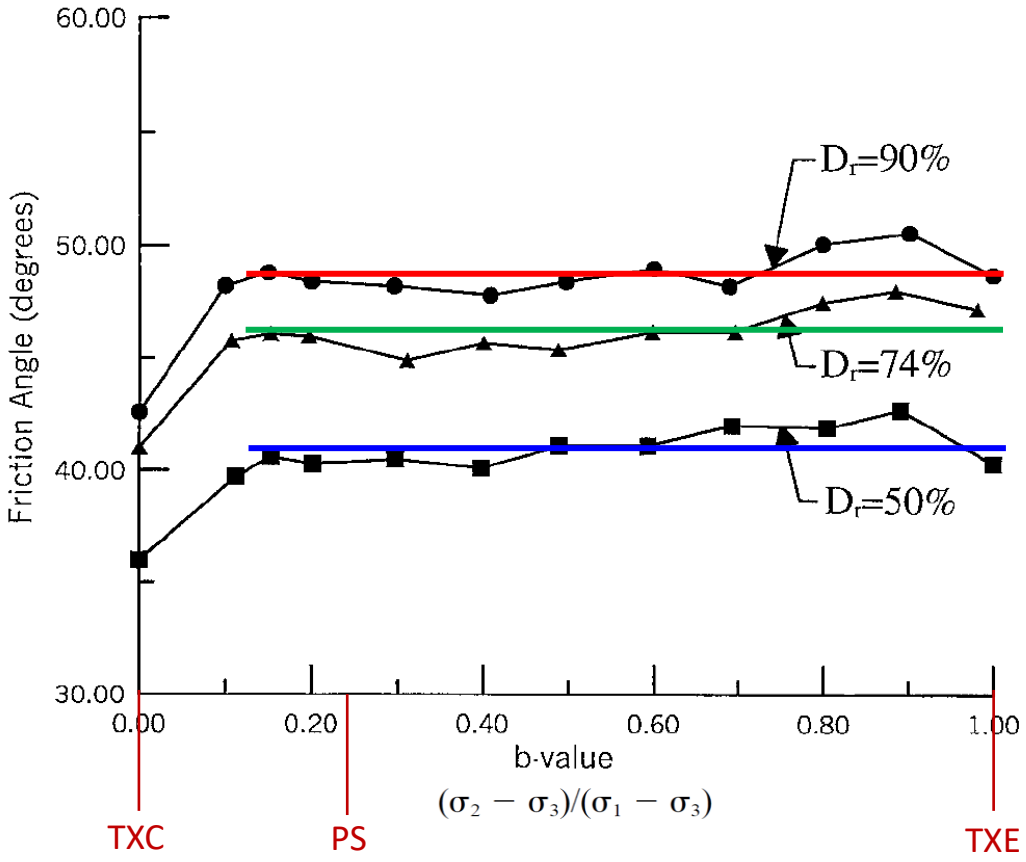
Experiments



Lade & Wang (2001)

Friction angle

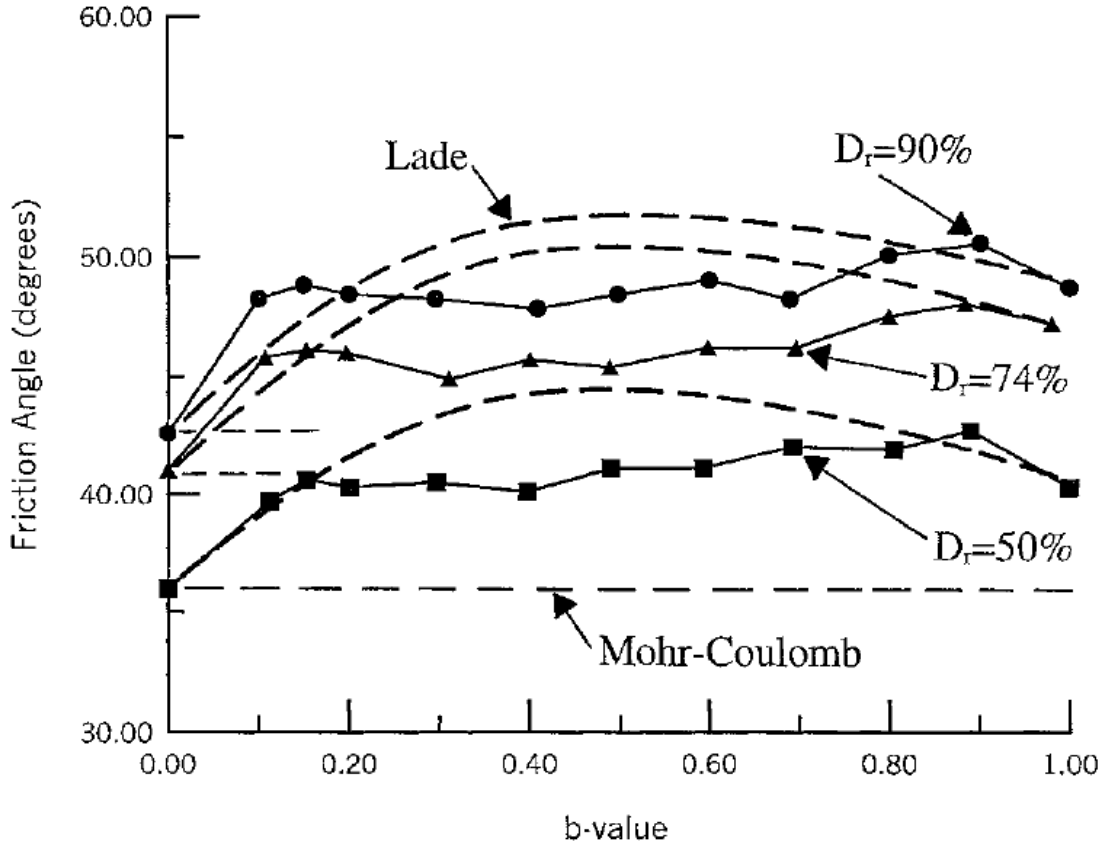
Experiments



Lade & Wang (2001)

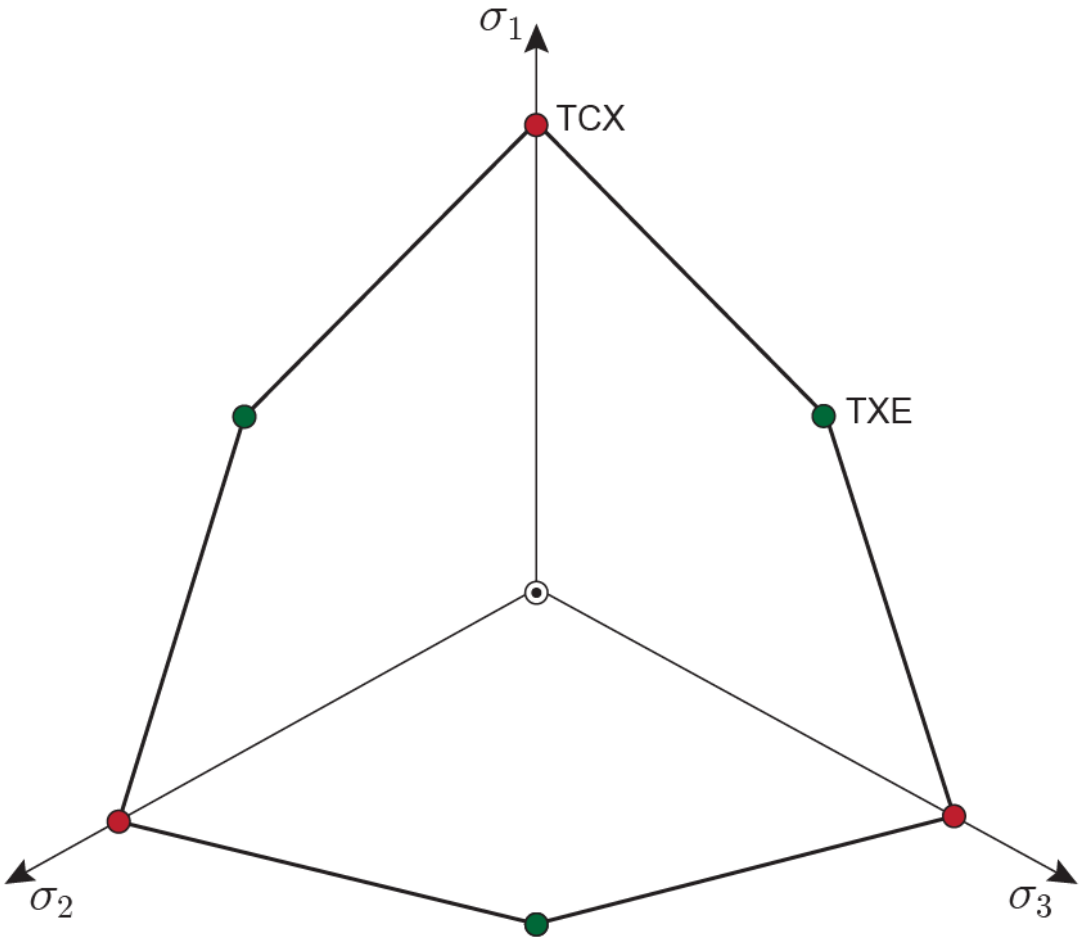
Friction angle

Experiments

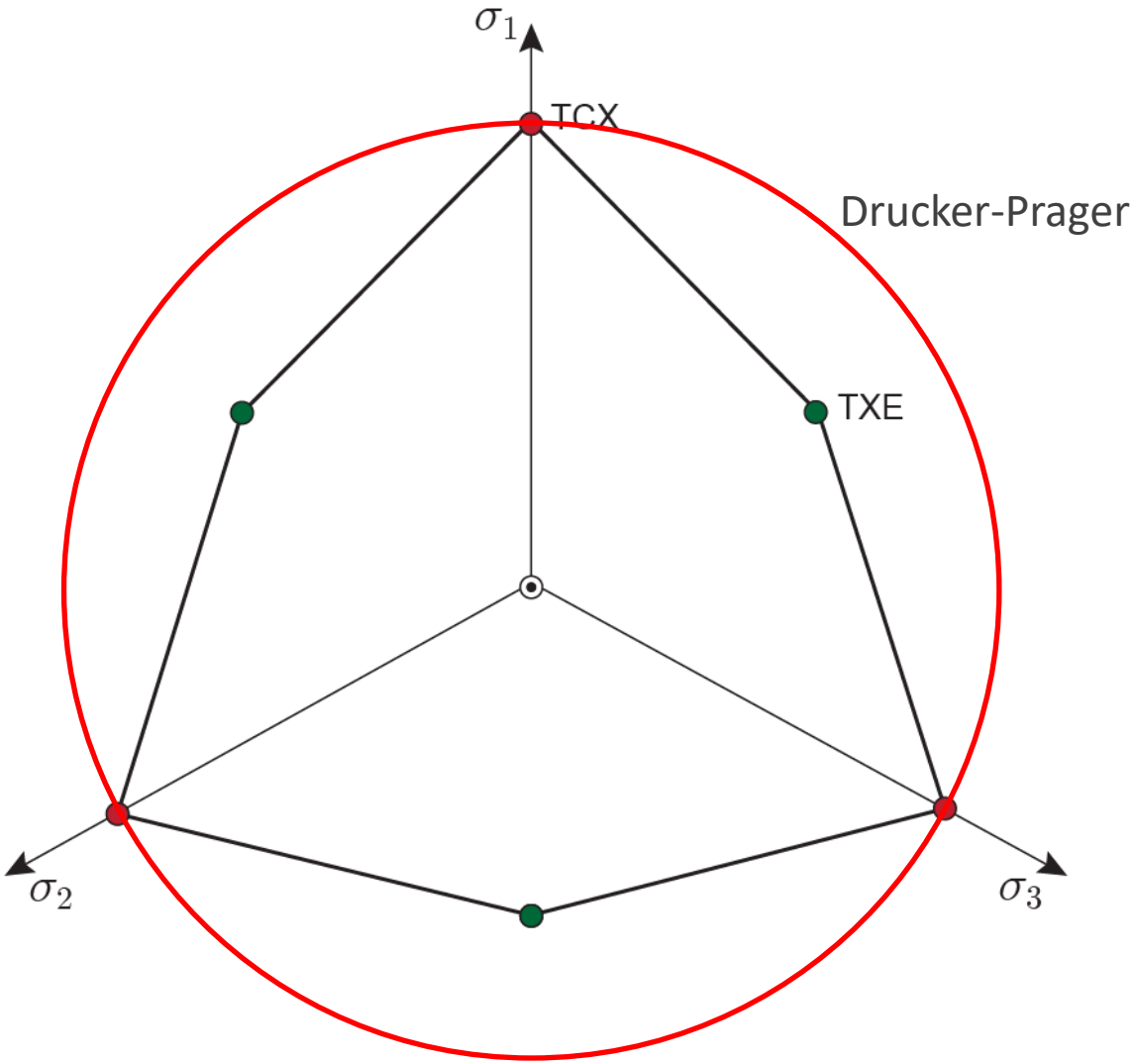


Lade & Wang (2001)

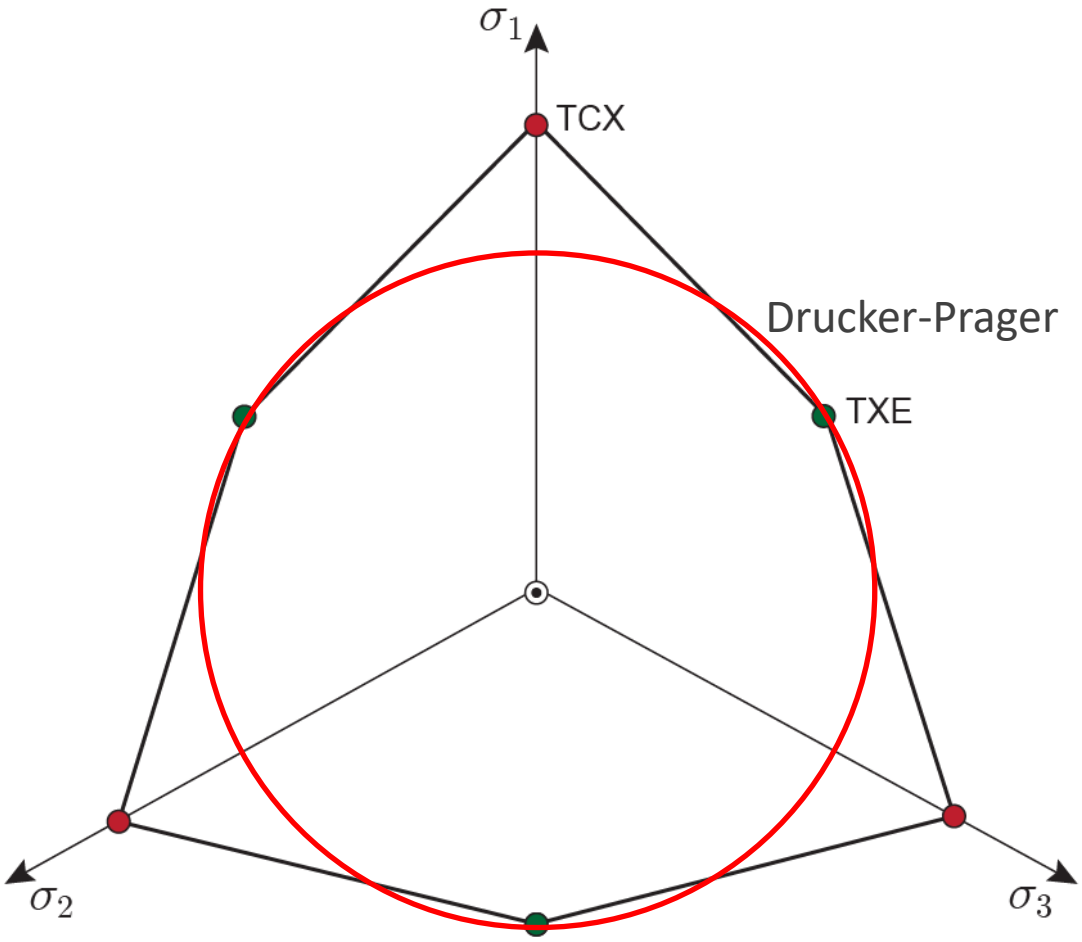
Failure criteria



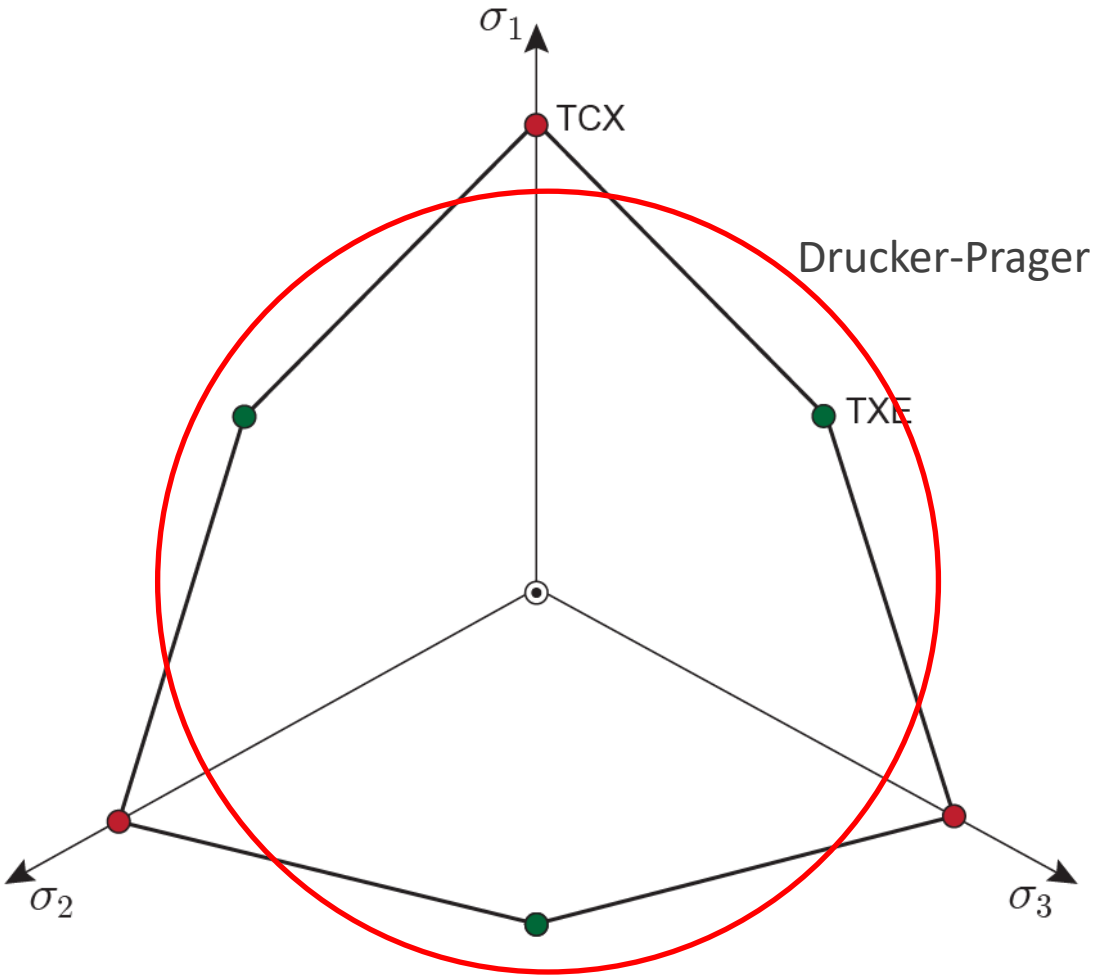
Failure criteria



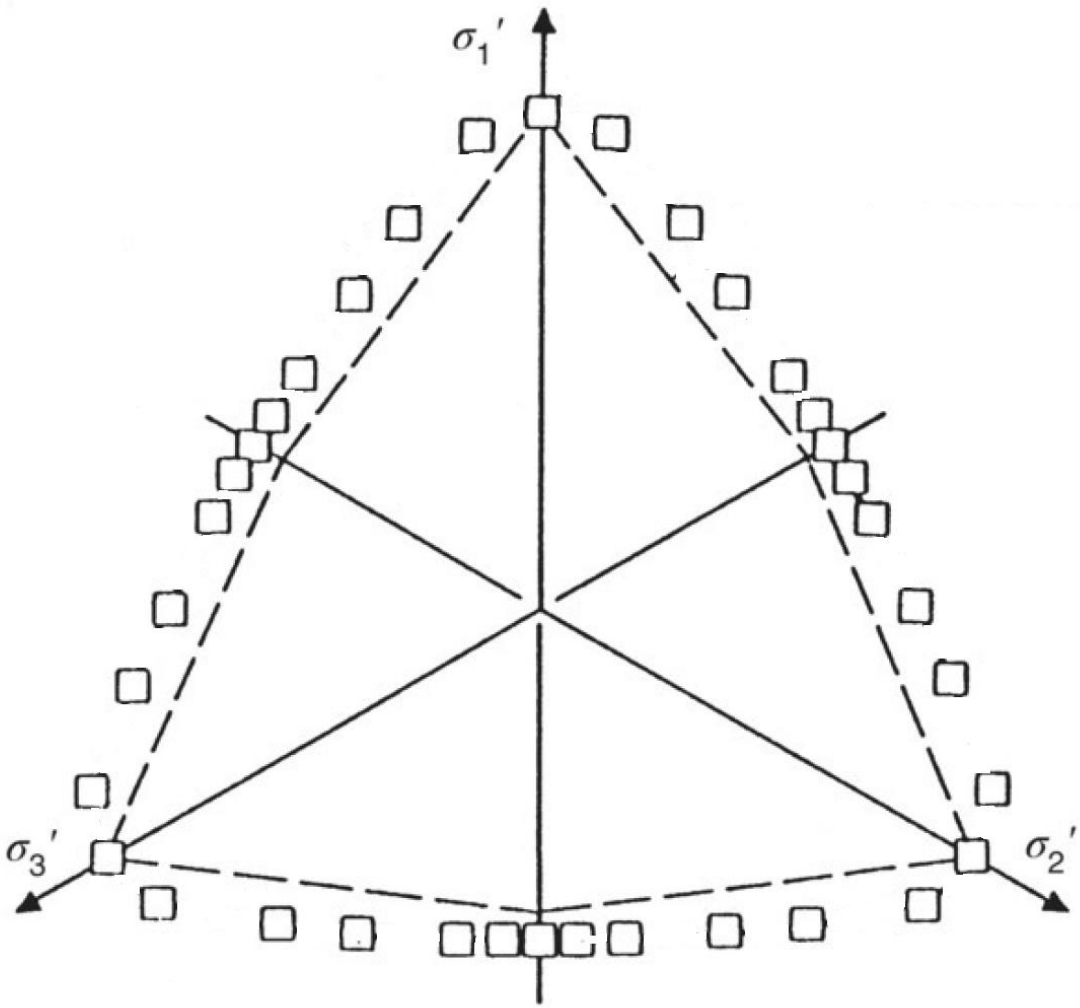
Failure criteria



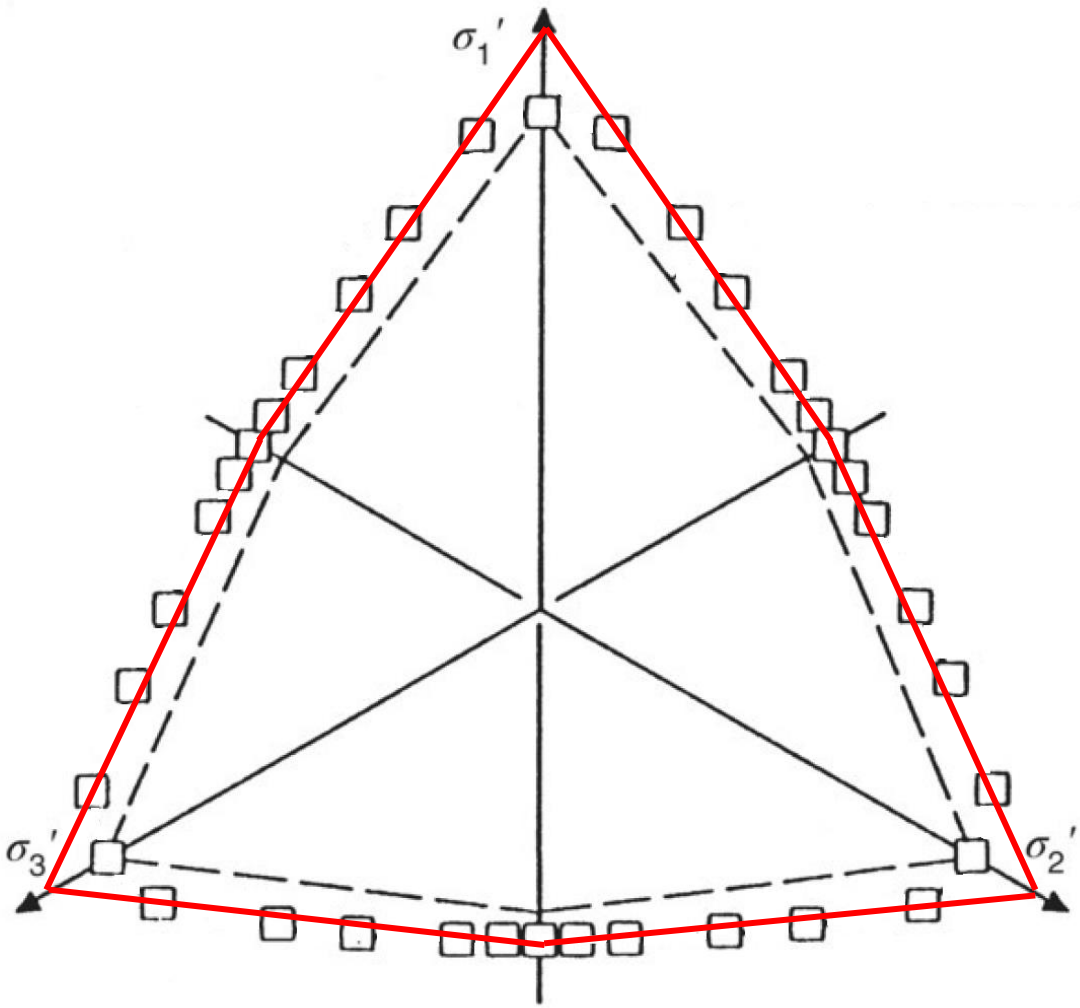
Failure criteria



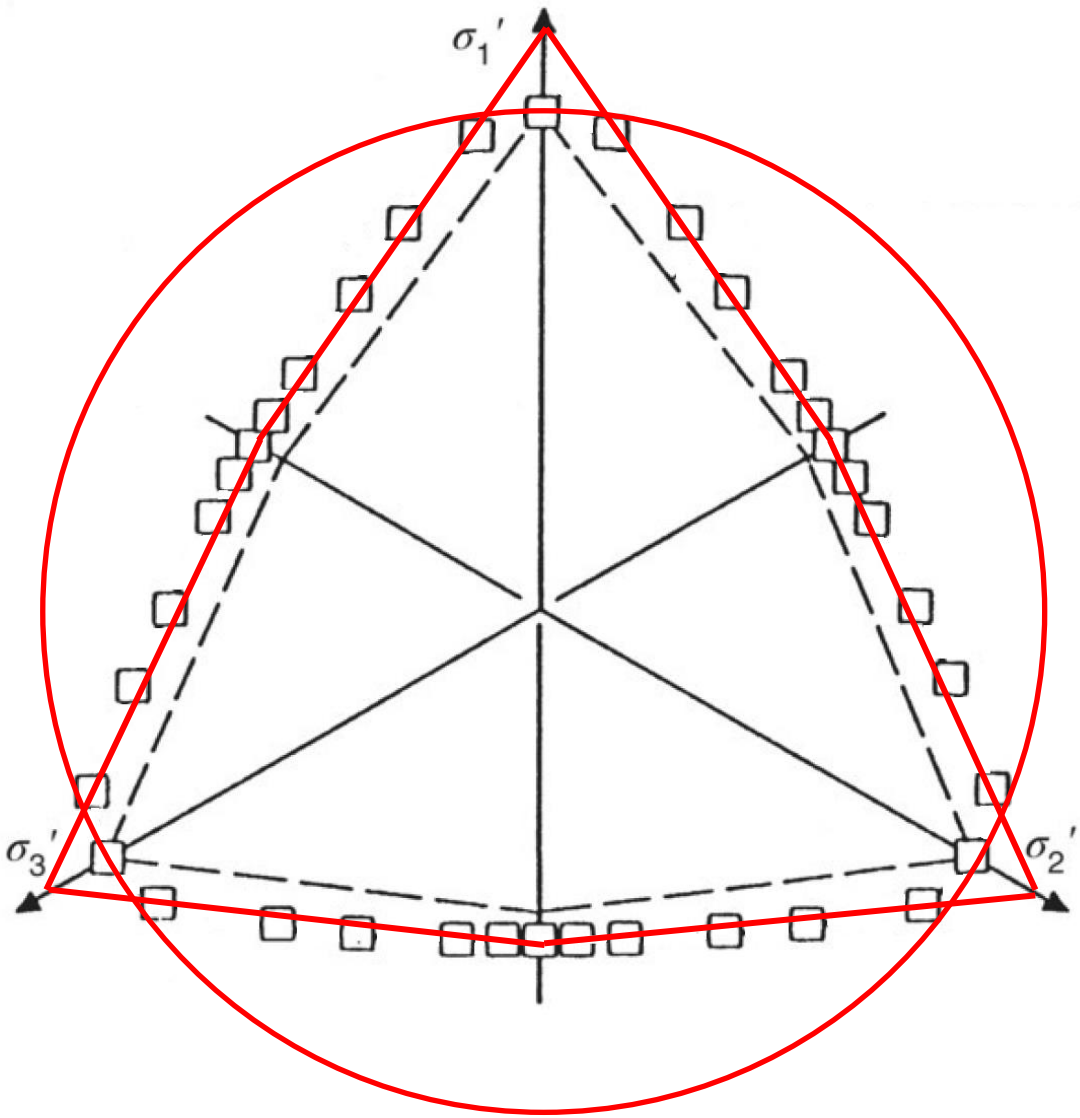
FASD Sand



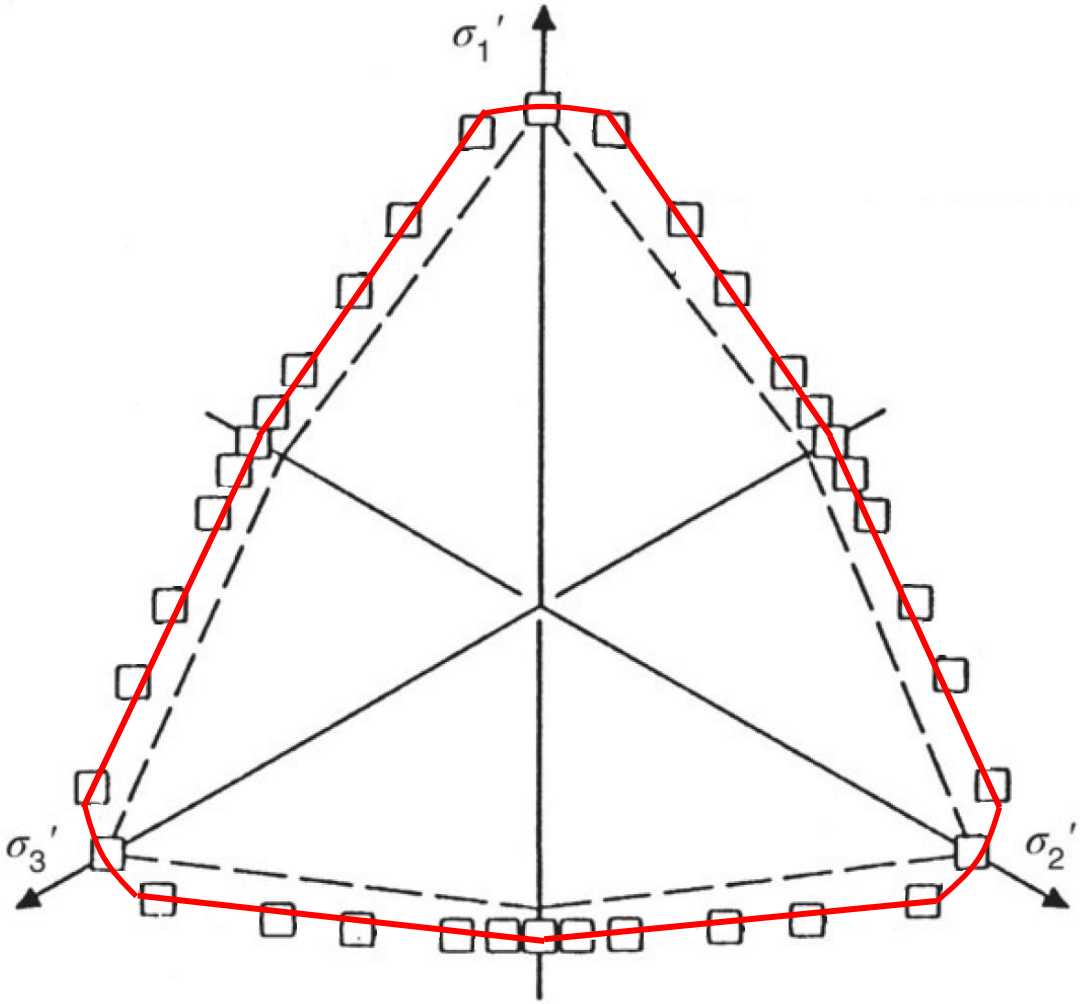
FASD Sand



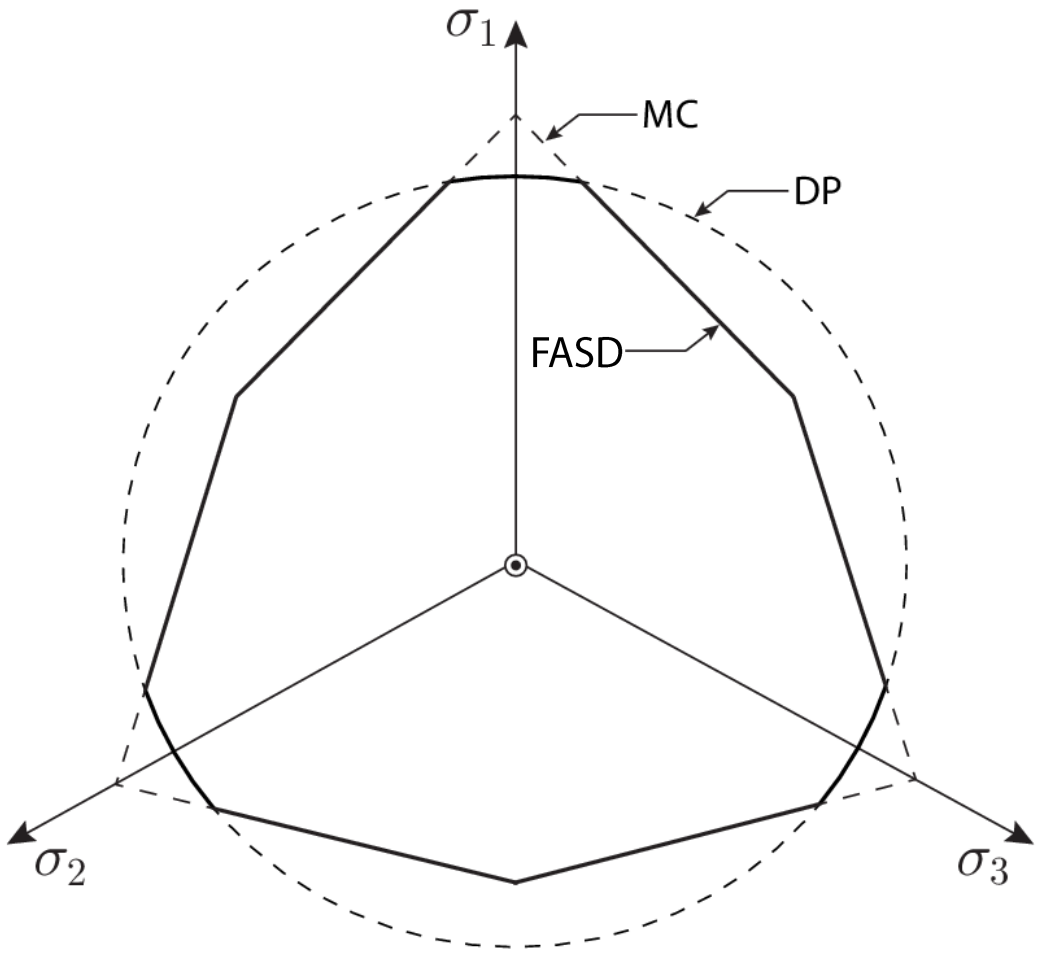
FASD Sand



FASD Sand

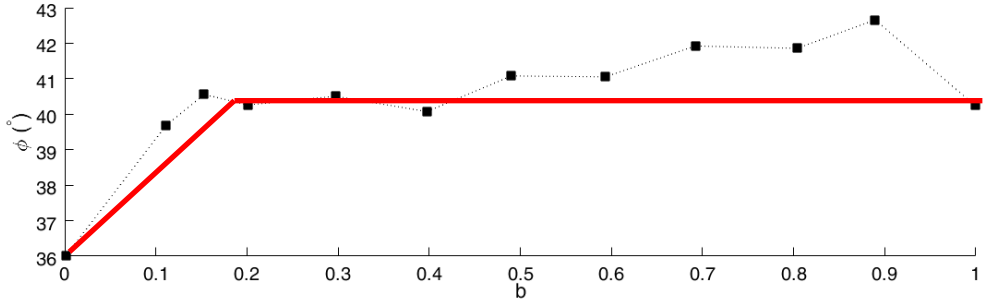


FASD Sand

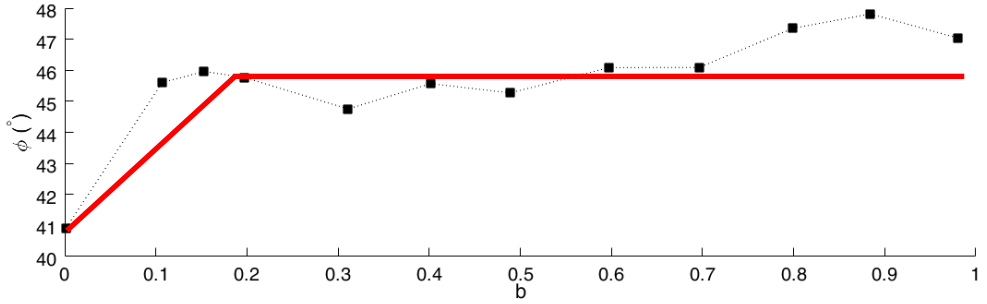


FASD

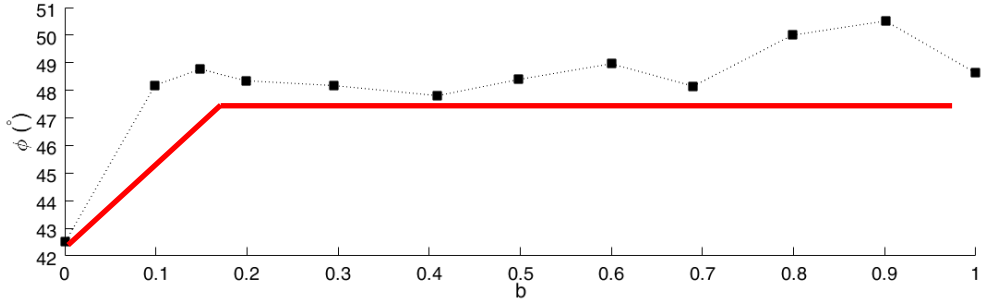
Loose



Medium



Dense



TXC PS TXE

Matched in TXC and:

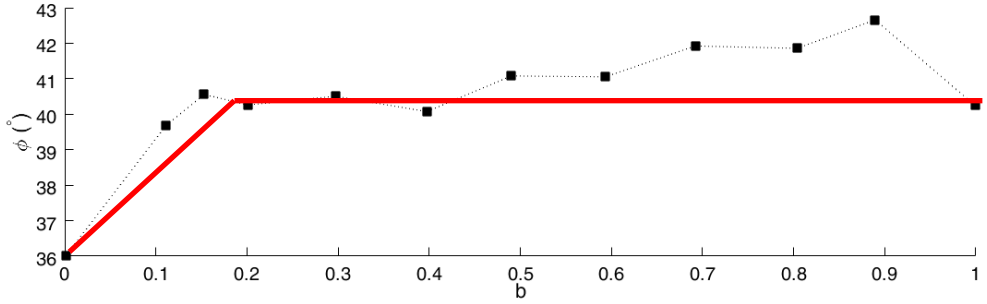
$$\phi_{ps} = 1.12\phi_{tc}$$

(Kulhawy & Mayne 1990)

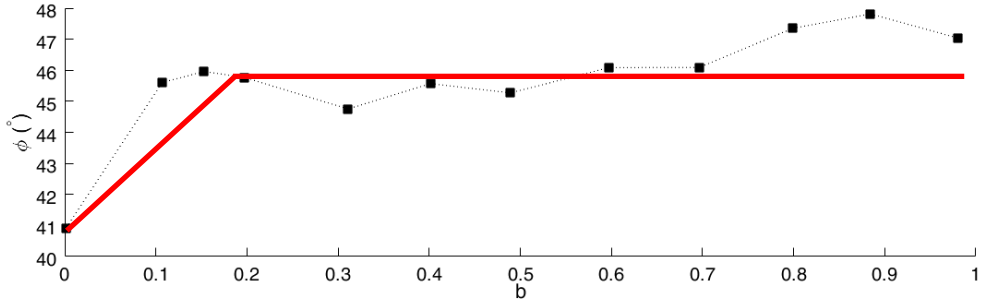
Data: Lade & Wang (2001)

FASD

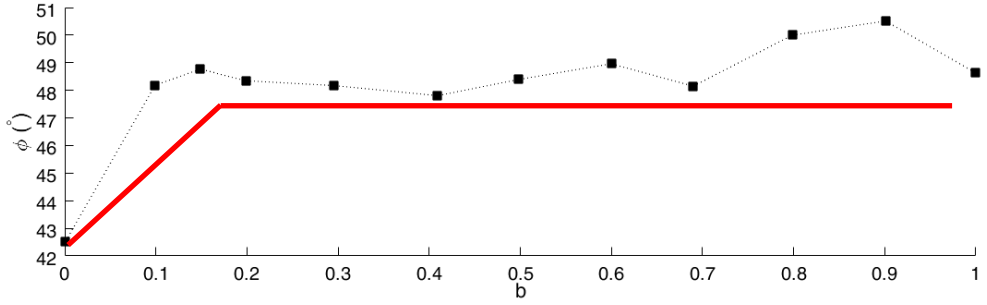
Loose



Medium



Dense



TXC PS TXE

Matched in TXC and:

$$\phi_{ps} = 1.12\phi_{tc}$$

(Kulhawy & Mayne 1990)

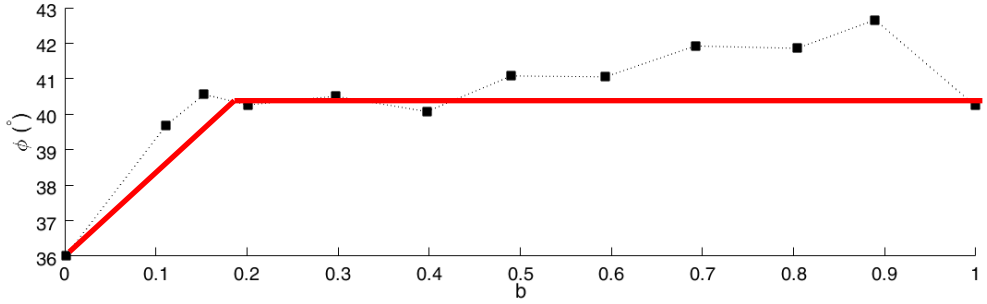
Old Danish:

$$\phi_{ps} = 1.1\phi_{tc}$$

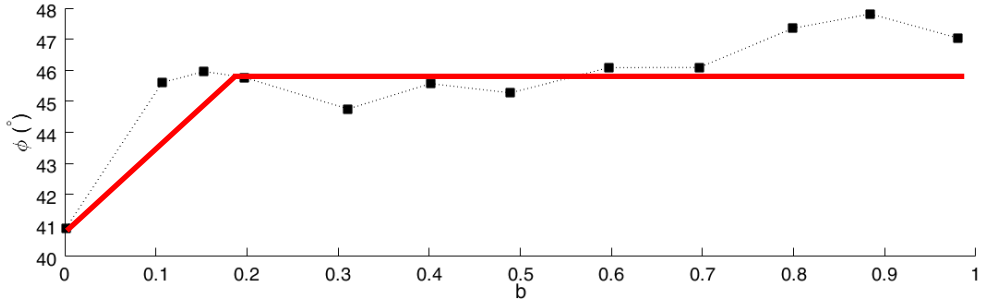
Data: Lade & Wang (2001)

FASD

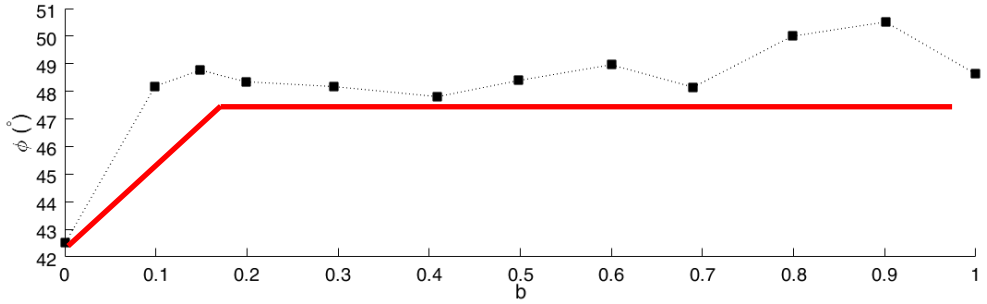
Loose



Medium



Dense



TXC PS TXE

Matched in TXC and:

$$\phi_{ps} = 1.12\phi_{tc}$$

(Kulhawy & Mayne 1990)

Old Danish:

$$\phi_{ps} = 1.1\phi_{tc}$$

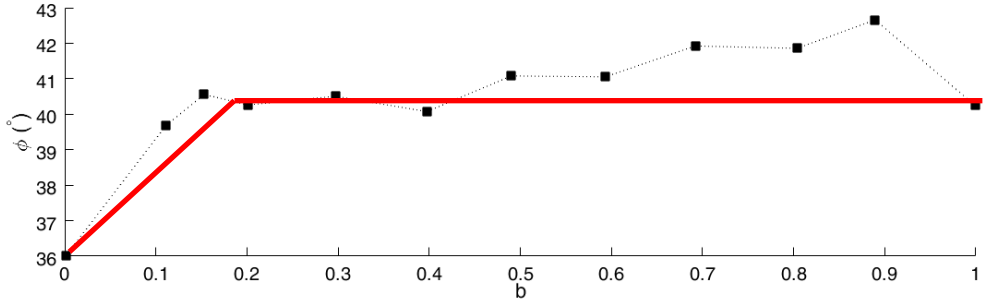
New Danish:

$$\phi_{ps} = (1 + 0.1I_D)\phi_{tc}$$

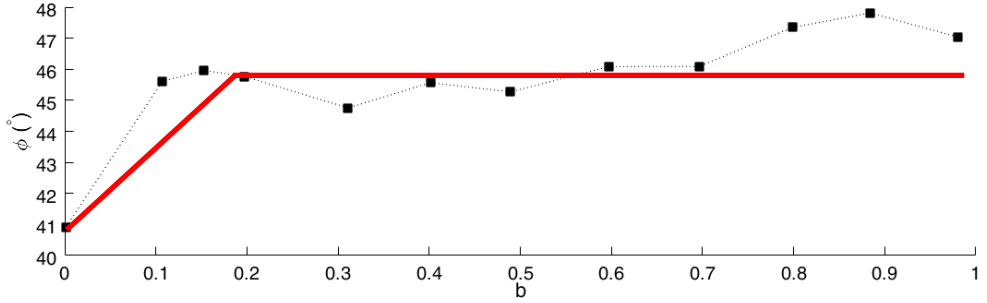
Data: Lade & Wang (2001)

FASD

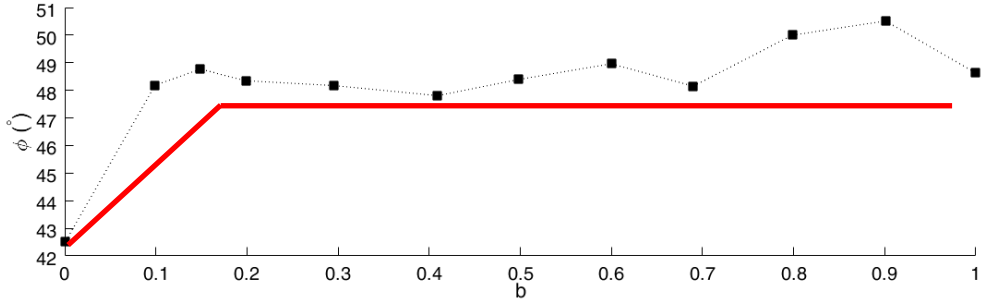
Loose



Medium



Dense



TXC PS TXE

Matched in TXC and:

$$\phi_{ps} = 1.12\phi_{tc}$$

(Kulhawy & Mayne 1990)

Old Danish:

$$\phi_{ps} = 1.1\phi_{tc}$$

New Danish:

$$\phi_{ps} = (1 + 0.1I_D)\phi_{tc}$$

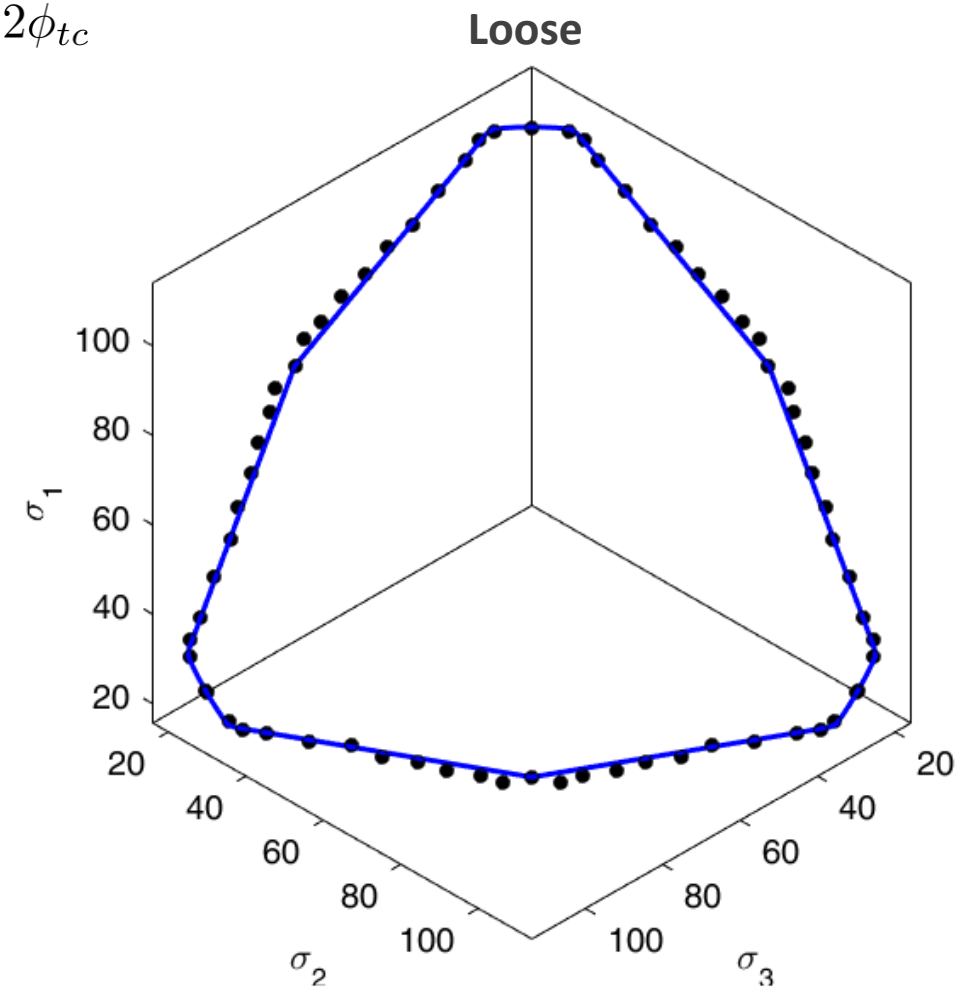
Stakemann (1976):

$$\phi_{ps} = (1 + 0.163I_D)\phi_{tc}$$

Data: Lade & Wang (2001)

FASD

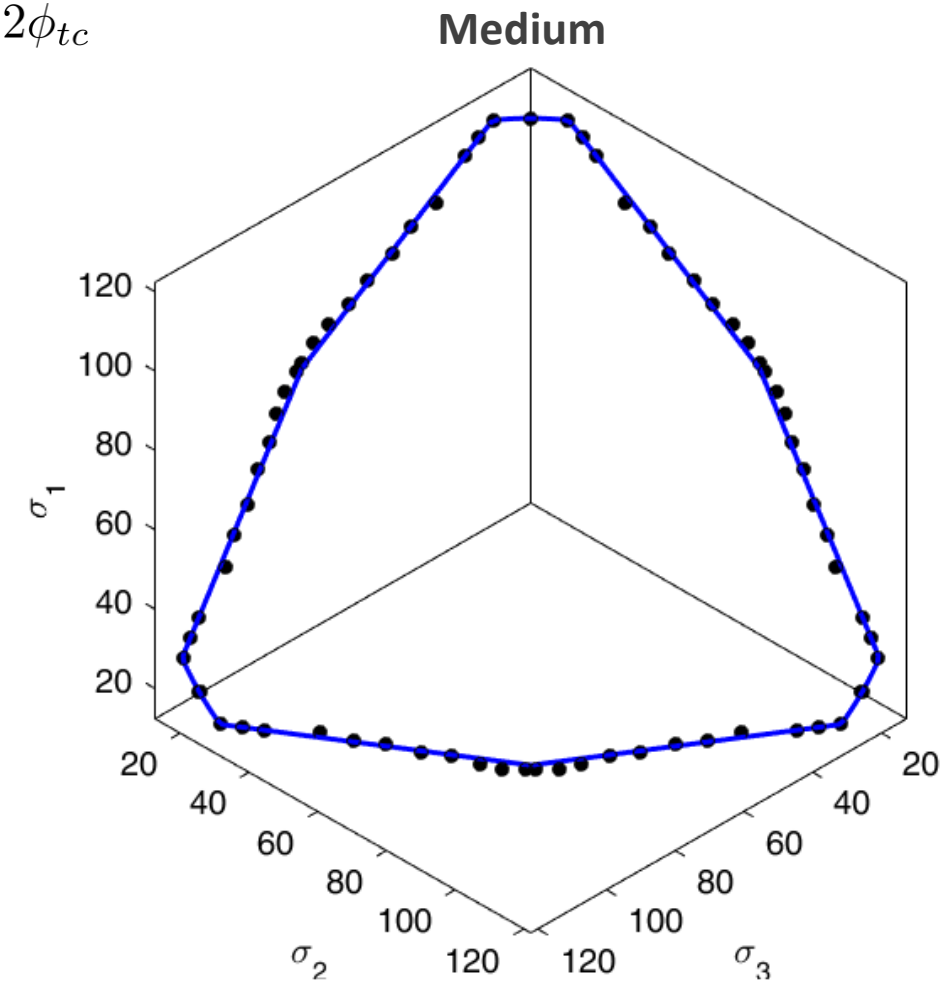
$$\phi_{ps} = 1.12\phi_{tc}$$



Optum CE

FASD

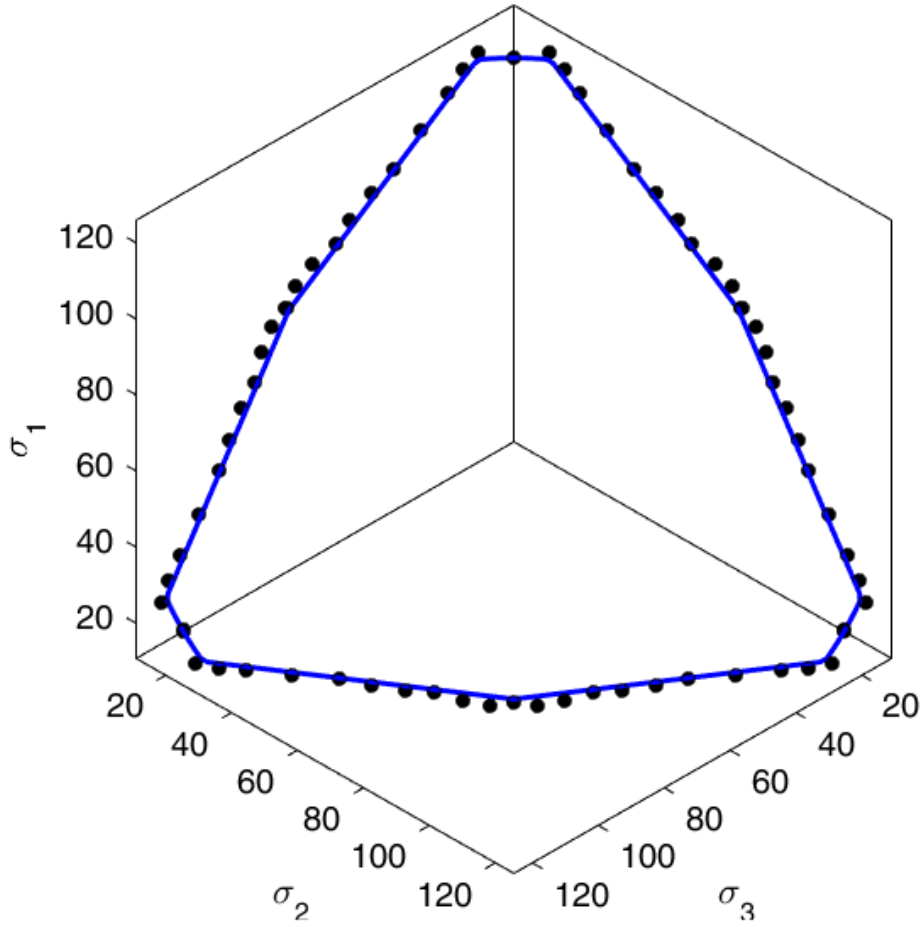
$$\phi_{ps} = 1.12\phi_{tc}$$



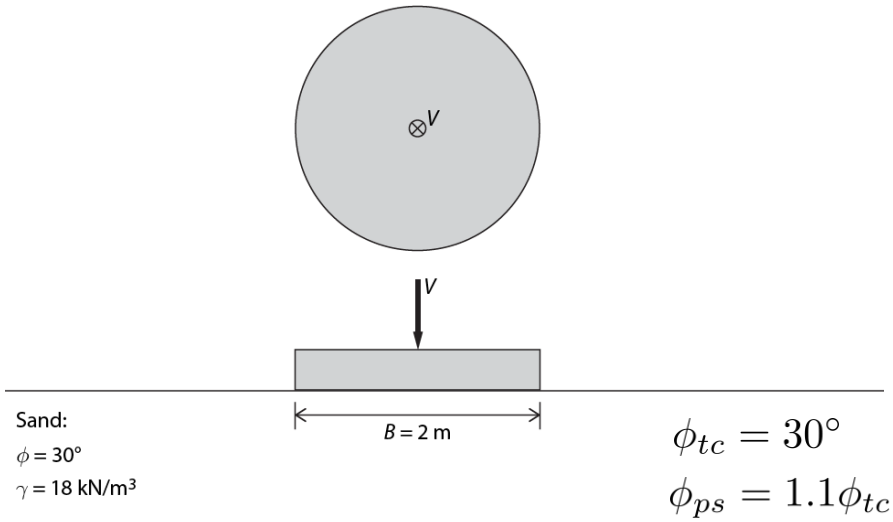
FASD

$$\phi_{ps} = 1.12\phi_{tc}$$

Dense



FASD



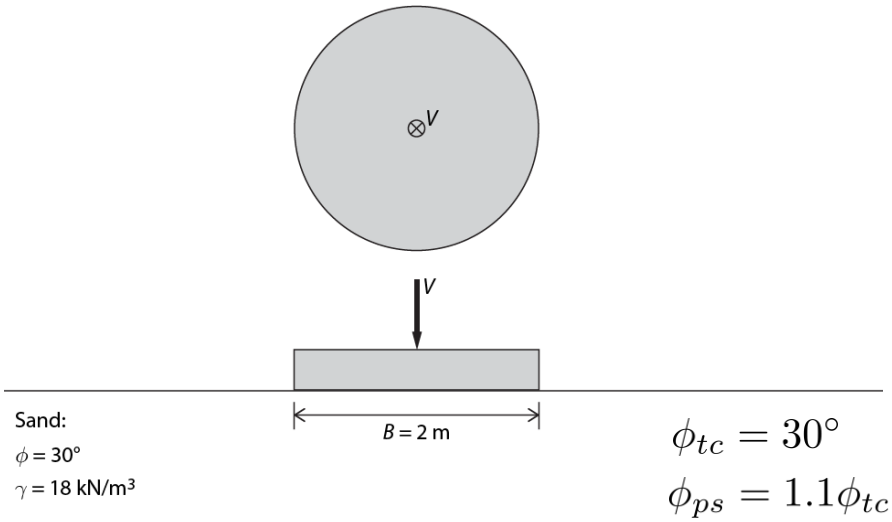
$$\phi_{tc} = 30°$$

$$\phi_{ps} = 1.1\phi_{tc} = 33°$$

G3 results (10,000 elements + 3 adaptive iterations):

	V/A (kPa)	Error (%)
MC, $f = f_{tc} = 30°$	274.1	-1.9
MC, $f = f_{ps} = 33°$	483.4	-3.8
FASD, $f_{tc} = 30°$, $f_{ps} = 33°$	379.5	

FASD



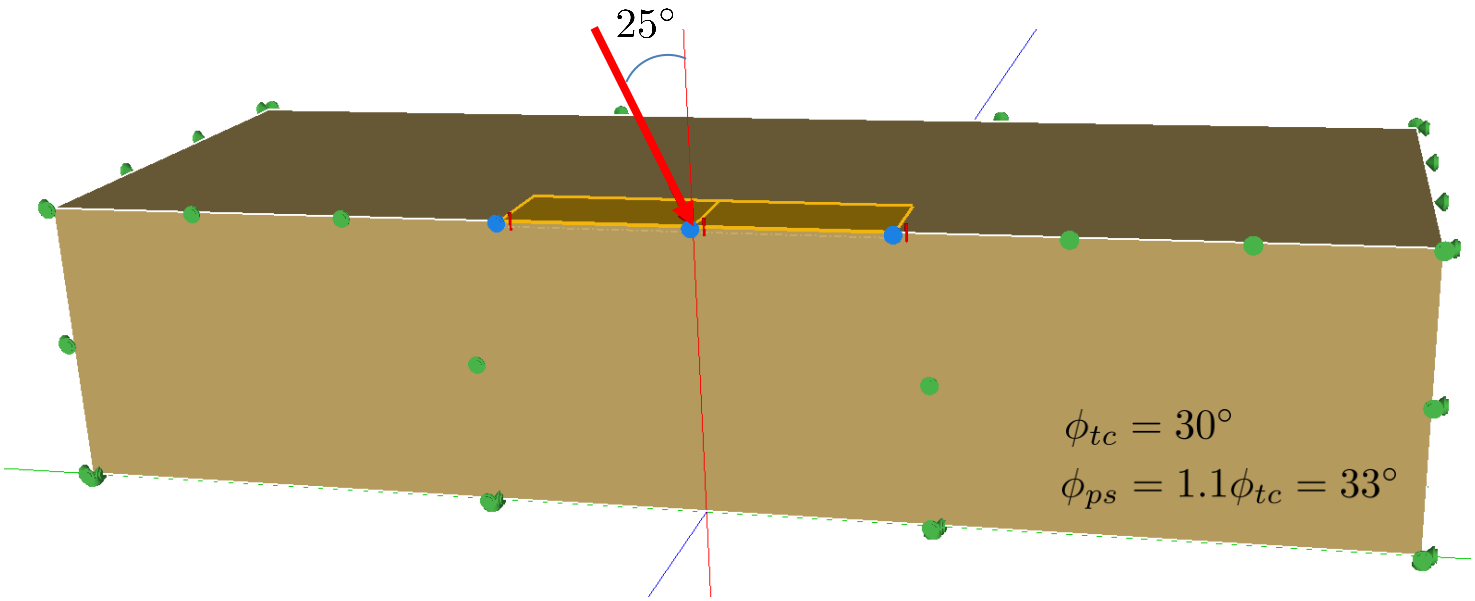
$$\phi_{tc} = 30°$$

$$\phi_{ps} = 1.1\phi_{tc} = 33°$$

G3 results (10,000 elements + 3 adaptive iterations):

	V/V_{FASD}	Error (%)
MC, $f = f_{tc} = 30°$	0.72	-1.9
MC, $f = f_{ps} = 33°$	1.27	-3.8
FASD, $f_{tc} = 30°$, $f_{ps} = 33°$	1.00	

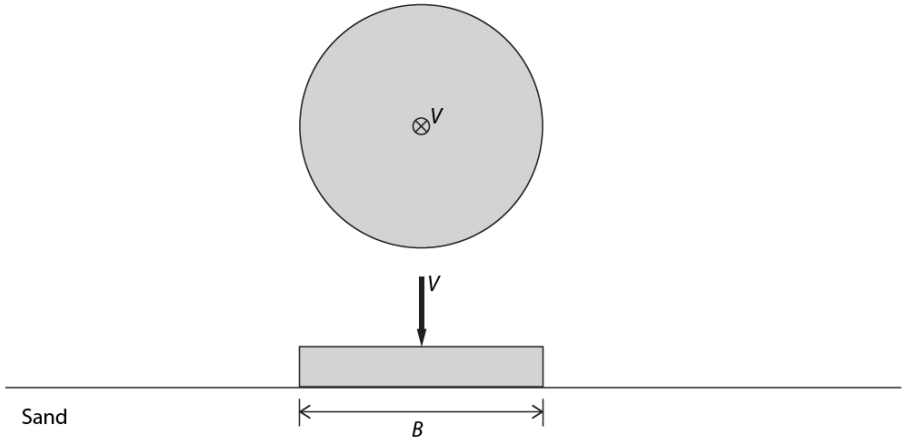
FASD



G3 results (10,000 elements + 3 adaptive iterations):

	V/V_{FASD}
MC, $f = f_{tc} = 30°$	0.55
MC, $f = f_{ps} = 33°$	1.09
FASD, $f_{tc} = 30°$, $f_{ps} = 33°$	1.00

The shape factor mystery



Circular/square footing:

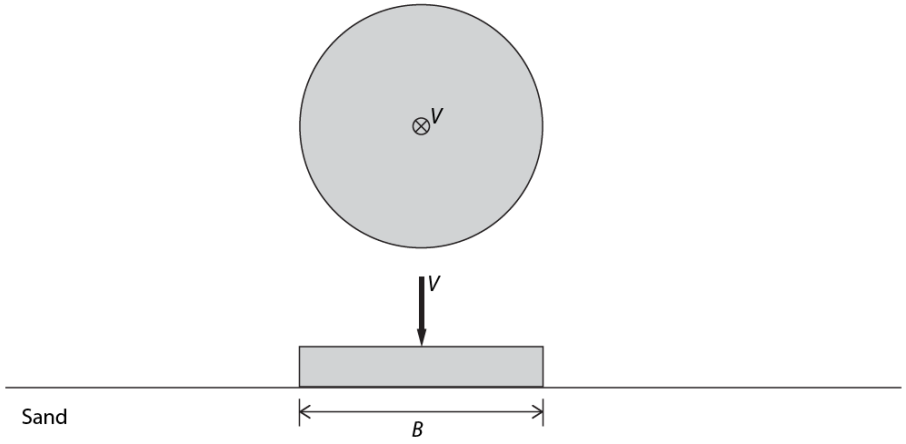
EC7: $s_\gamma = 0.7$

Danish annex: $s_\gamma = 0.6$

Briaud (2007): $s_\gamma = 1.25$

Meyerhof: $s_\gamma = 1 + 0.1 \tan^2(45 + \frac{1}{2}\phi) = 1.3 \text{ for } \phi = 30°$

The shape factor mystery



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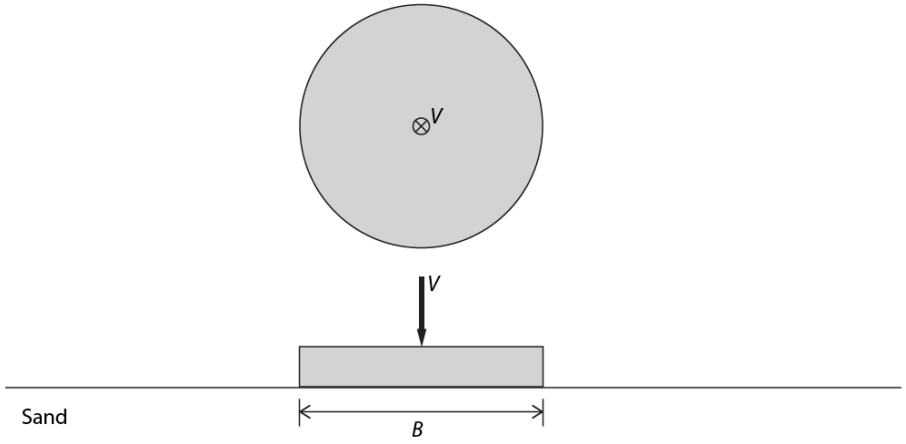

Circular/square footing:

Experimental:

De Beer (1970):	$s_\gamma = 0.6$	(model)
Muhs (1970):	$s_\gamma = 0.84$	(full scale)
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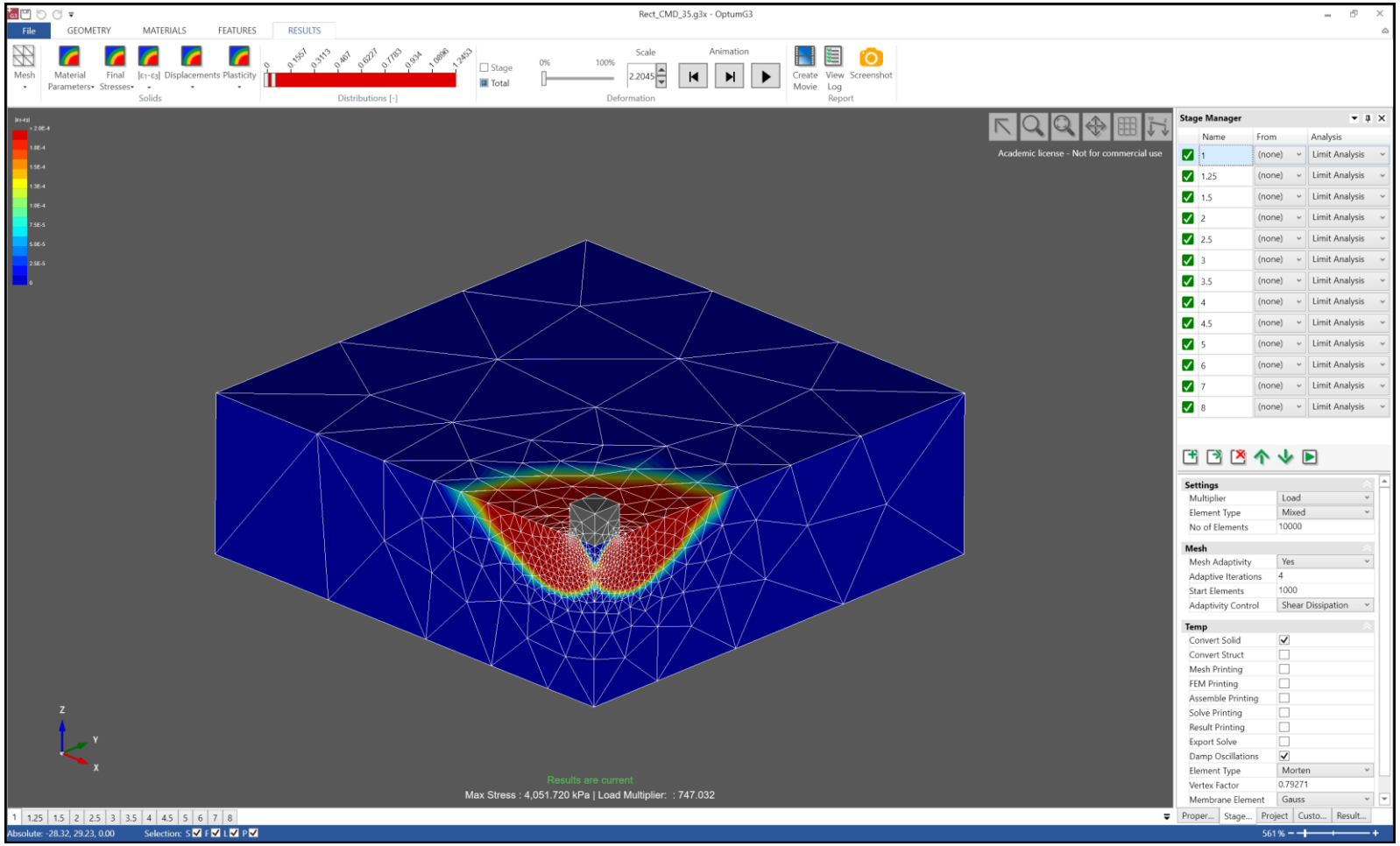
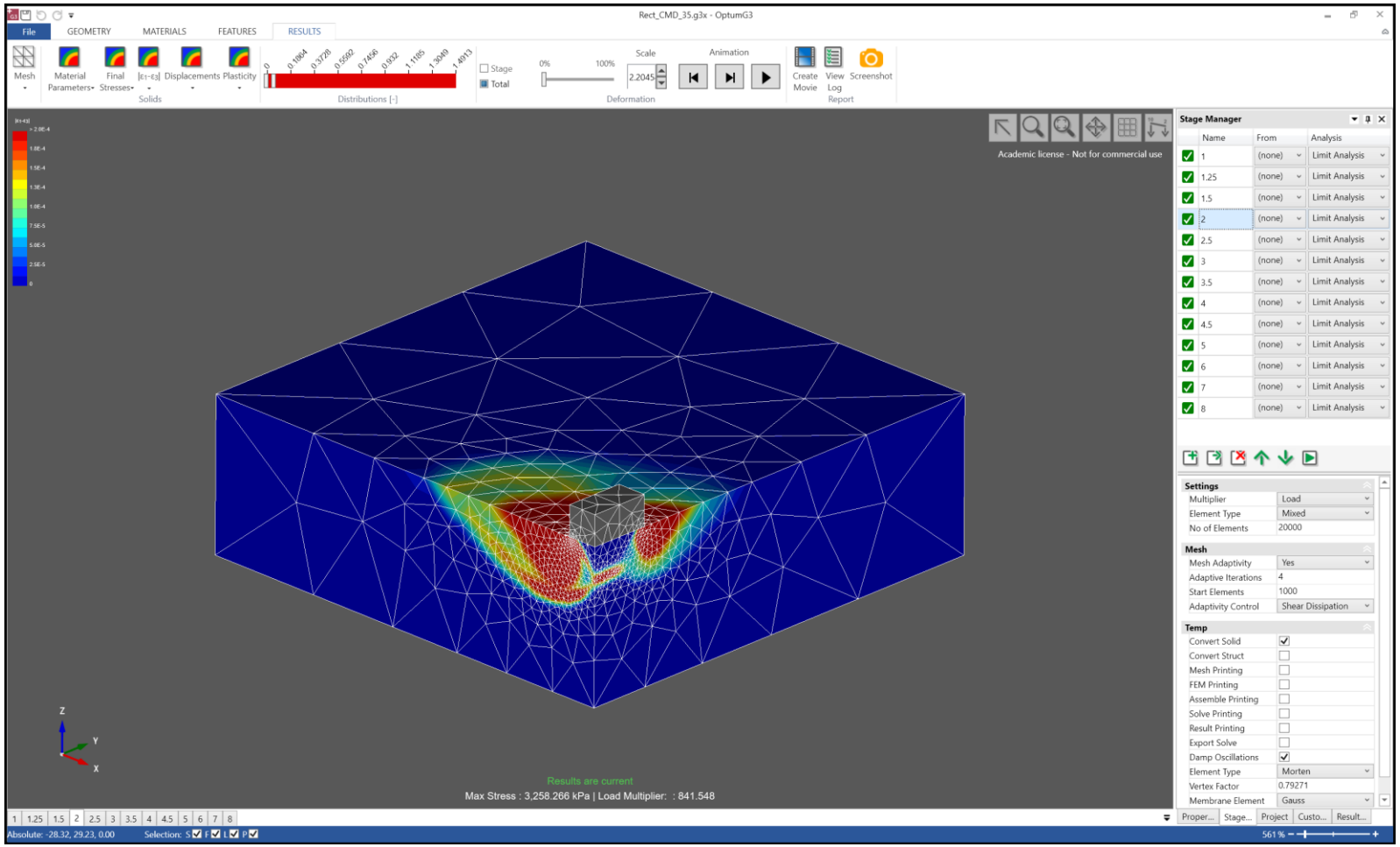
Terzaghi (1943):	$s_\gamma = 0.8$	(Golder's experiments)

The shape factor mystery



The shape factor mystery



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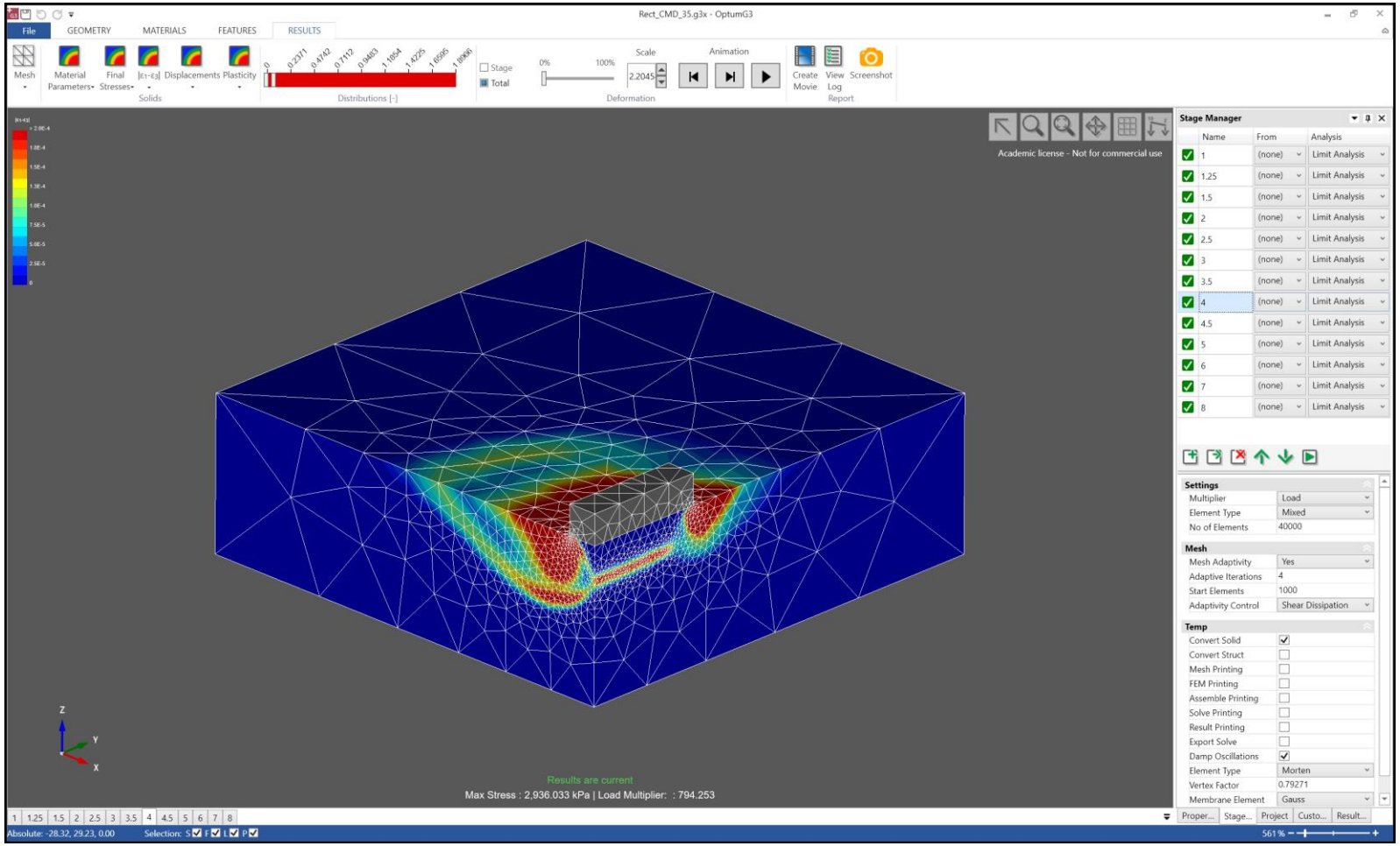

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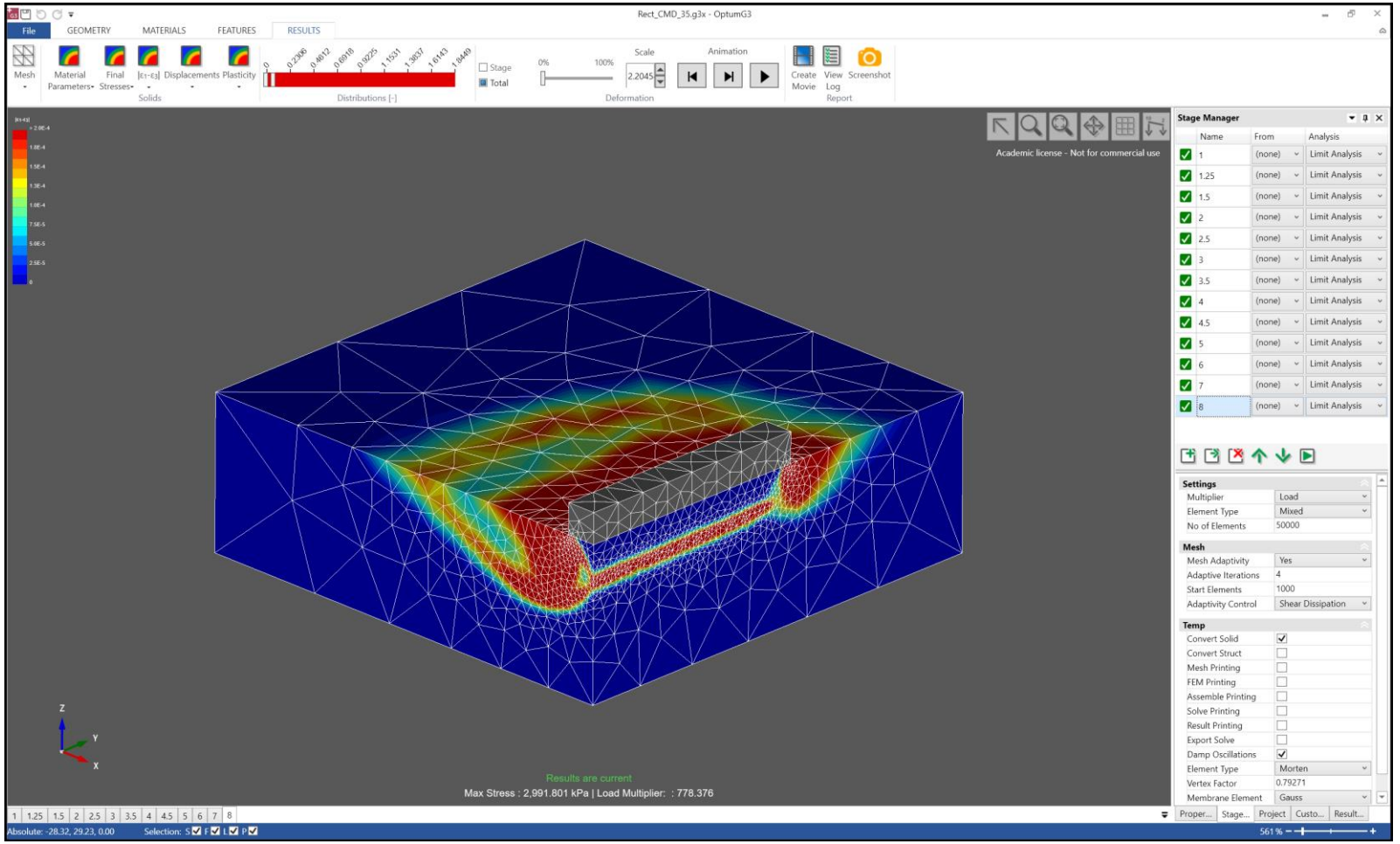

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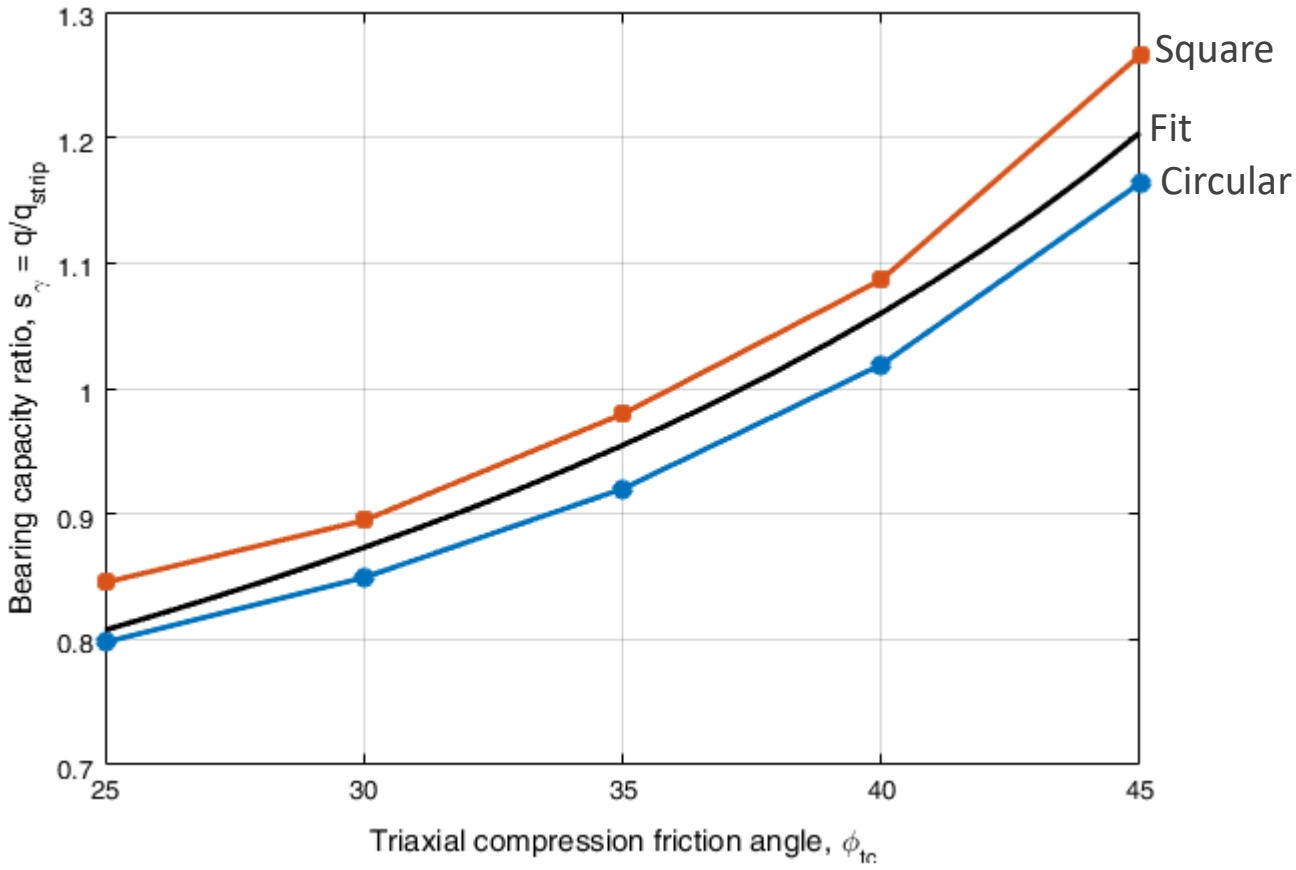
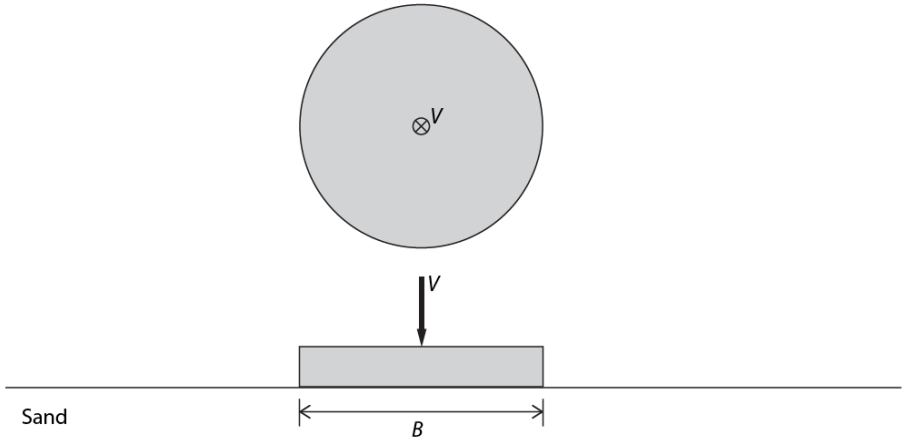


Fit: $s_\gamma = 0.59e^{0.59\tan(1.12\phi_{tc})}$

The shape factor mystery



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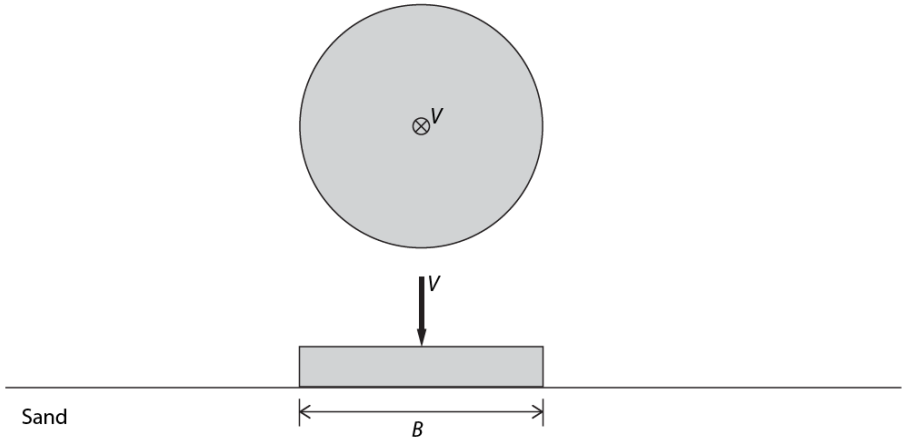

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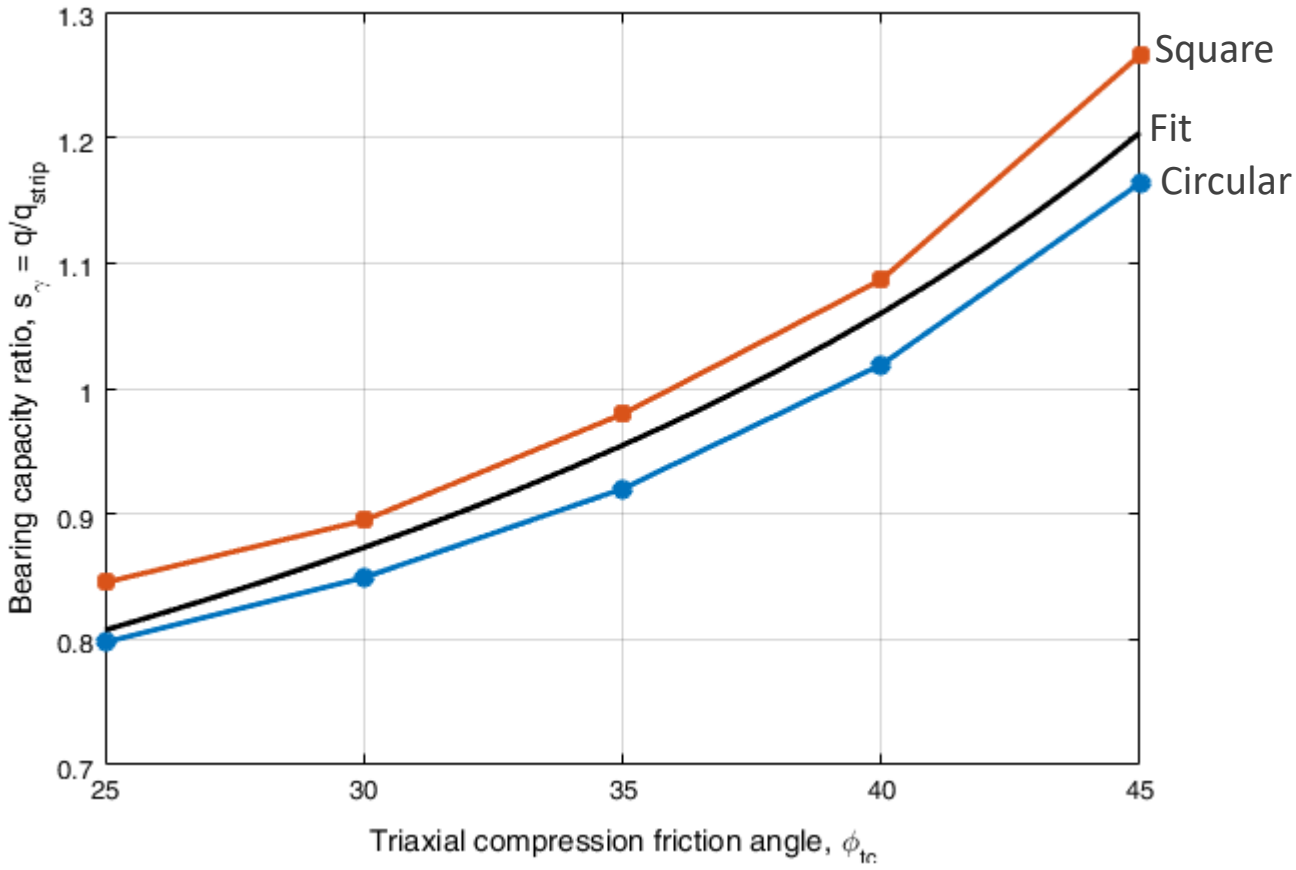

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