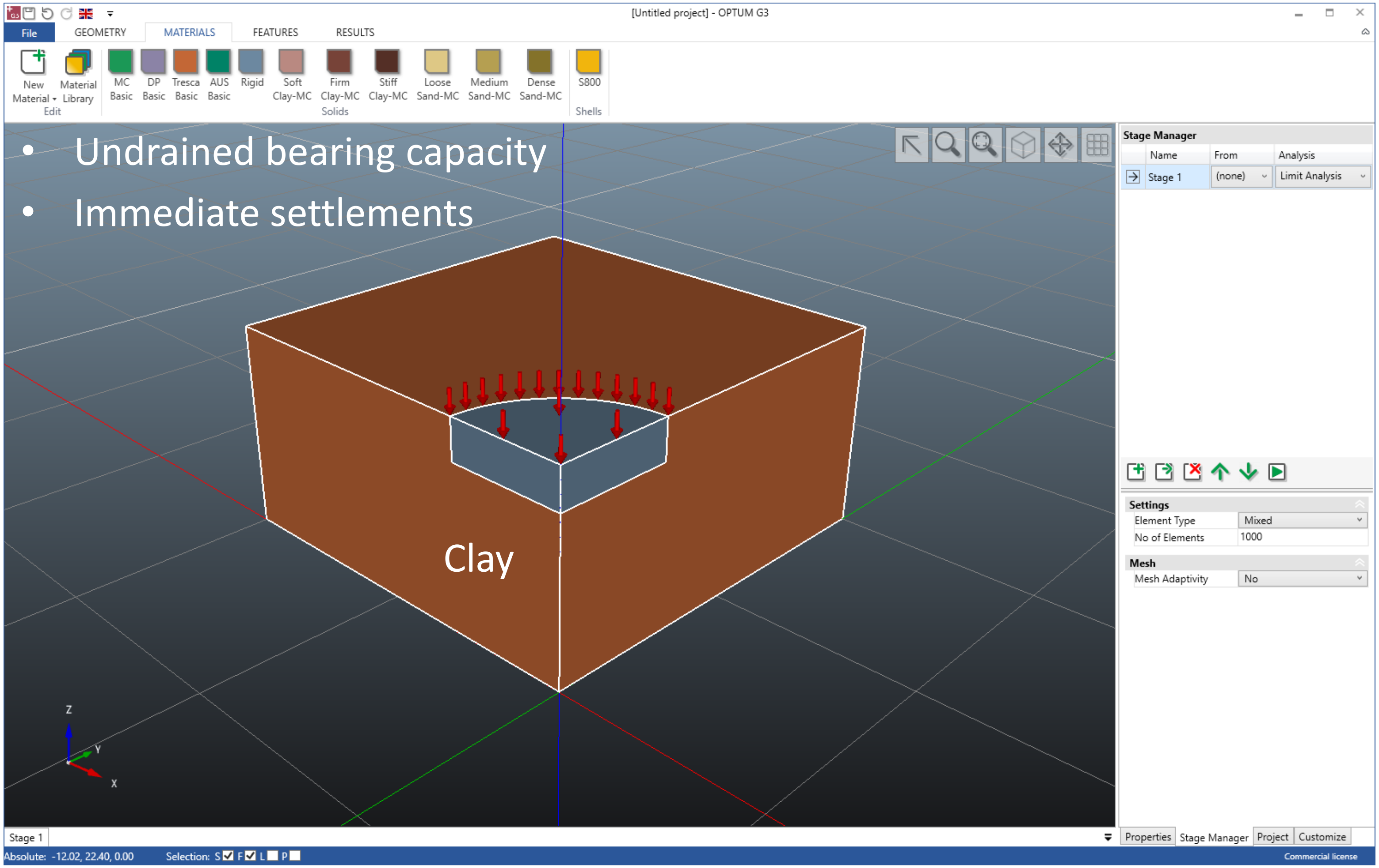
AUS: ANISOTROPIC UNDRAINED SHEAR STRENGTH MODEL FOR CLAY

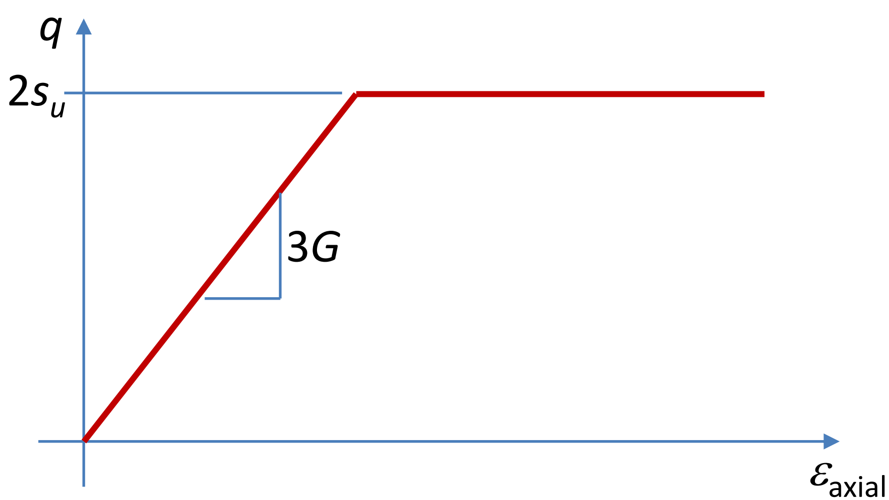
KRISTIAN KRABBENHOFT
Optum Computational Engineering

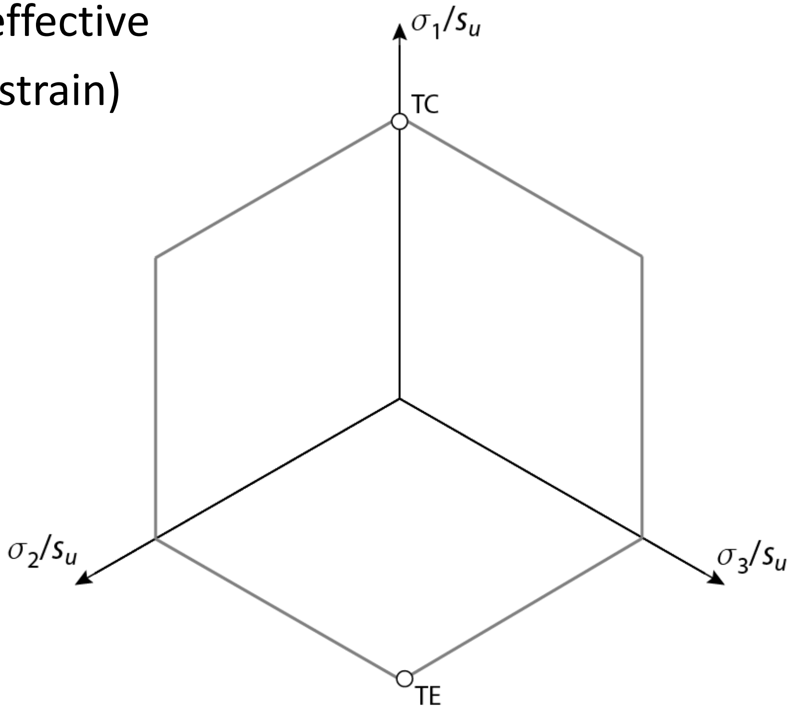
Undrained analysis



Tresca model

- Clay under undrained conditions
- Two parameters: s_u and G (may vary with depth)
- Reasonable for both deformations (with adequate G, e.g. G_{50}) and ultimate capacity
- Theoretical basis: can be derived from effective stress Mohr-Coulomb model (for plane strain)





Optum CE

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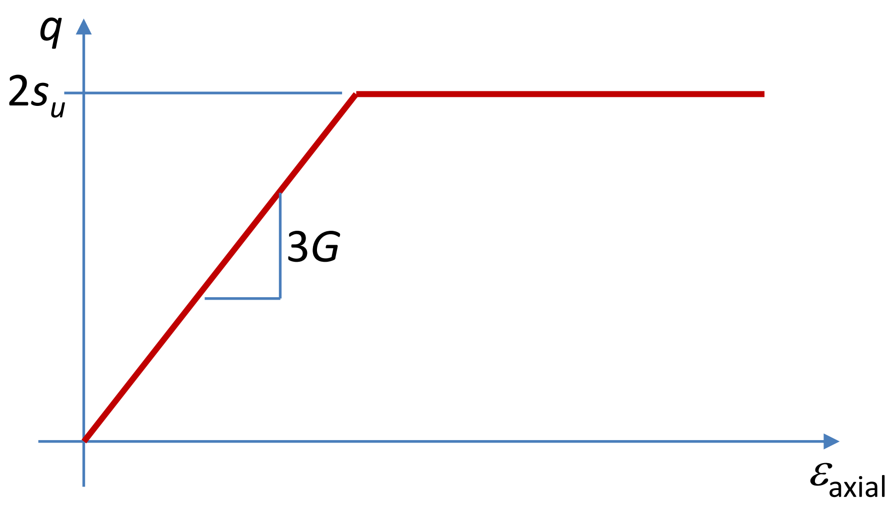

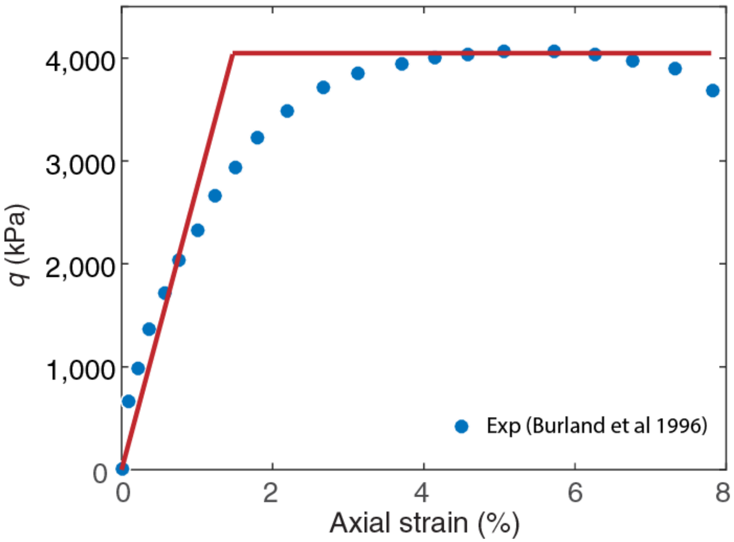

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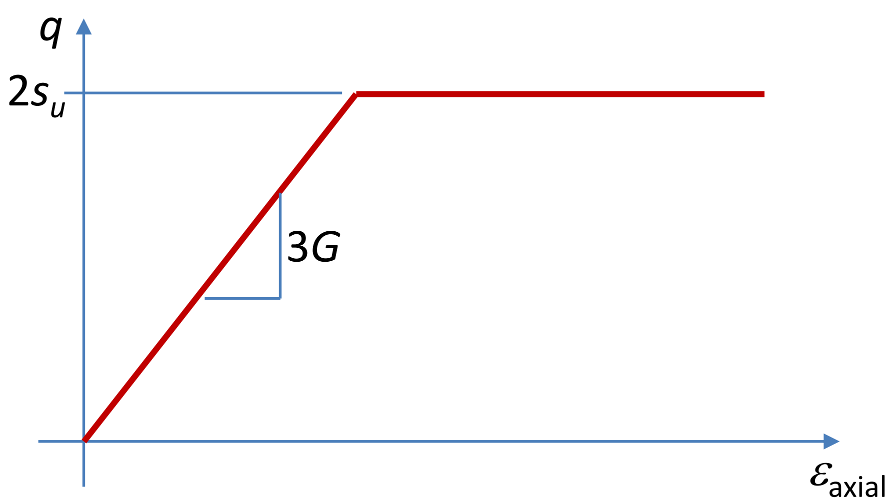

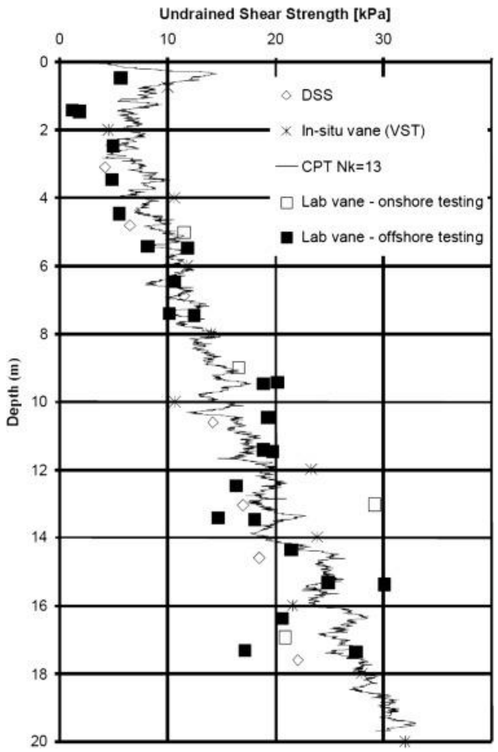

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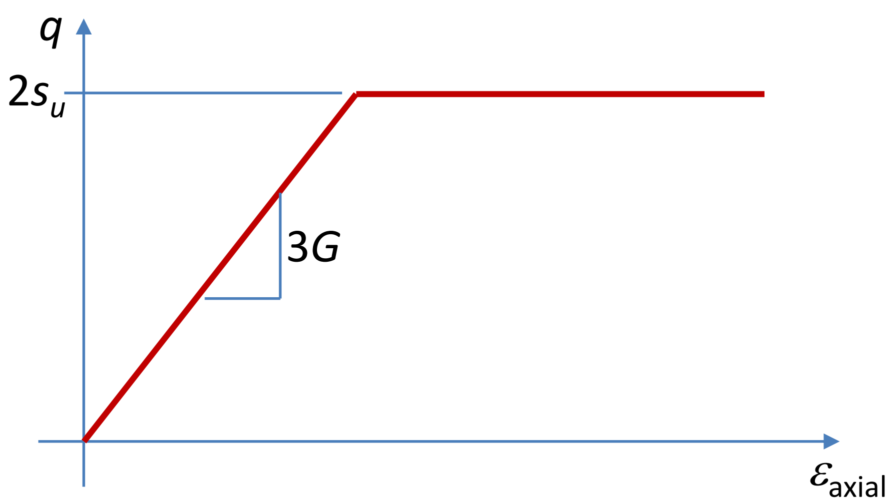

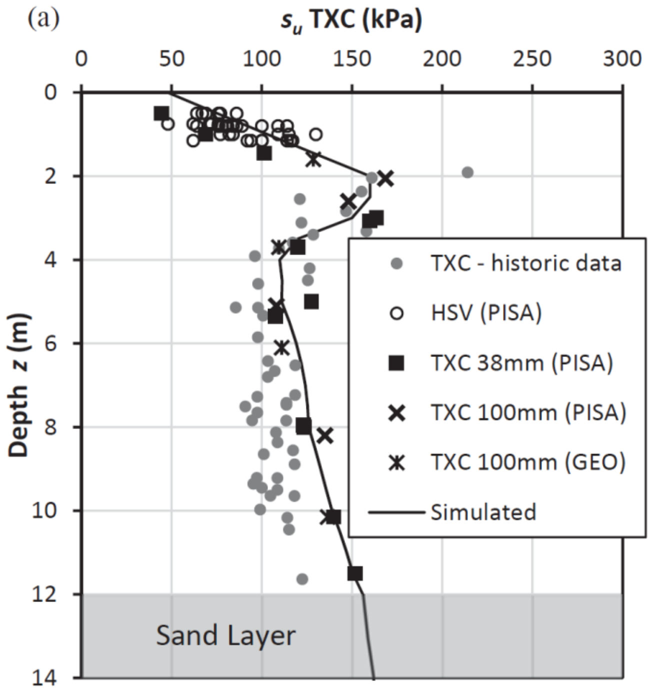

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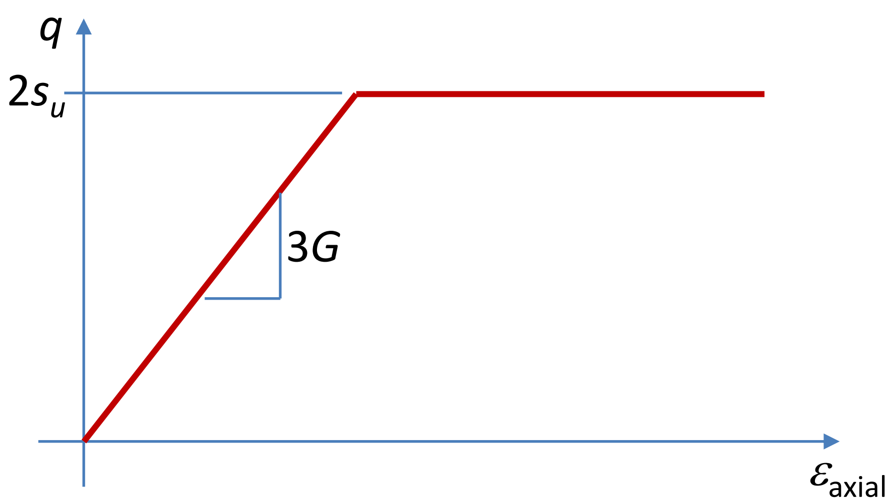
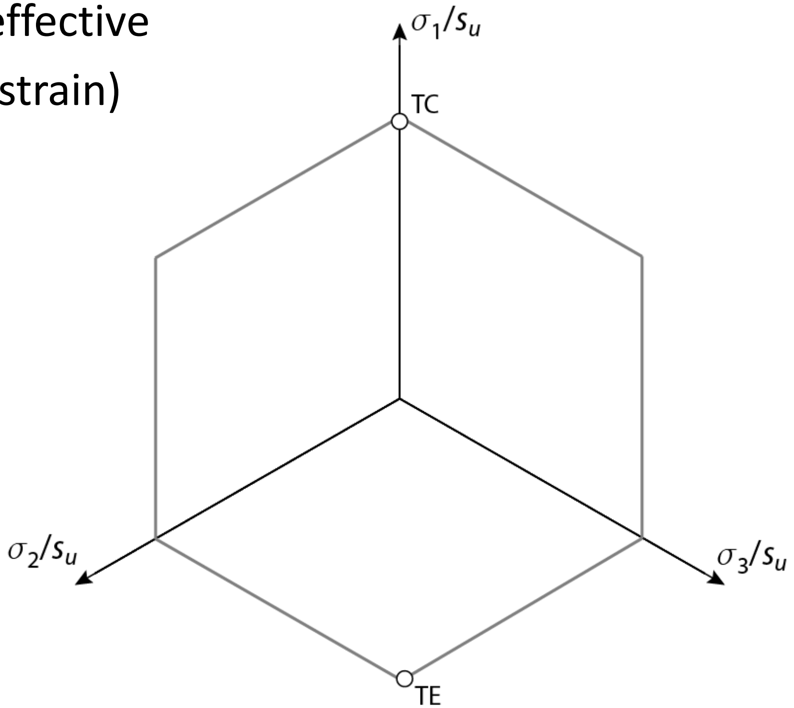
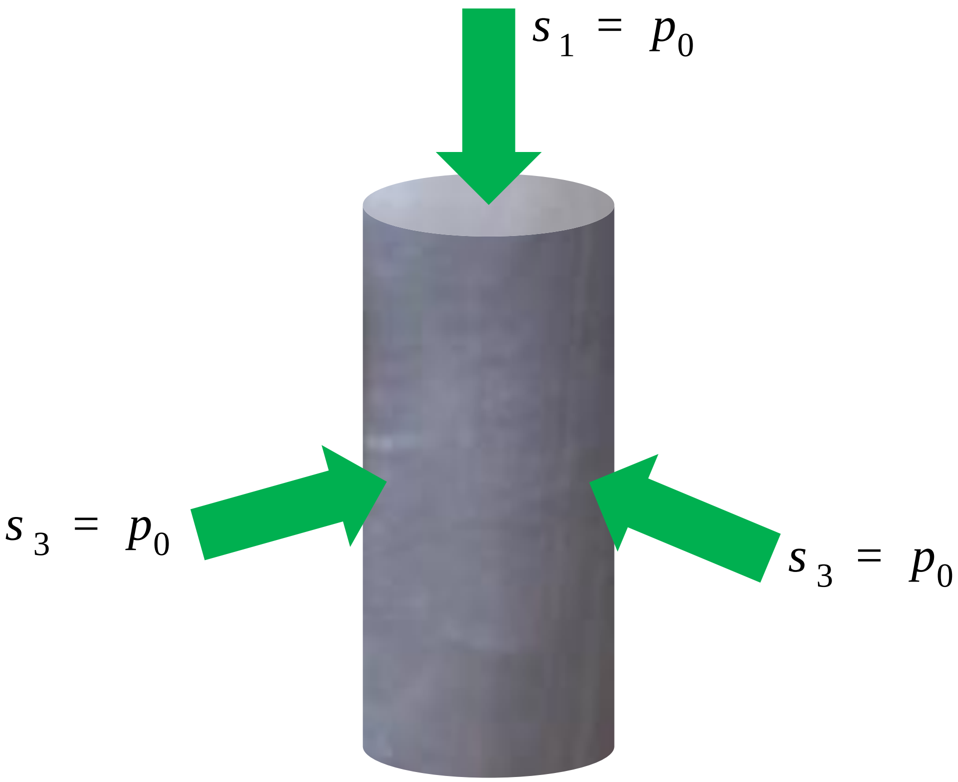




Optum CE

Undrained shear strength

Triaxial compression test



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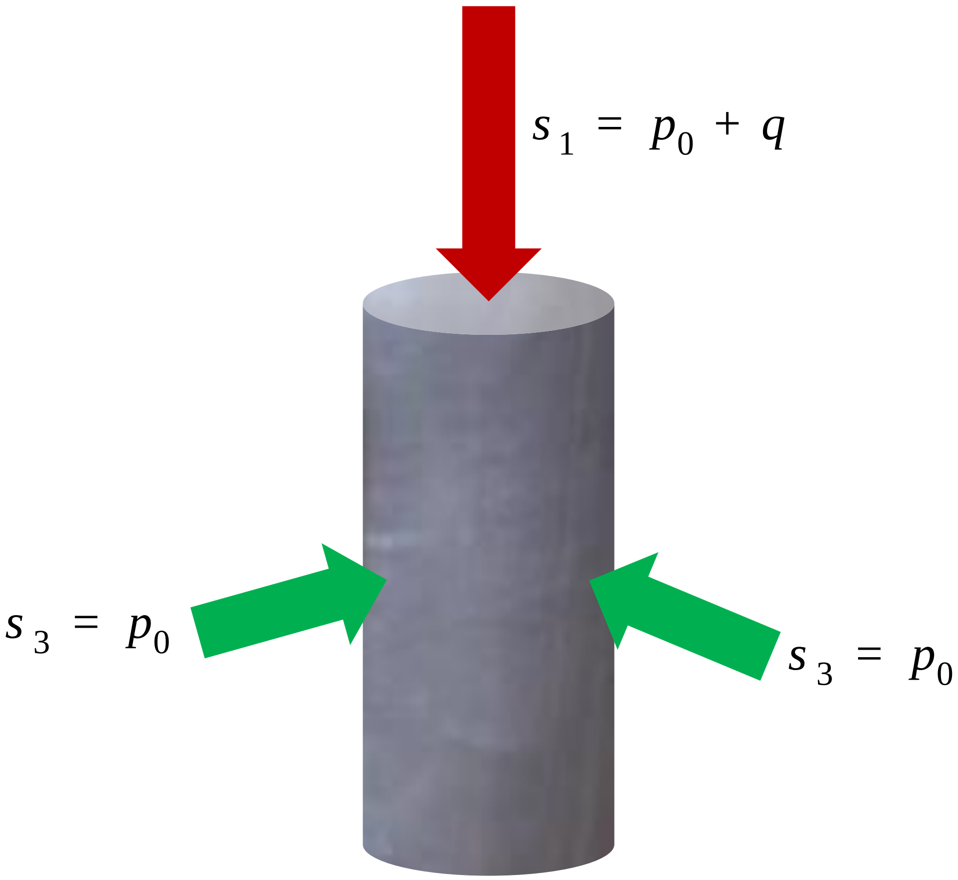

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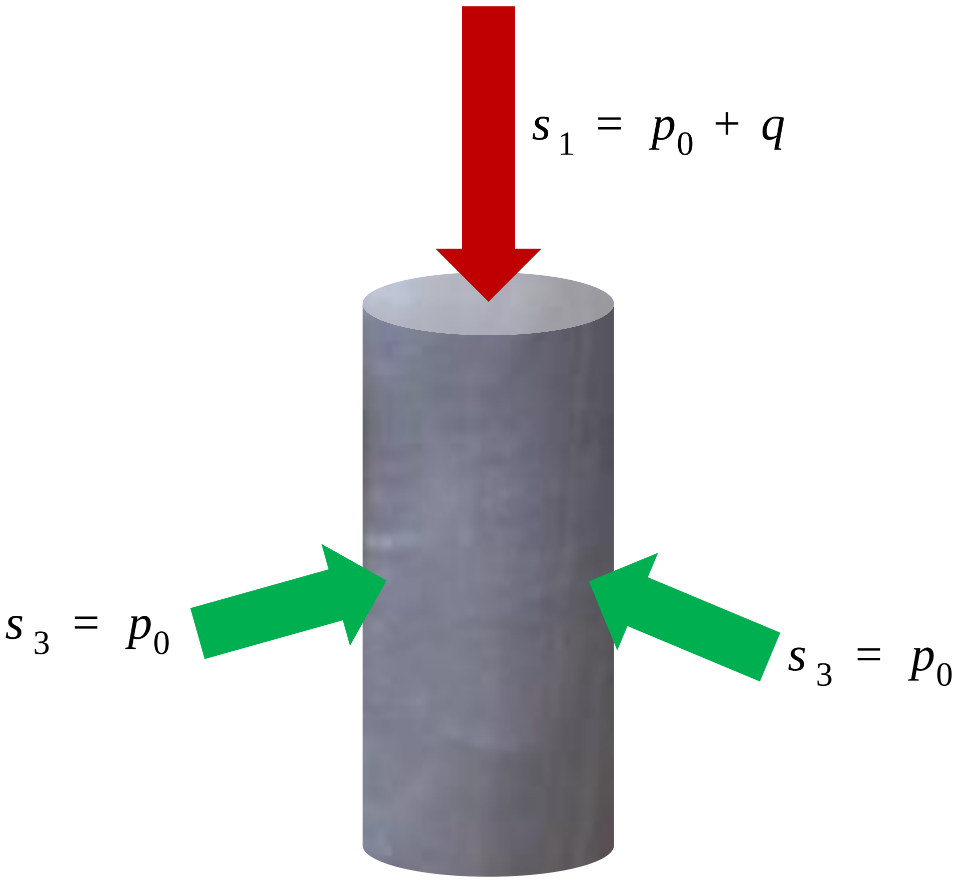


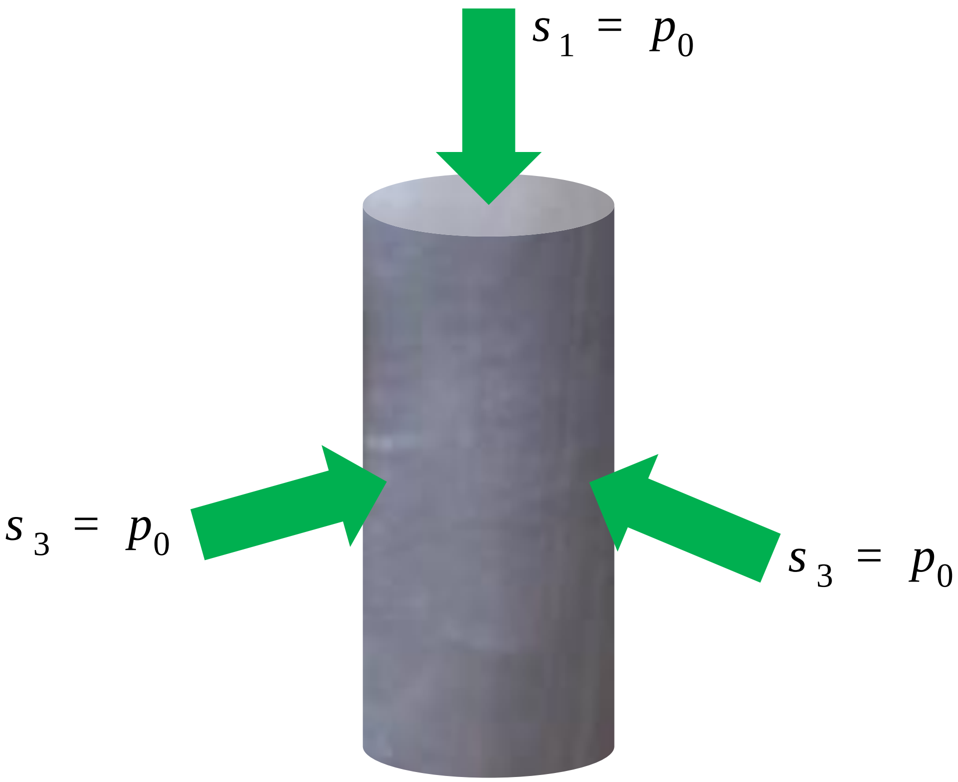
Failure (TC):

$$\tfrac{1}{2}\left|s_1 - s_3\right| = \tfrac{1}{2}q = s_{uc}$$

AUS

Undrained shear strength

Triaxial extension test



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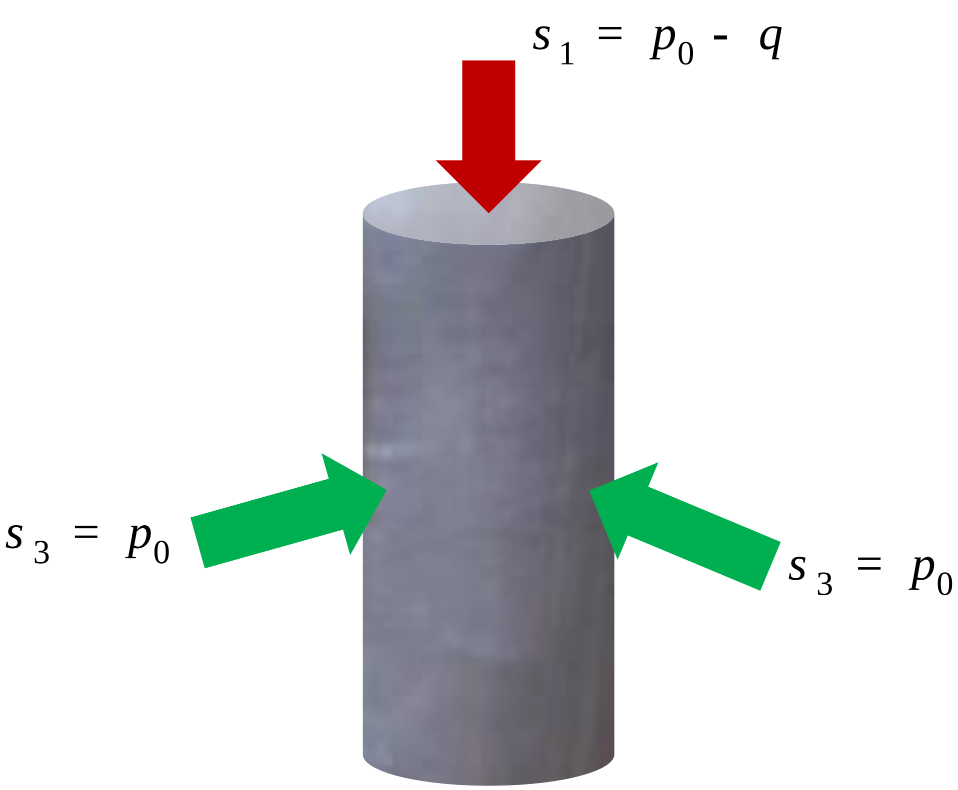

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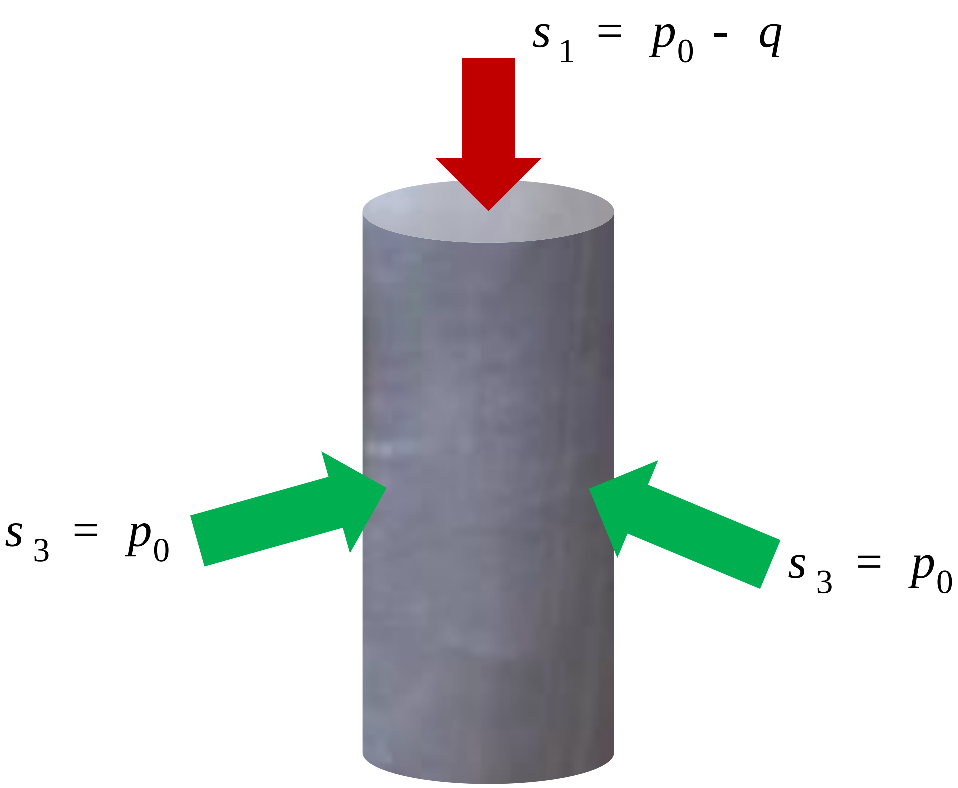
Undrained shear strength

Triaxial extension test



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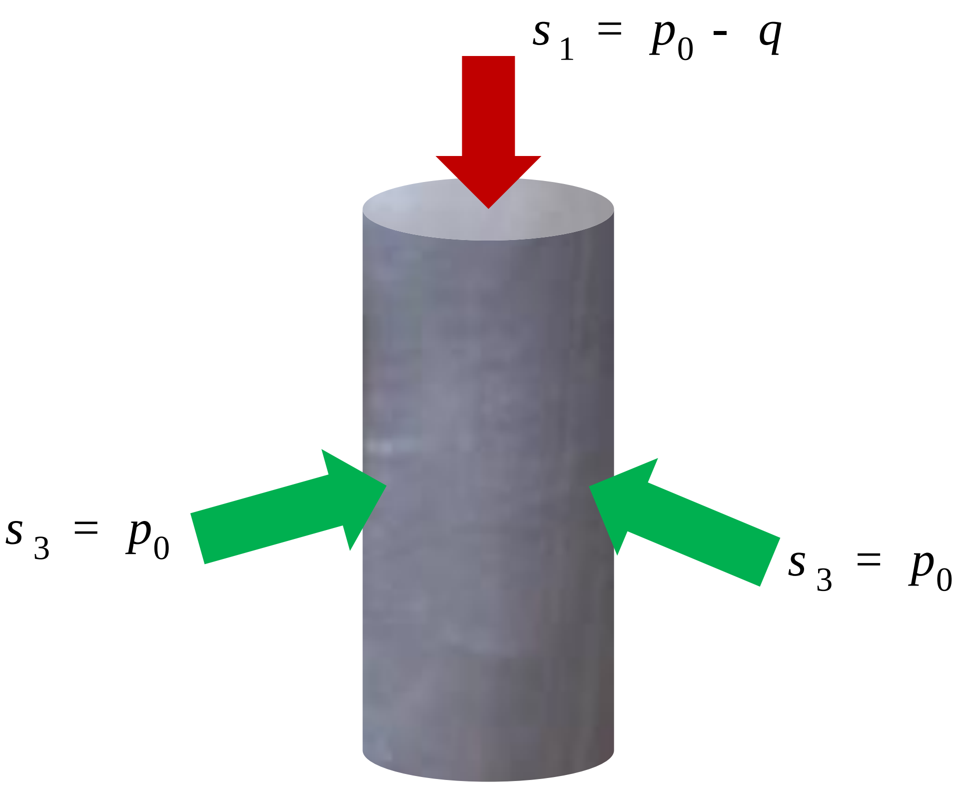
Questions:

1. Do we have $s_{ue} = s_{uc}$?

Undrained shear strength

Triaxial extension test



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$$\tfrac{1}{2}\left|s_1 - s_3\right| = \tfrac{1}{2}q = s_{uc}$$

Failure (TE):

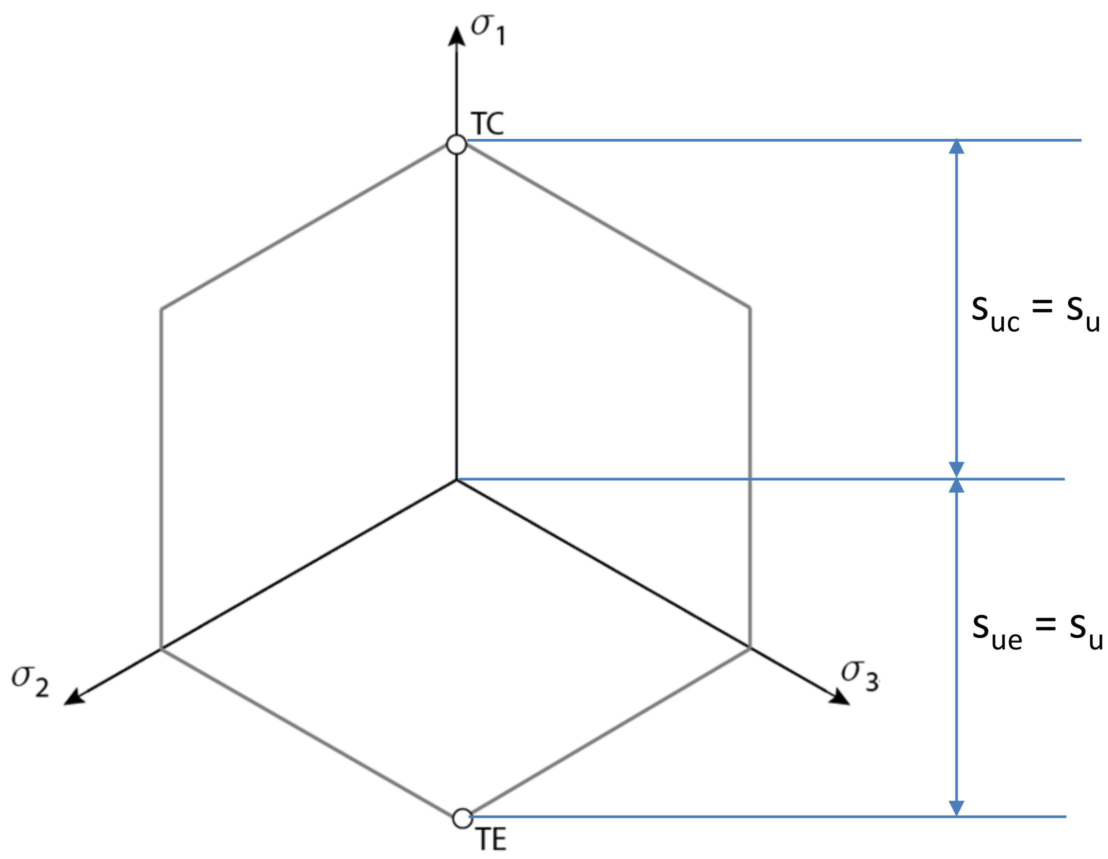
$$\tfrac{1}{2}\left|s_1 - s_3\right| = \tfrac{1}{2}q = s_{ue}$$

Questions:

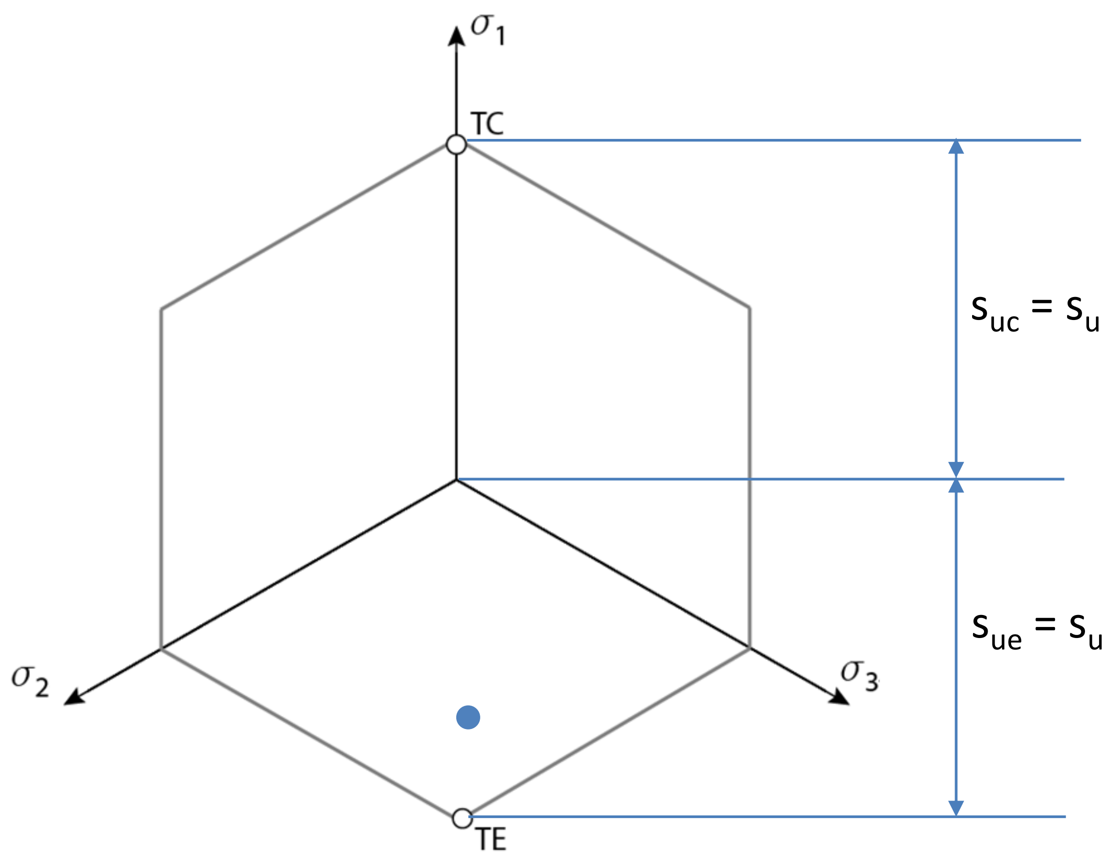
1. Do we have $s_{ue} = s_{uc}$?
2. Do we expect $s_{ue} = s_{uc}$?

Optum^{CE}

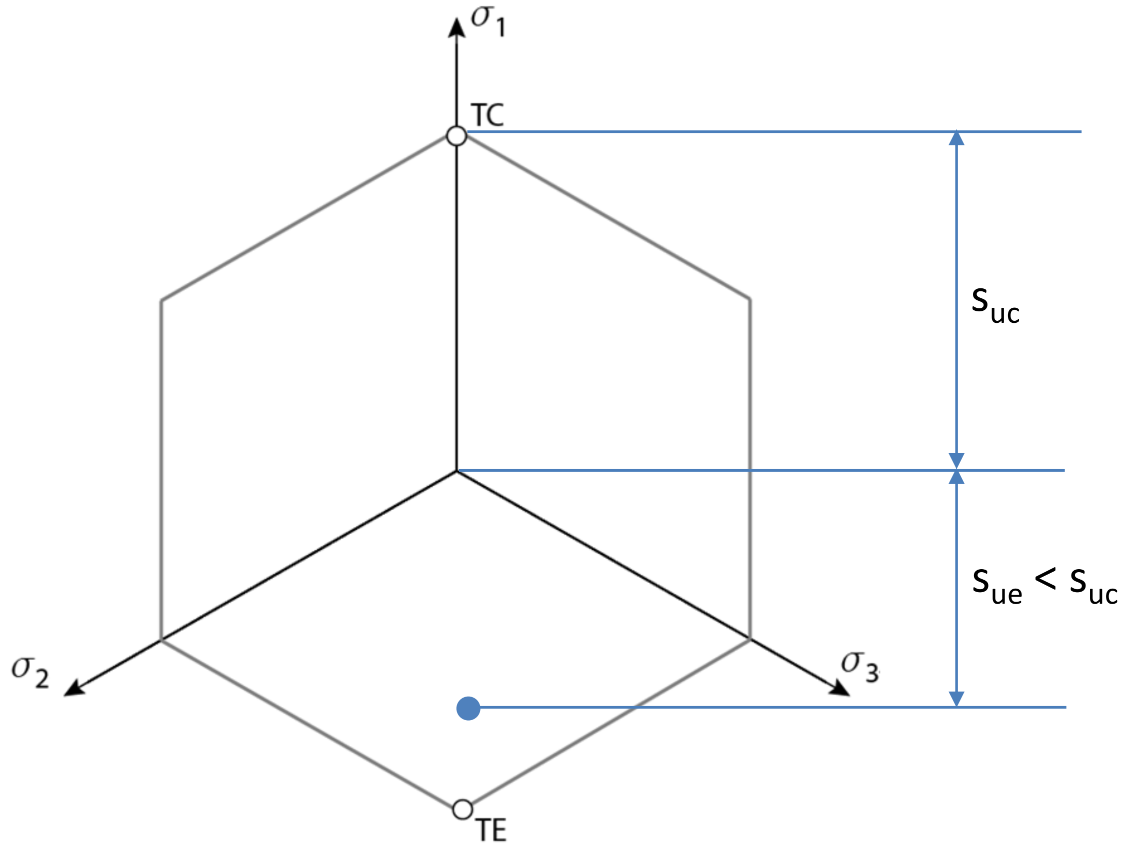
Tresca



Tresca

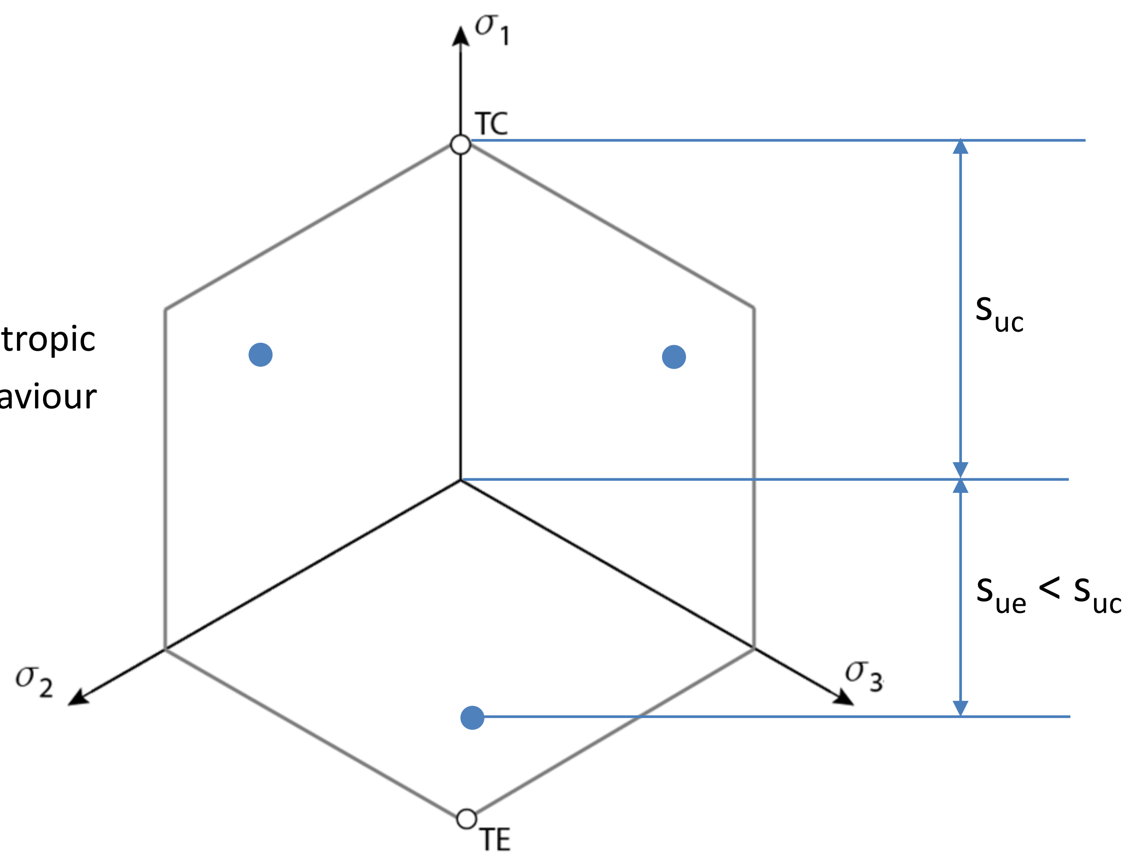


Tresca



Tresca

Assuming isotropic material behaviour



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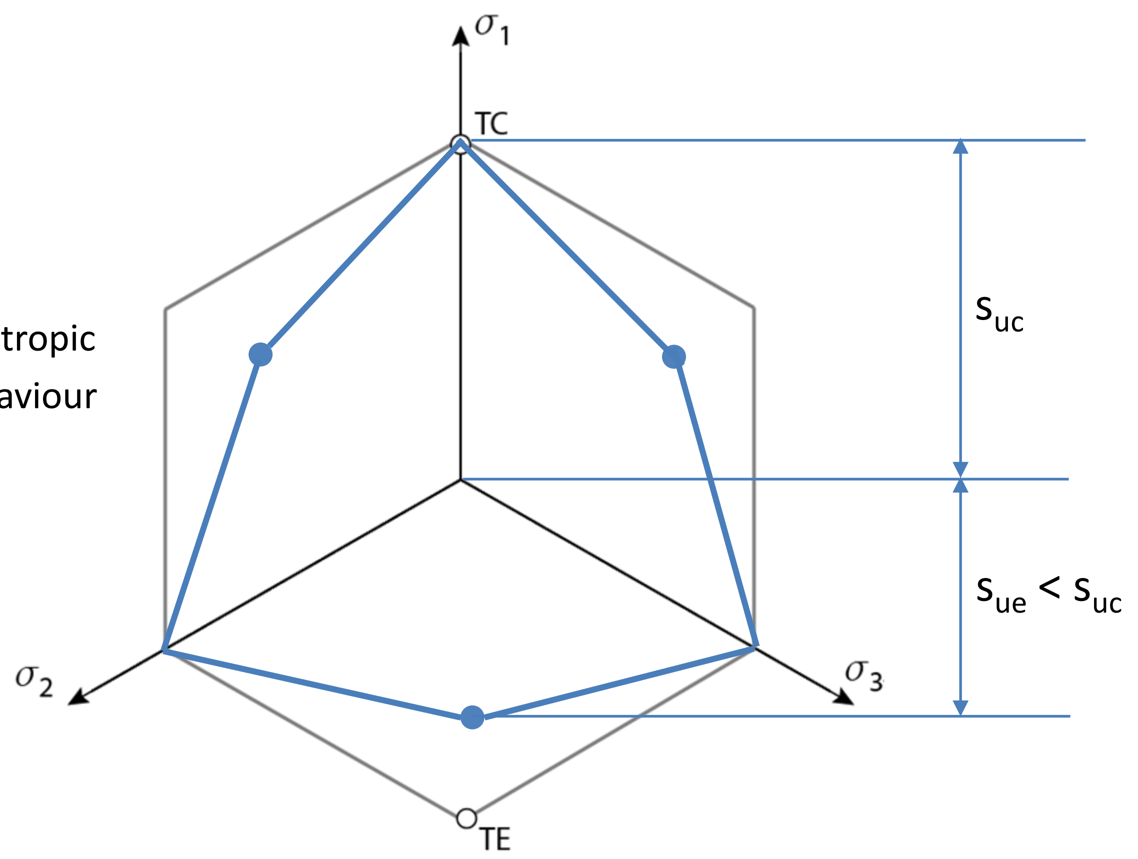

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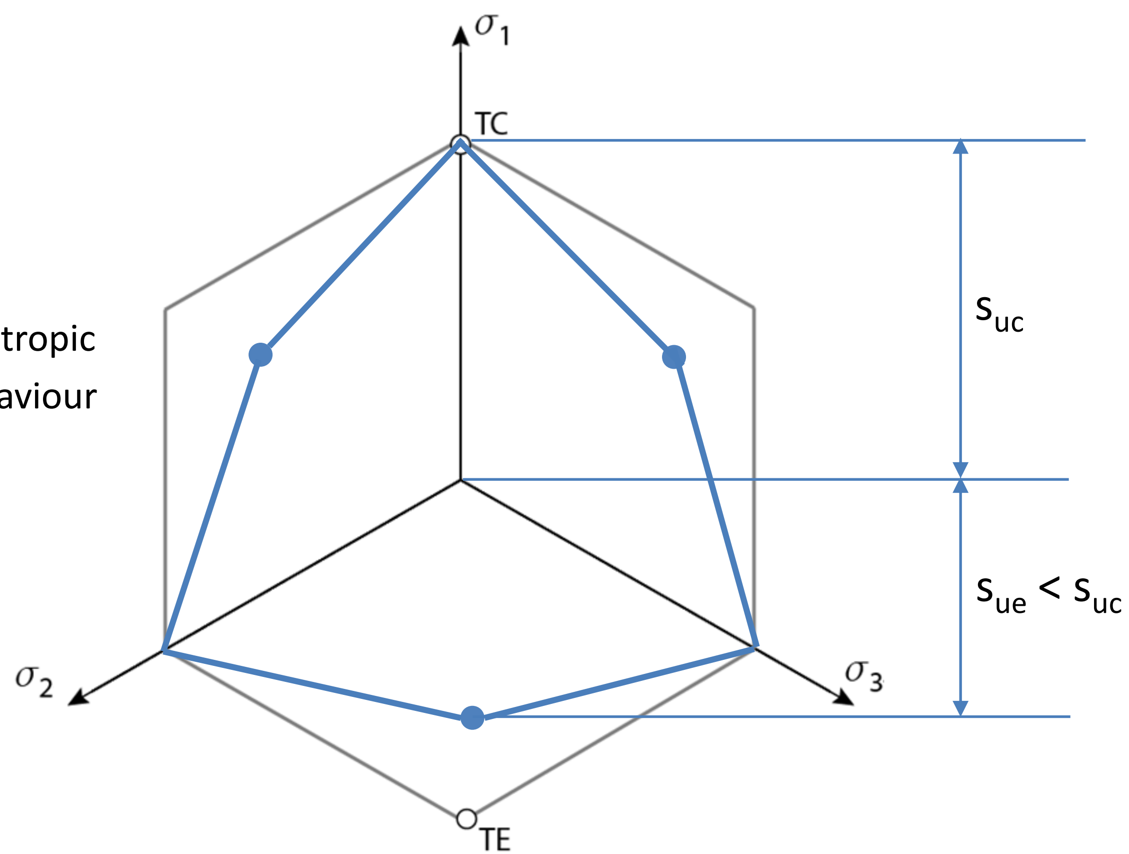
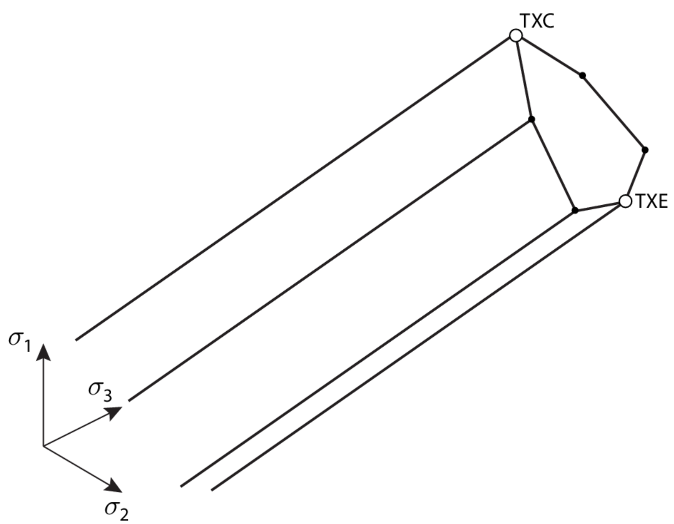
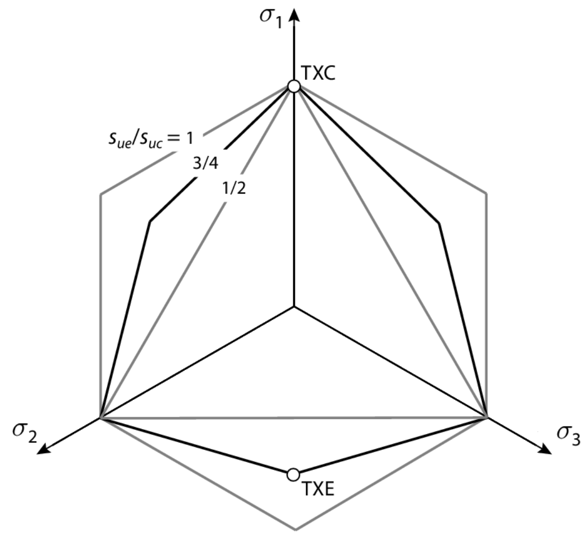


Generalized Tresca (s_{uc}, s_{ue})

Generalized Tresca

$$F_u = s_1 - s_3 + (s_{ue} / s_{uc} - 1)(s_1 - s_2) - 2s_{uc}$$





Krabbenhoft, K. and Lyamin, A. V. (2015) **Géotechnique Letters 5**, 313–317, http://dx.doi.org/10.1680/jgele.15.00120

Generalised Tresca criterion for undrained total stress analysis

K. KRABBENHOFT* and A. V. LYAMIN*

A new failure criterion, the generalised Tresca criterion, for undrained total stress analysis is presented. The criterion is consistent with an underlying effective stress Mohr–Coulomb model. It involves two parameters: the undrained shear strengths in triaxial compression and extension. As such, the model predicts different strengths in compression and extension without introducing any physical anisotropy due to layering, direction of deposition and so on. The model is applied to a number of boundary value problems, where the effects of an extension/compression shear strength ratio less than unity generally turns out to be relatively moderate.

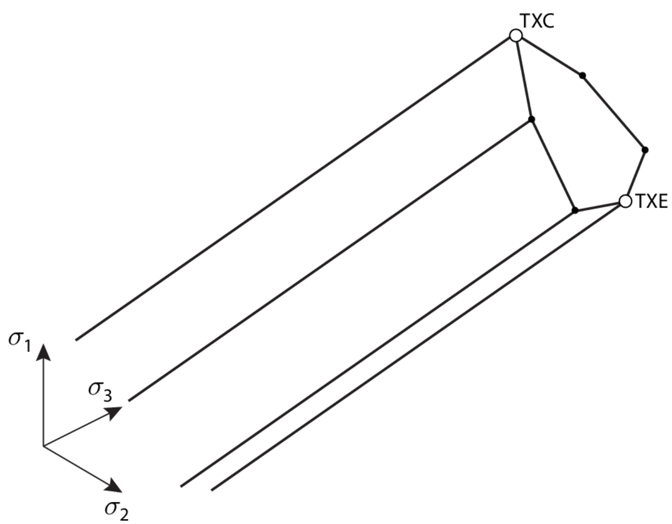
KEYWORDS: clays; geotechnical engineering; solid mechanics

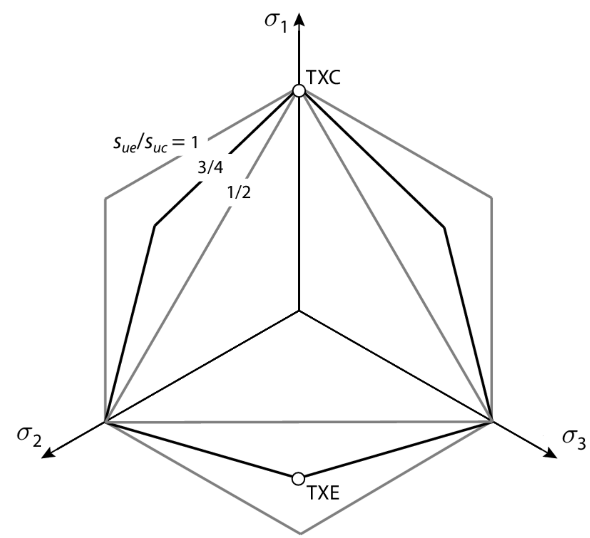
Limitation: $\frac{1}{2} \leq \frac{s_{ue}}{s_{uc}} \leq 1$

Generalized Tresca

Shape Size

$$F_u = s_1 - s_3 + (s_{ue}/s_{uc} - 1)(s_1 - s_2) - 2s_{uc}$$





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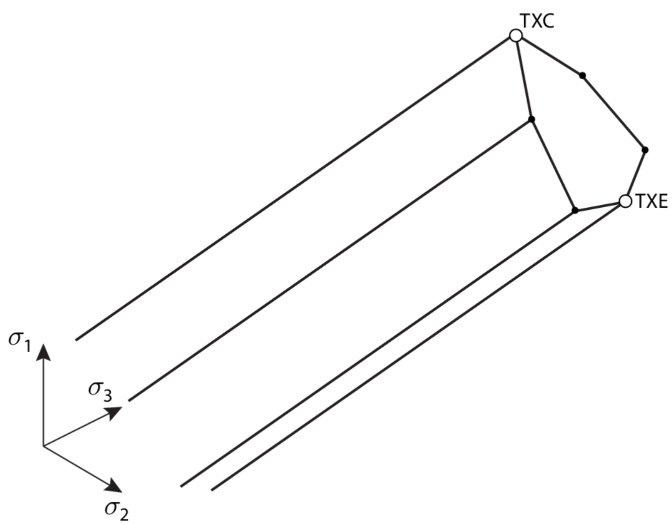
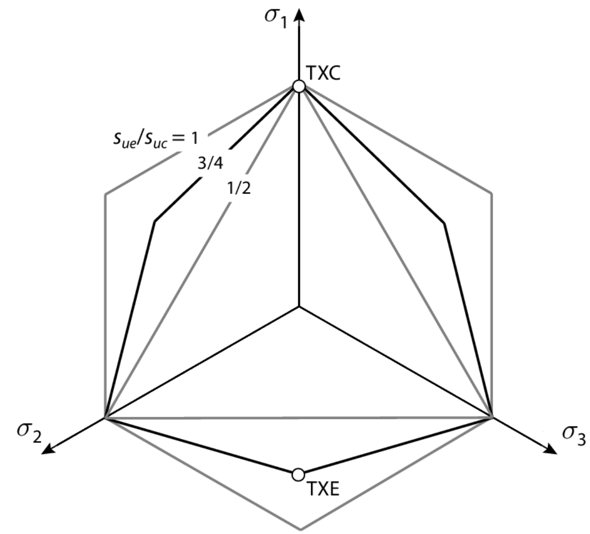




Material	
Name	Tresca Basic
Material Model	Tresca
Color	click to change
Reducible Strength	Yes

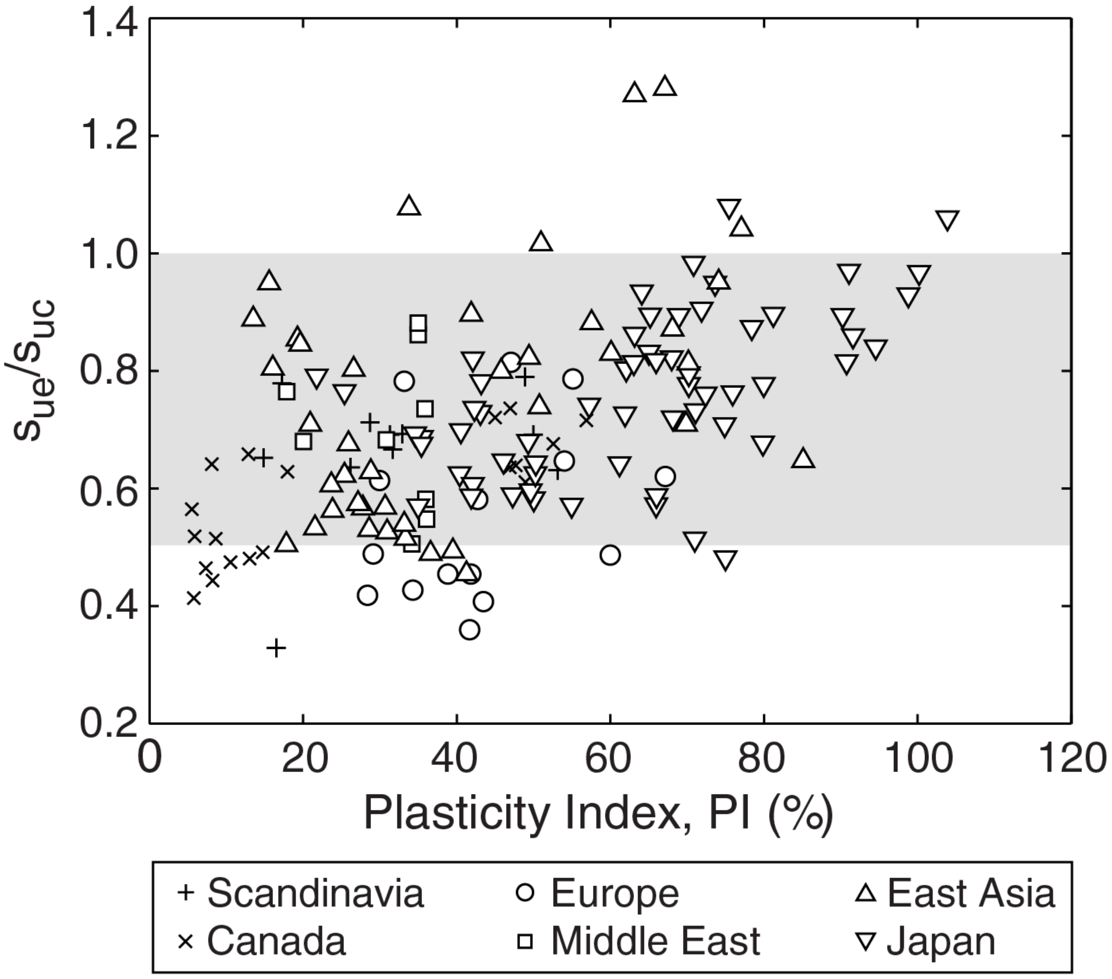
Strength	
Option	Standard
s_u (kPa)	100

Material	
Name	Tresca Basic
Material Model	Tresca
Color	click to change
Reducible Strength	Yes

Strength	
Option	Generalized
s_{uc} (kPa)	30
s_{ue} (kPa)	20

AUS

sue to suc (Won 2013)



Optum CE

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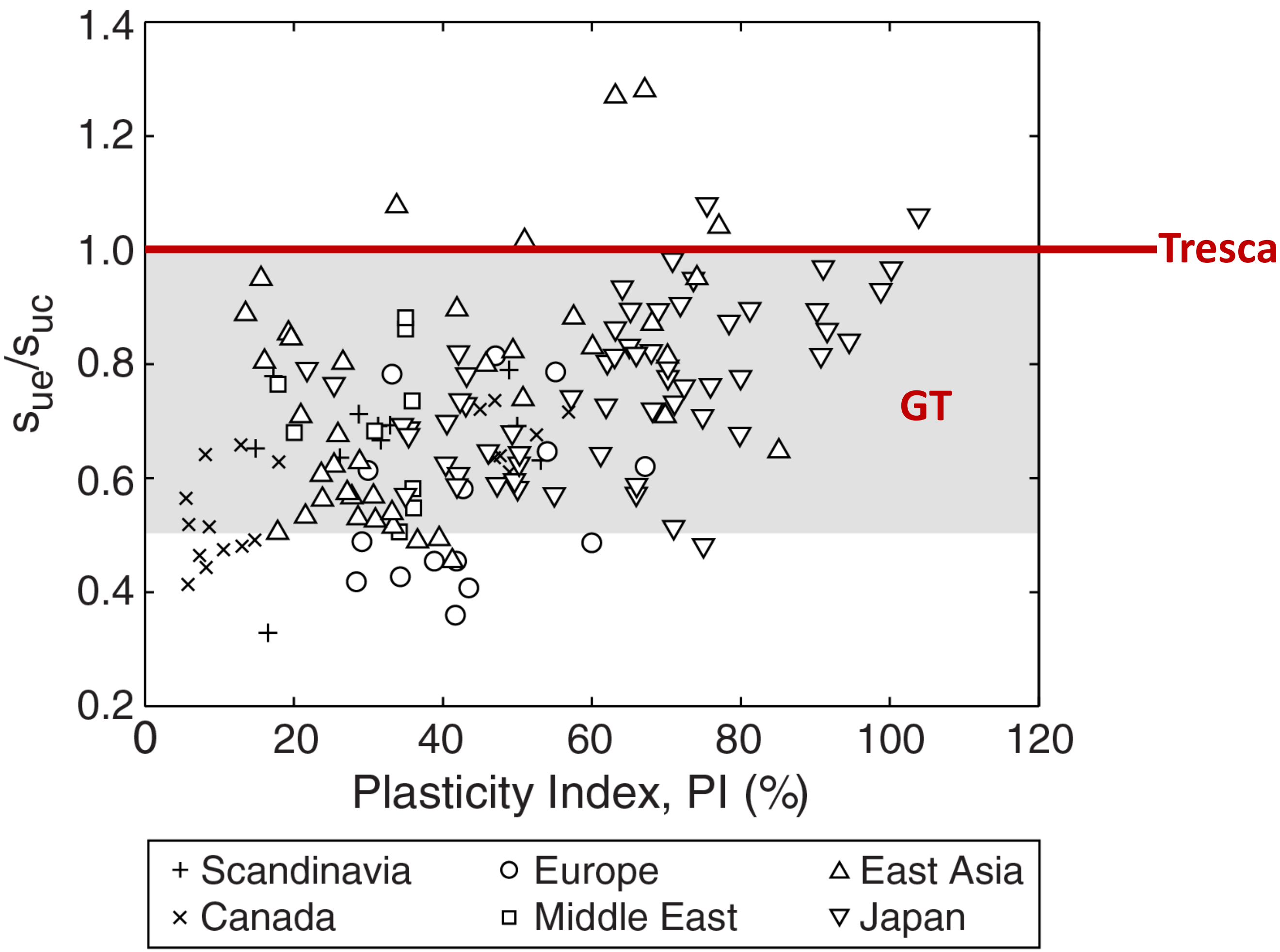

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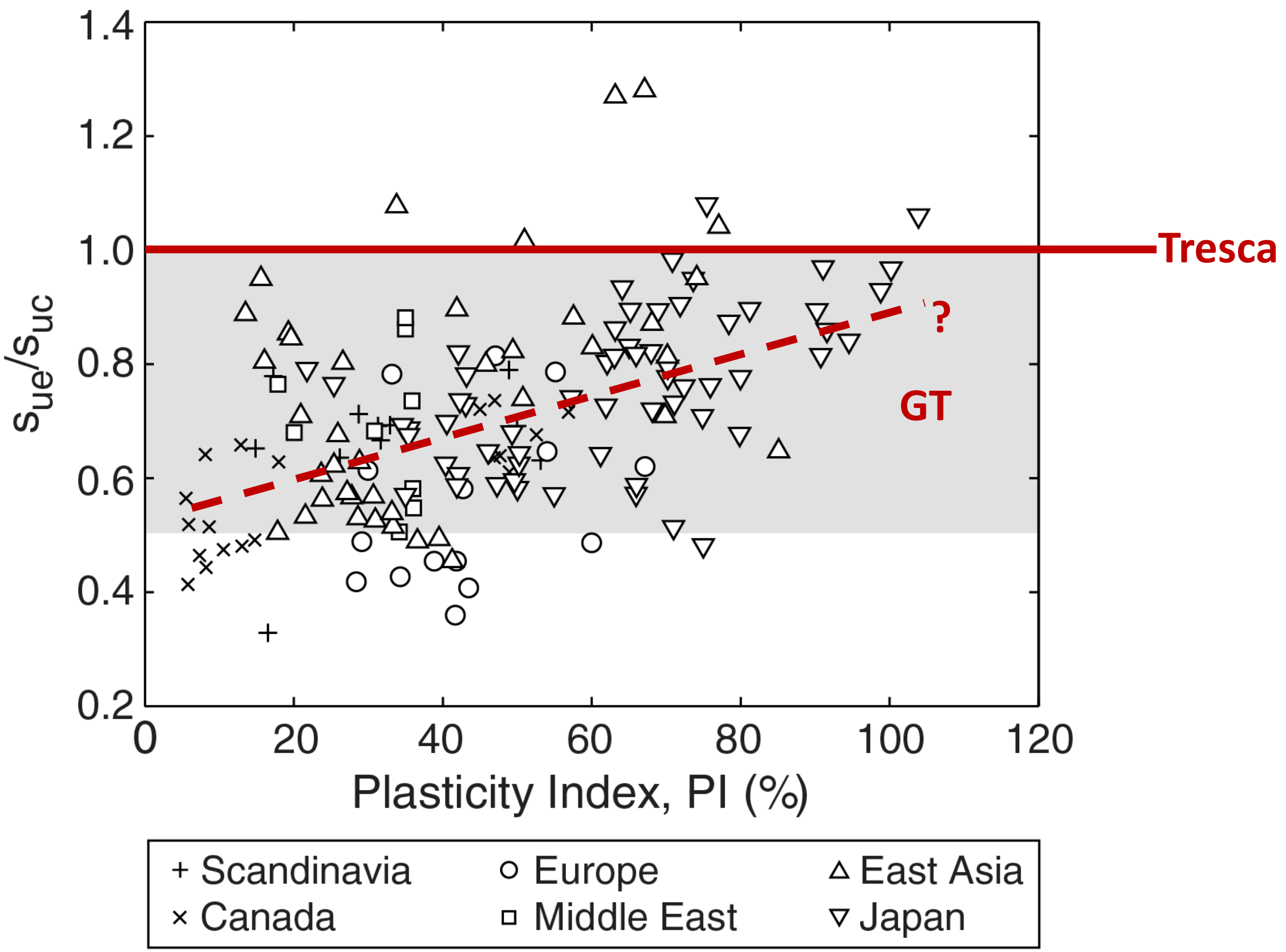
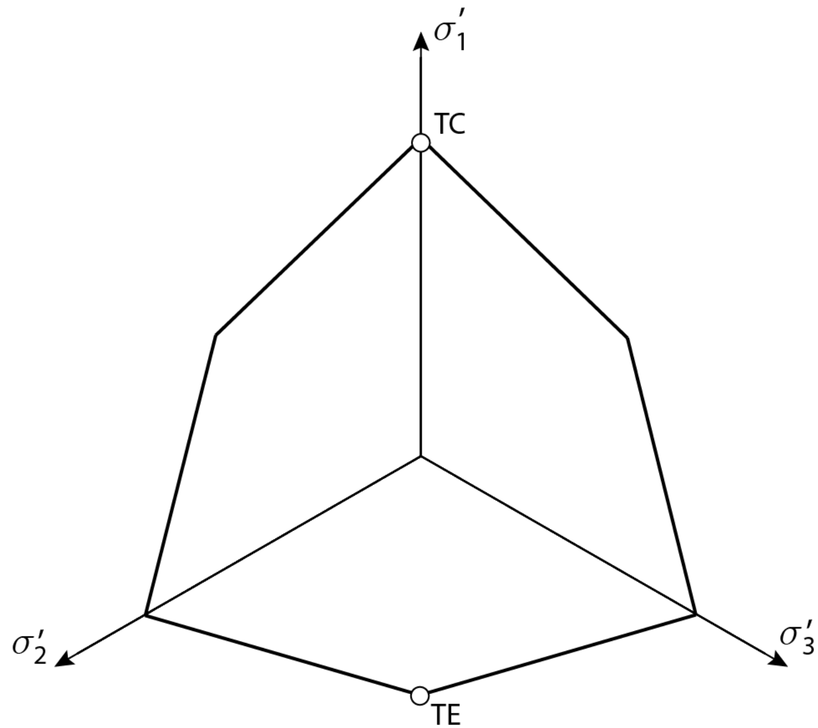


Mohr-Coulomb

$$F = s_1 - s_3 - (s_1' + s_3')\sin f - 2c\cos f$$

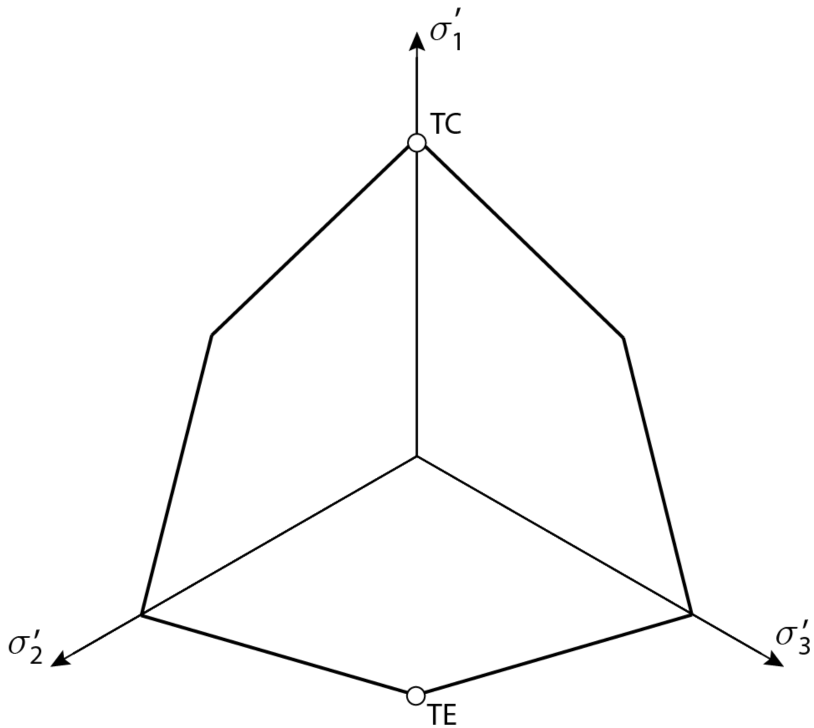


Mohr-Coulomb

$$F = \sigma_1 - \sigma_3 - (\sigma_1^{'} + \sigma_3^{'})\sin\phi - 2c\cos\phi$$

Undrained (linear elastic/perfectly plastic, zero dilation):

$$p' = p_0^{'}$$



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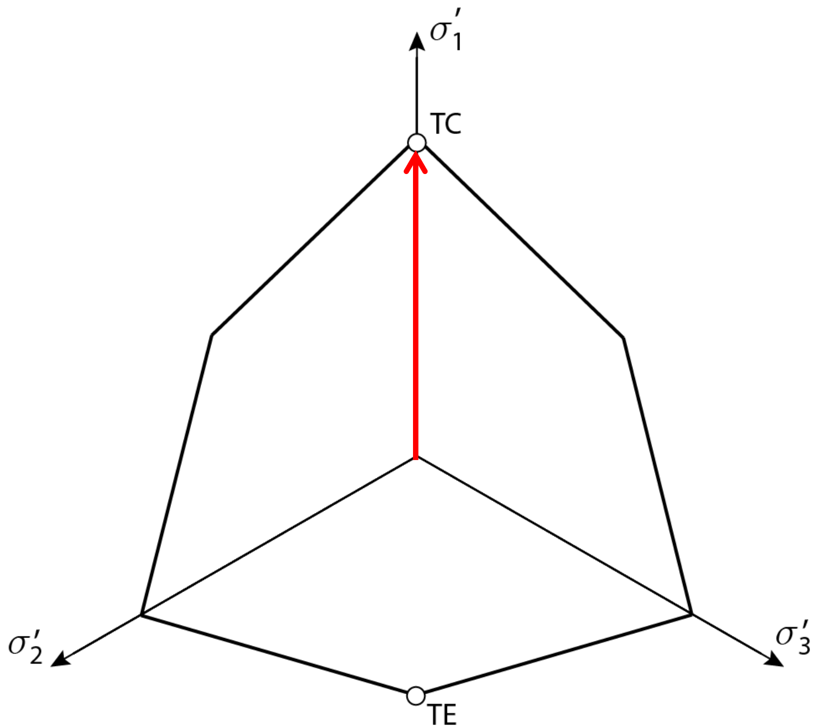

AUS

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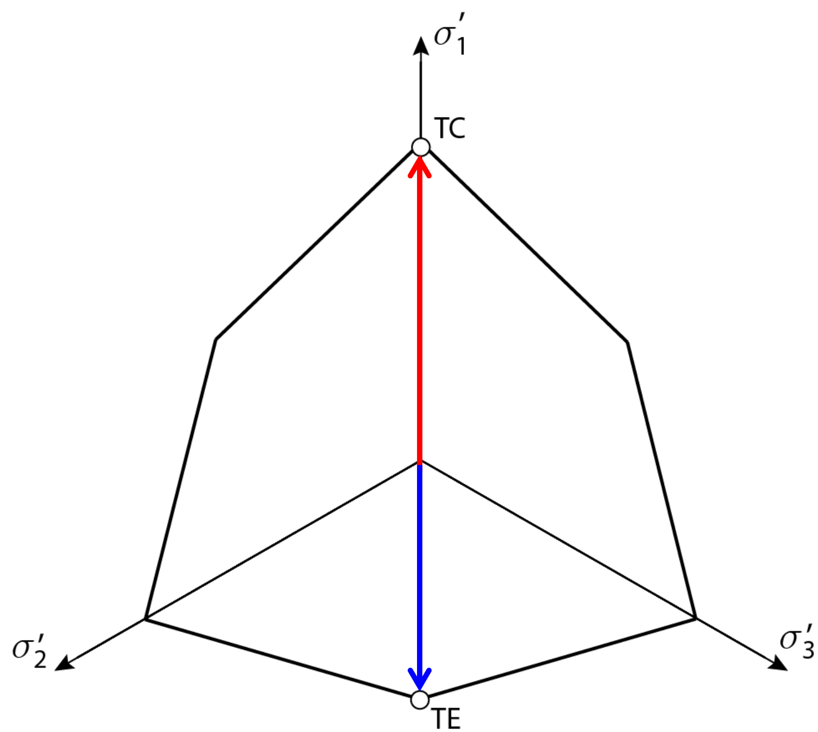

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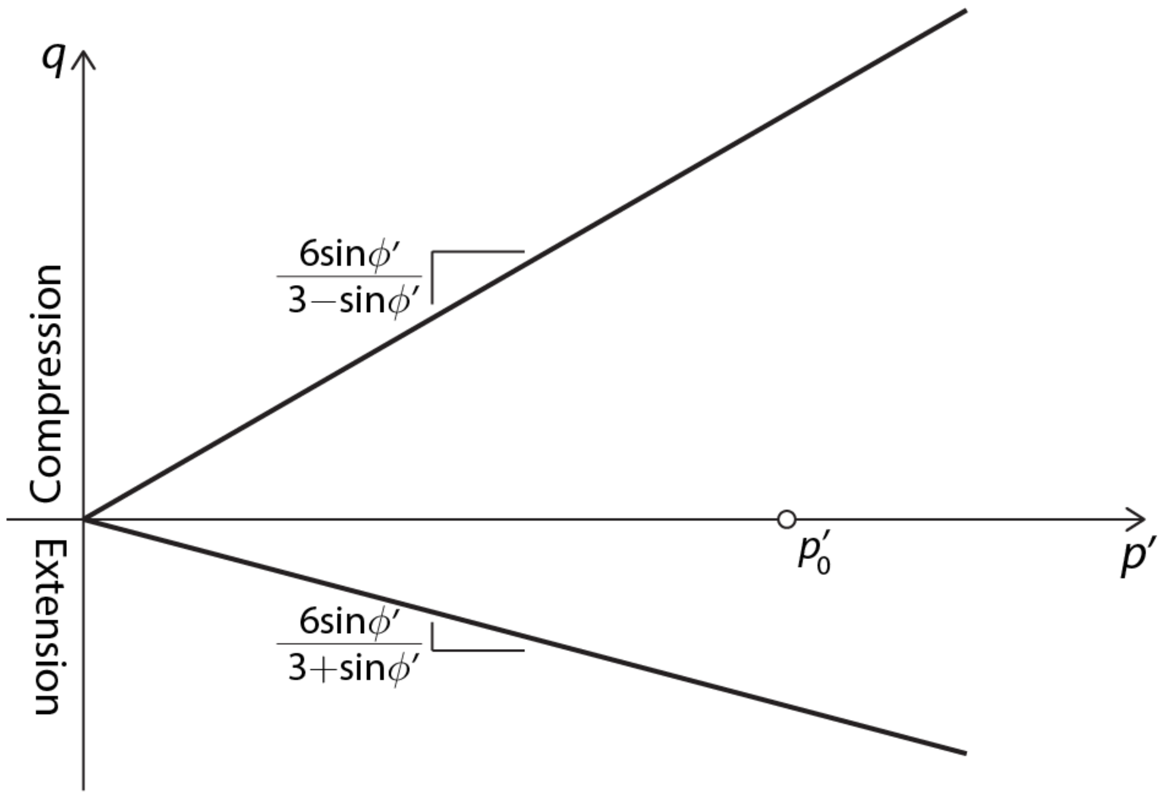

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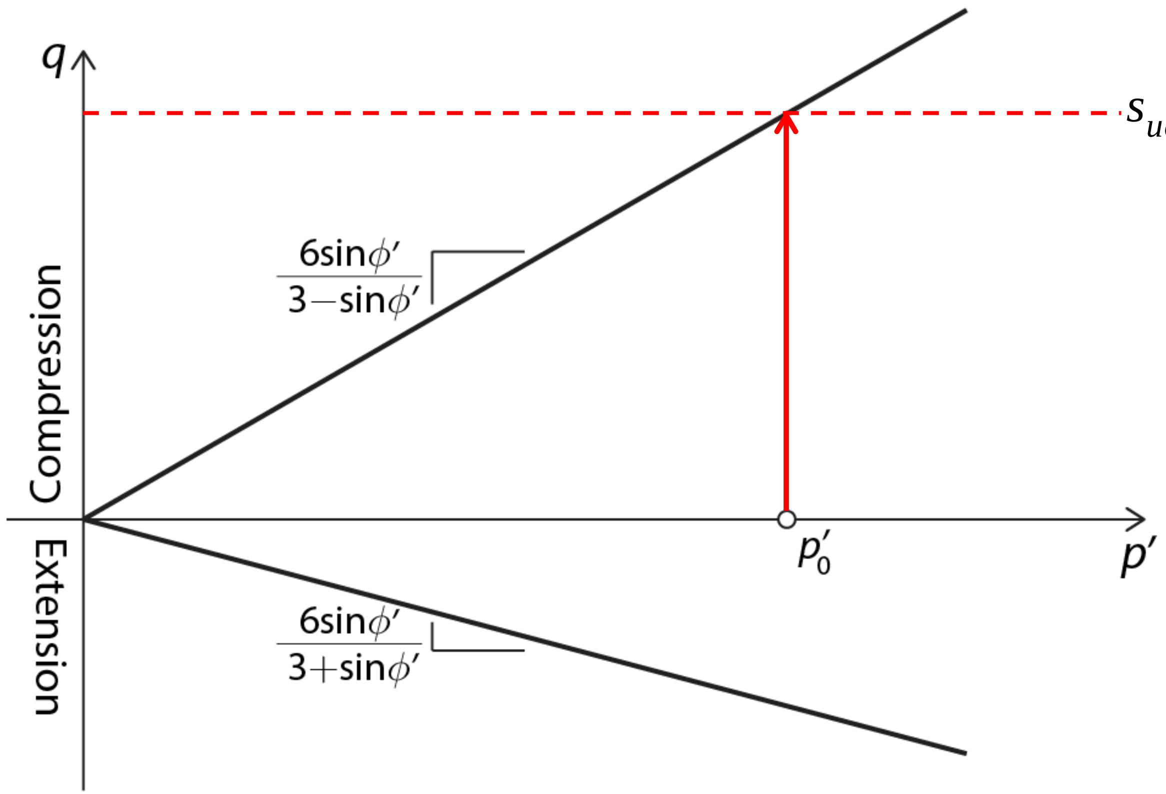
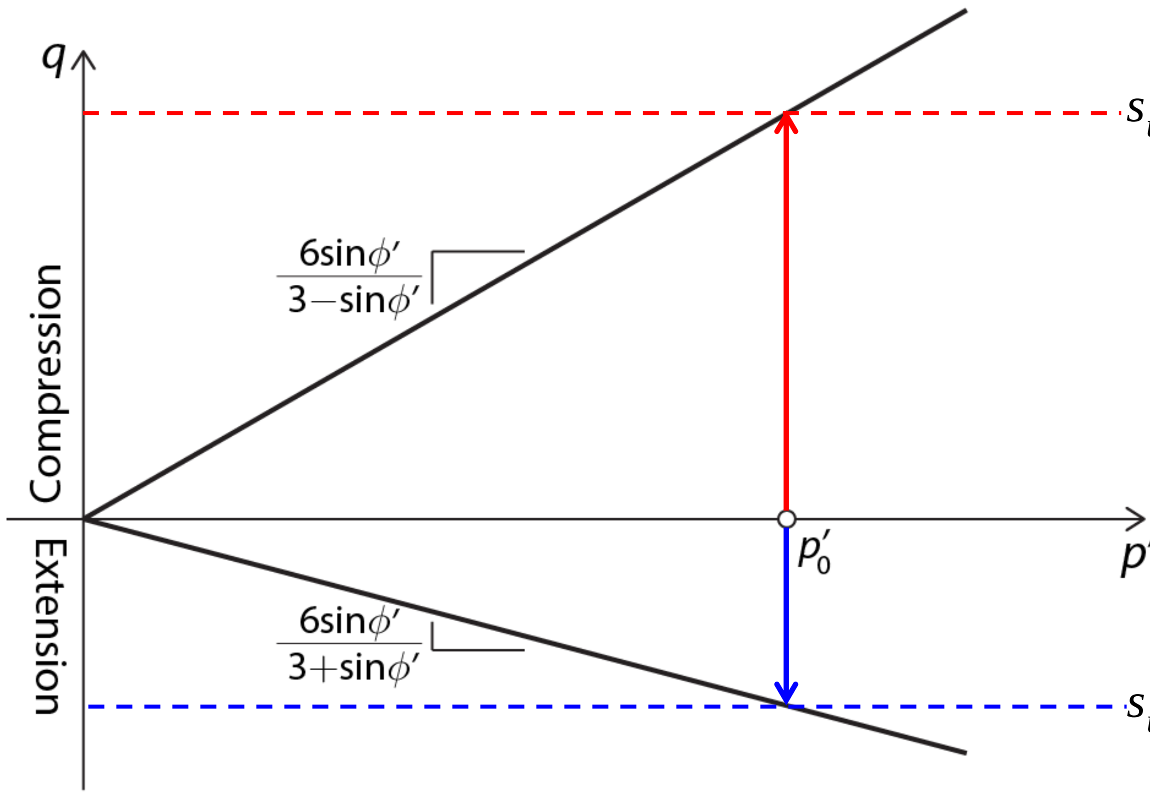


$$s_{uc} = \frac{3\sin f}{3 - \sin f} p_0^{'}$$

Mohr-Coulomb

$$F = \sigma_1 - \sigma_3 - (\sigma_1' + \sigma_3')\sin\phi - 2c\cos\phi$$

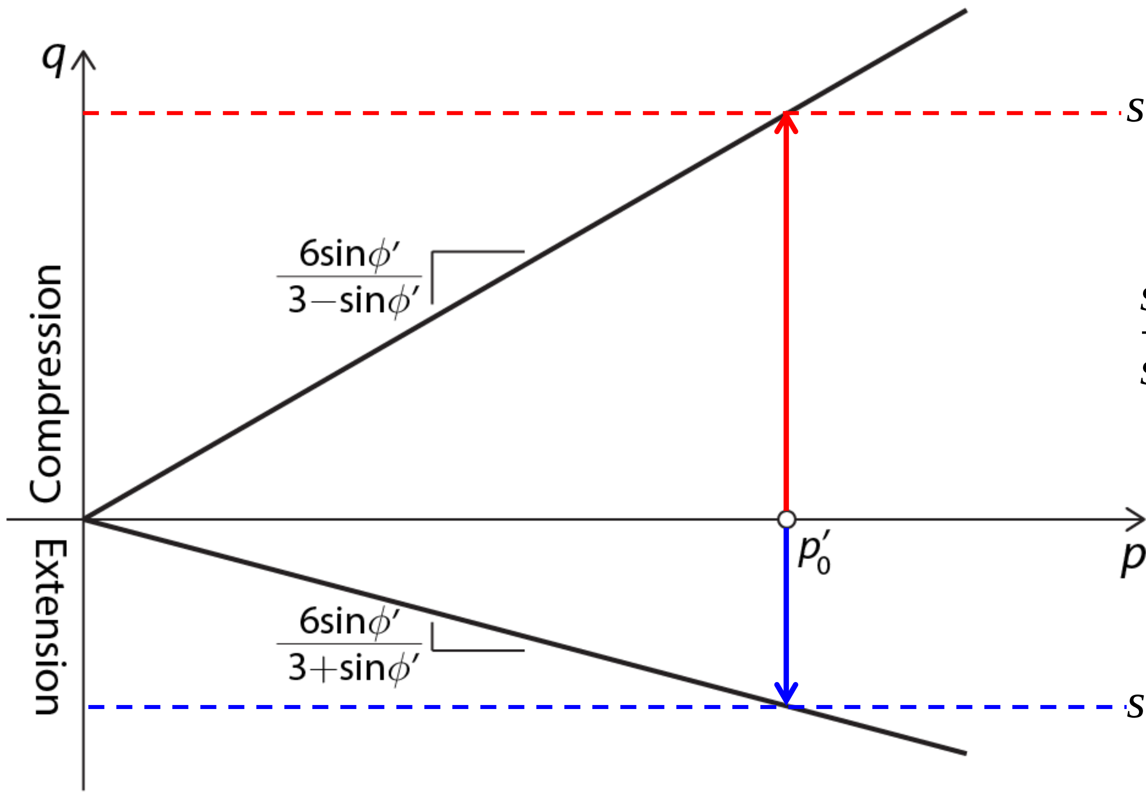


$$\sigma_{uc} = \frac{3\sin\phi}{3-\sin\phi}p_0'$$

$$\sigma_{ue} = \frac{3\sin\phi}{3+\sin\phi}p_0'$$

Mohr-Coulomb

$$F = s_1 - s_3 - (s_1^{'} + s_3^{'})\sin f - 2c\cos f$$



$$s_{uc} = \frac{3\sin f}{3 - \sin f} p_0^{'}$$

$$\frac{s_{ue}}{s_{uc}} = \frac{3 - \sin f}{3 + \sin f}$$

$$s_{ue} = \frac{3\sin f}{3 + \sin f} p_0^{'}$$

Mohr-Coulomb

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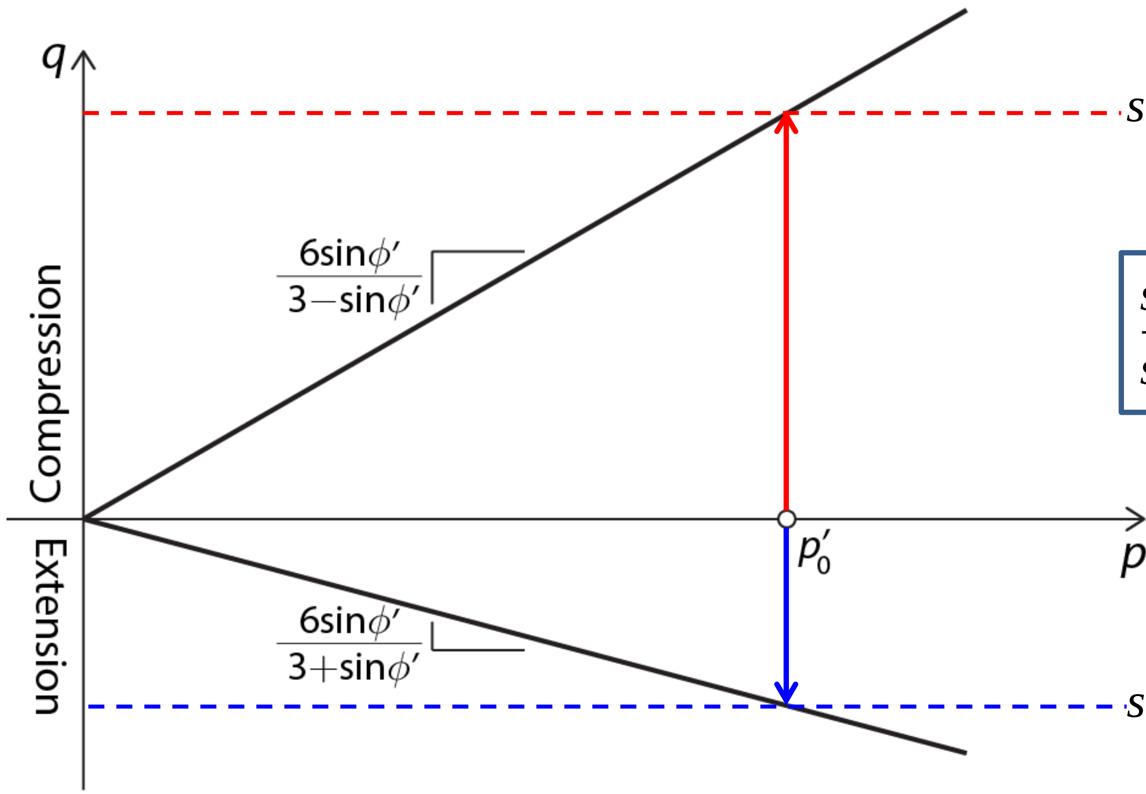

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Undrained (p' = p_0')

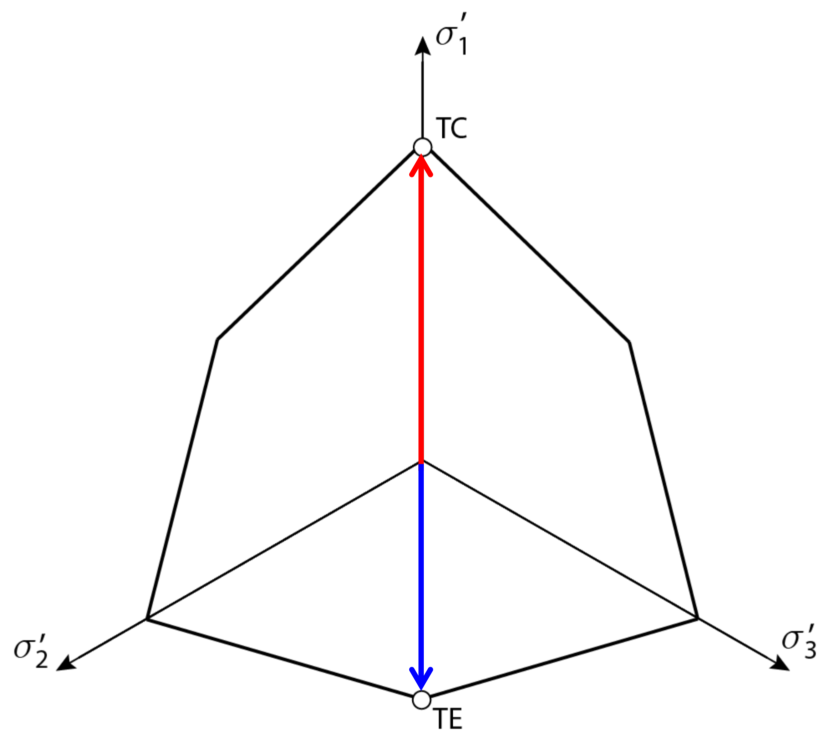
$$F_u = \sigma_1 - \sigma_3 + (s_{ue}/s_{uc} - 1)(\sigma_1 - \sigma_2) - 2s_{uc}$$

← Generalized Tresca

$$s_{ue} = \frac{3\sin\phi}{3+\sin\phi}p_0'$$

$$s_{uc} = \frac{3\sin\phi}{3-\sin\phi}p_0'$$

$$\frac{s_{ue}}{s_{uc}} = \frac{3-\sin\phi}{3+\sin\phi}$$



Optum[CE]

Mohr-Coulomb

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Undrained (p’ = p_0’)

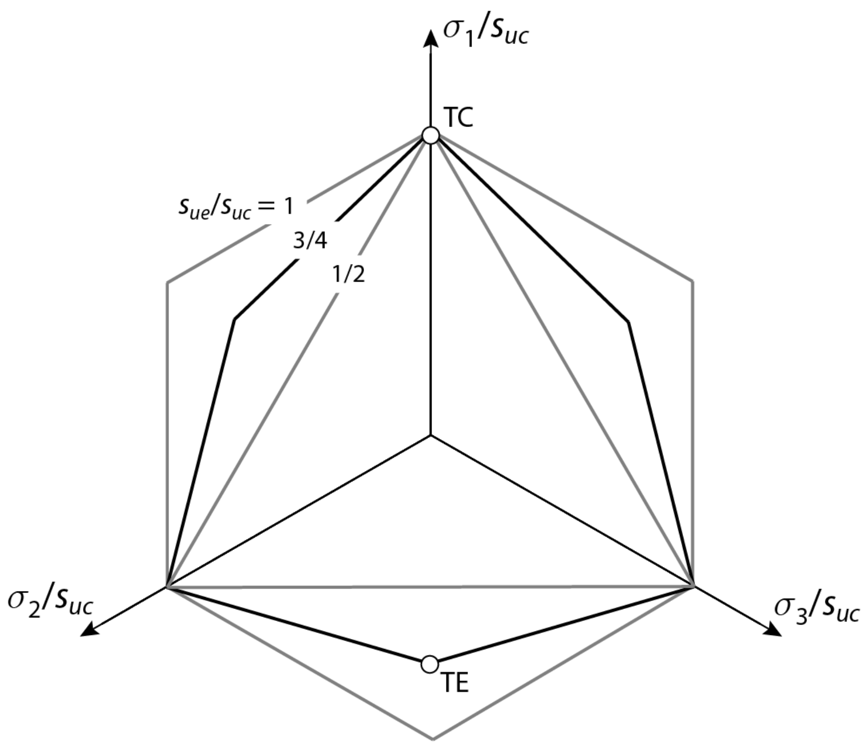
$$F_u = s_1 - s_3 + (s_{ue}/s_{uc} - 1)(s_1 - s_2) - 2s_{uc}$$

← **Generalized Tresca**

$$s_{ue} = \frac{3\sin f}{3 + \sin f} p_0^{'}$$

$$s_{uc} = \frac{3\sin f}{3 - \sin f} p_0^{'}$$

$$\frac{s_{ue}}{s_{uc}} = \frac{3 - \sin f}{3 + \sin f}$$



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← Generalized Tresca

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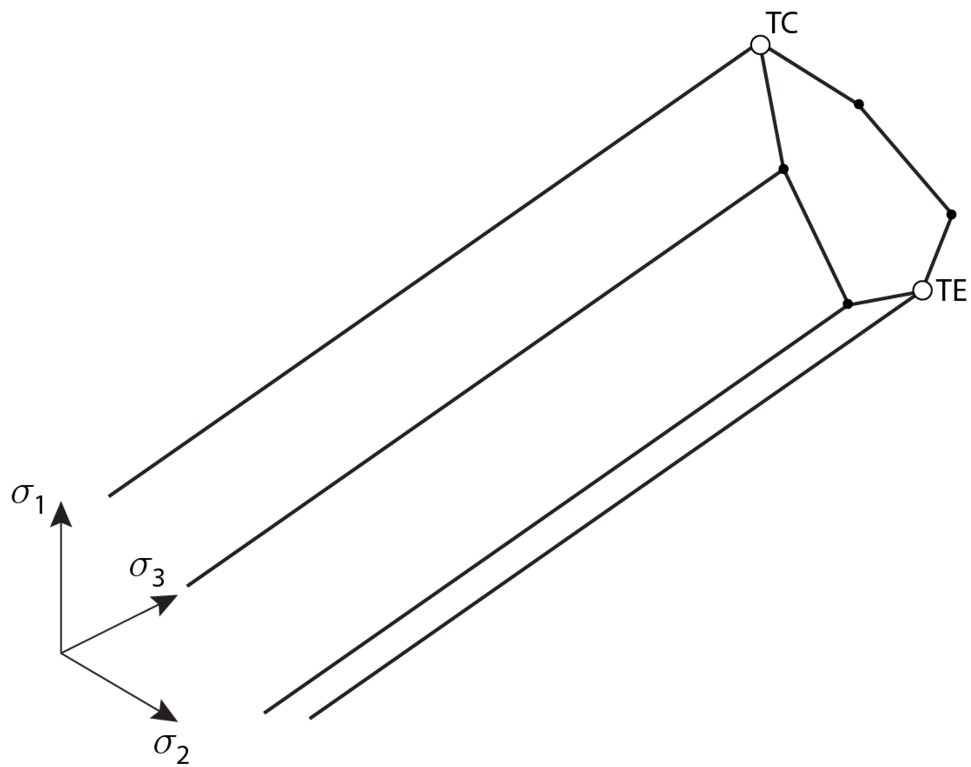
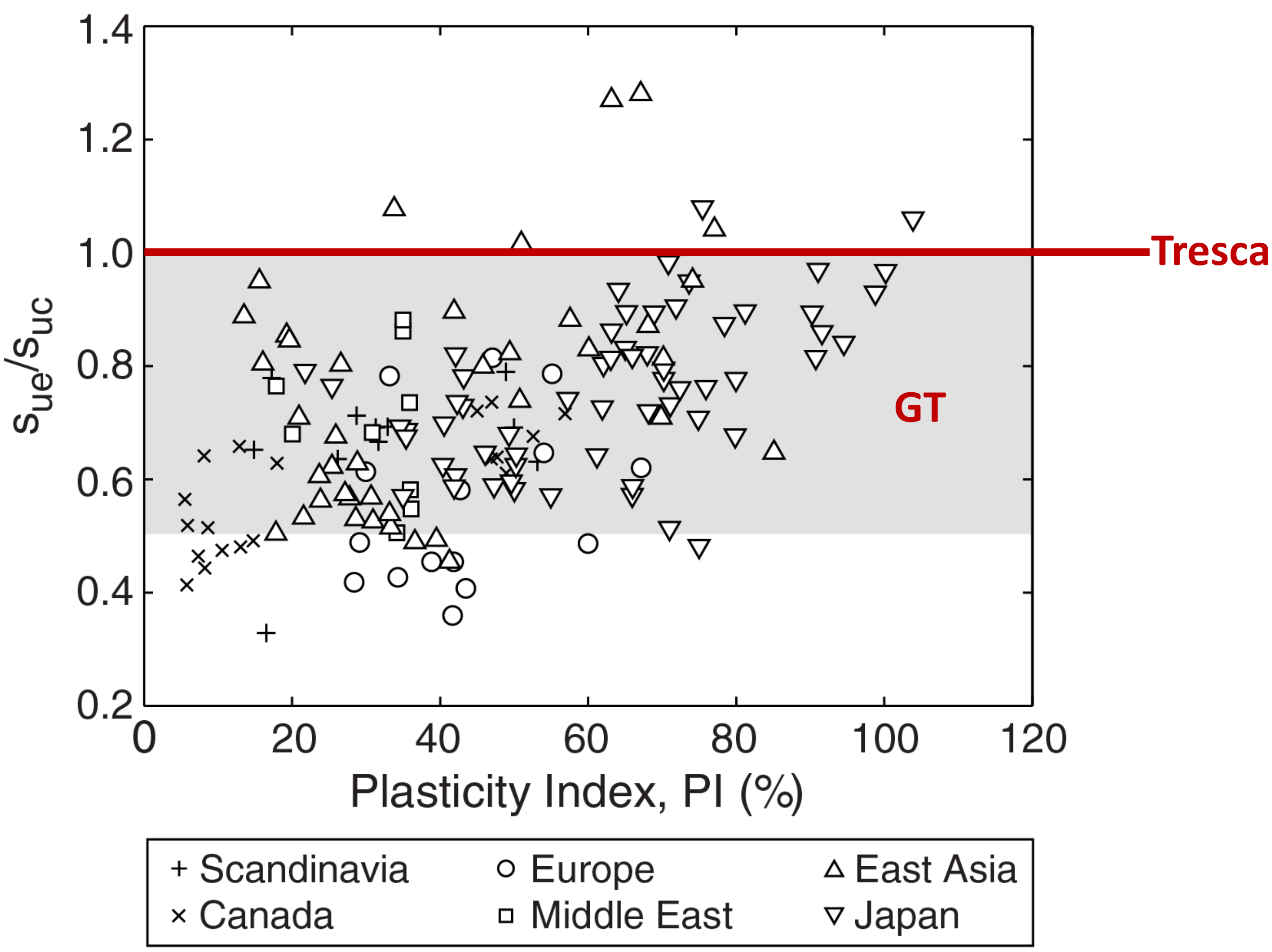
$$s_{uc} = \frac{3\sin f}{3 - \sin f}p_0'$$

$$\frac{s_{ue}}{s_{uc}} = \frac{3 - \sin f}{3 + \sin f}$$



OptumCE

sue to suc (Won 2013)



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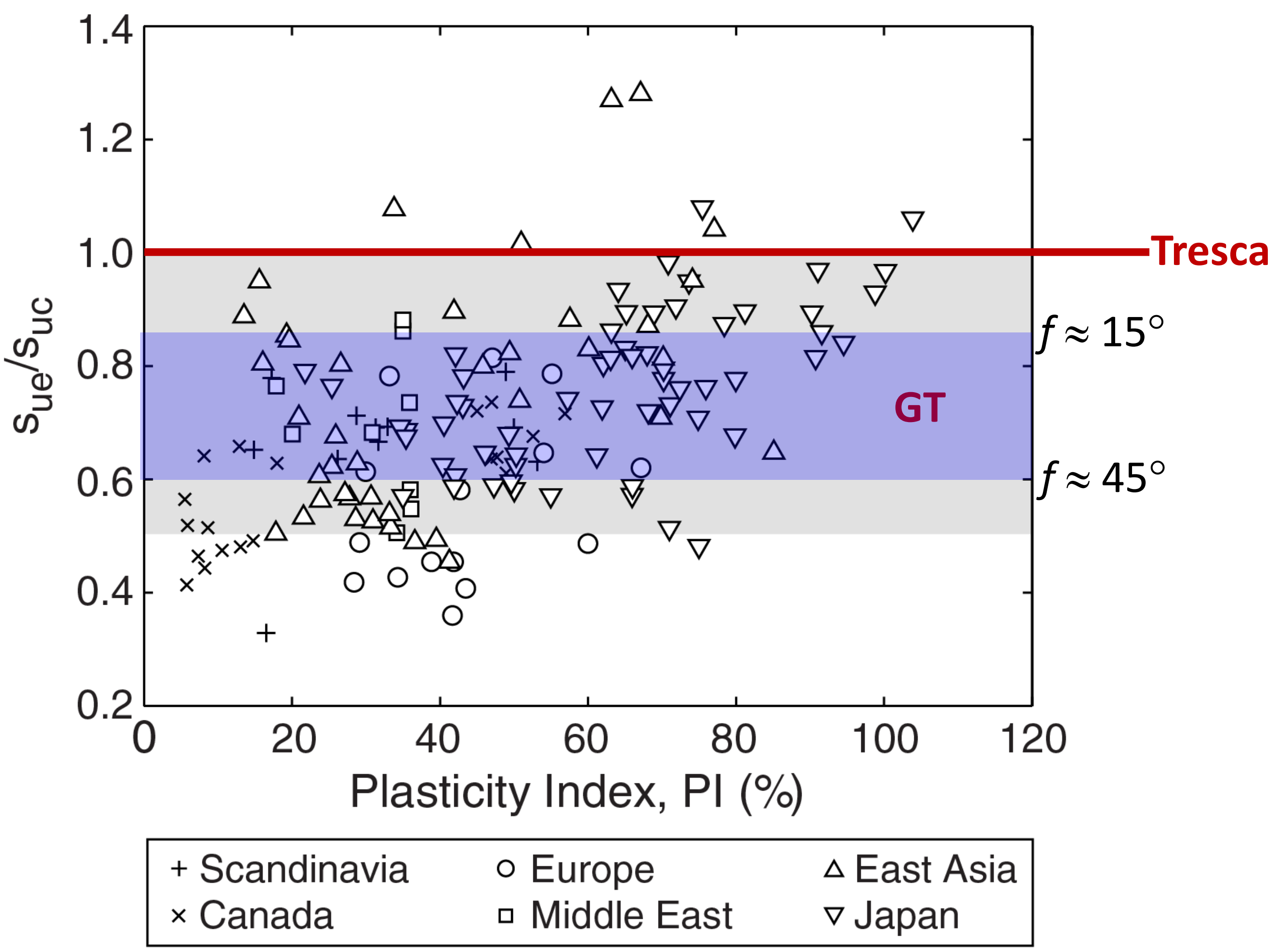
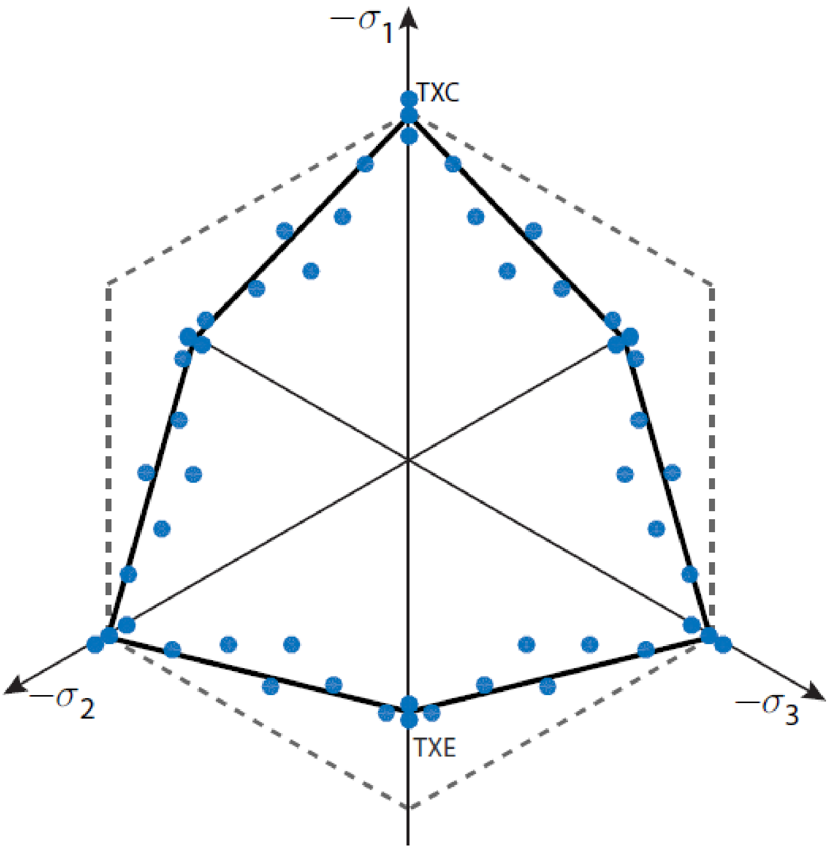


Yield surface

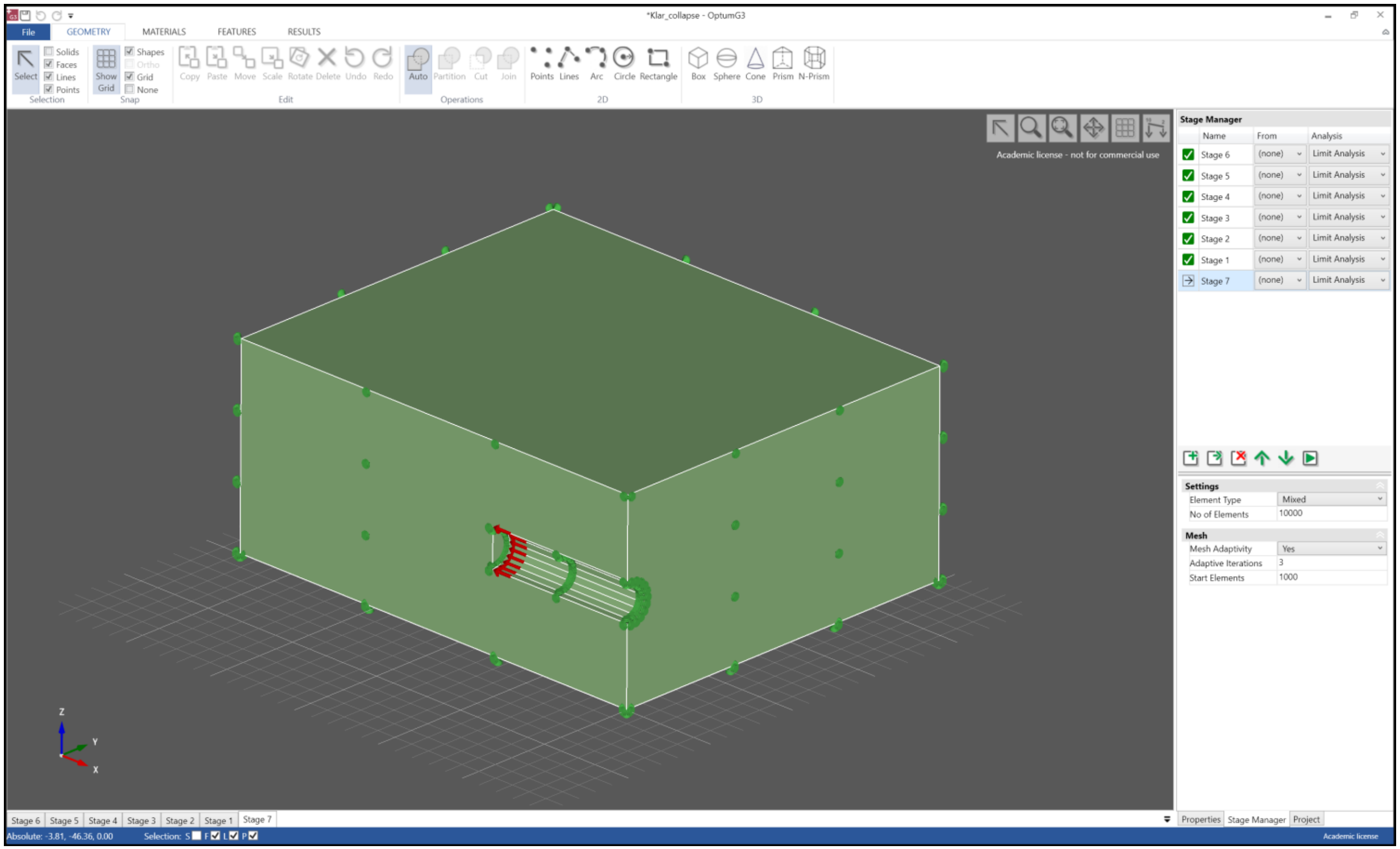
Undrained true triaxial tests on NC Edgar Plastic Kaolinite (Lade 1990)

Assuming isotropic material behaviour

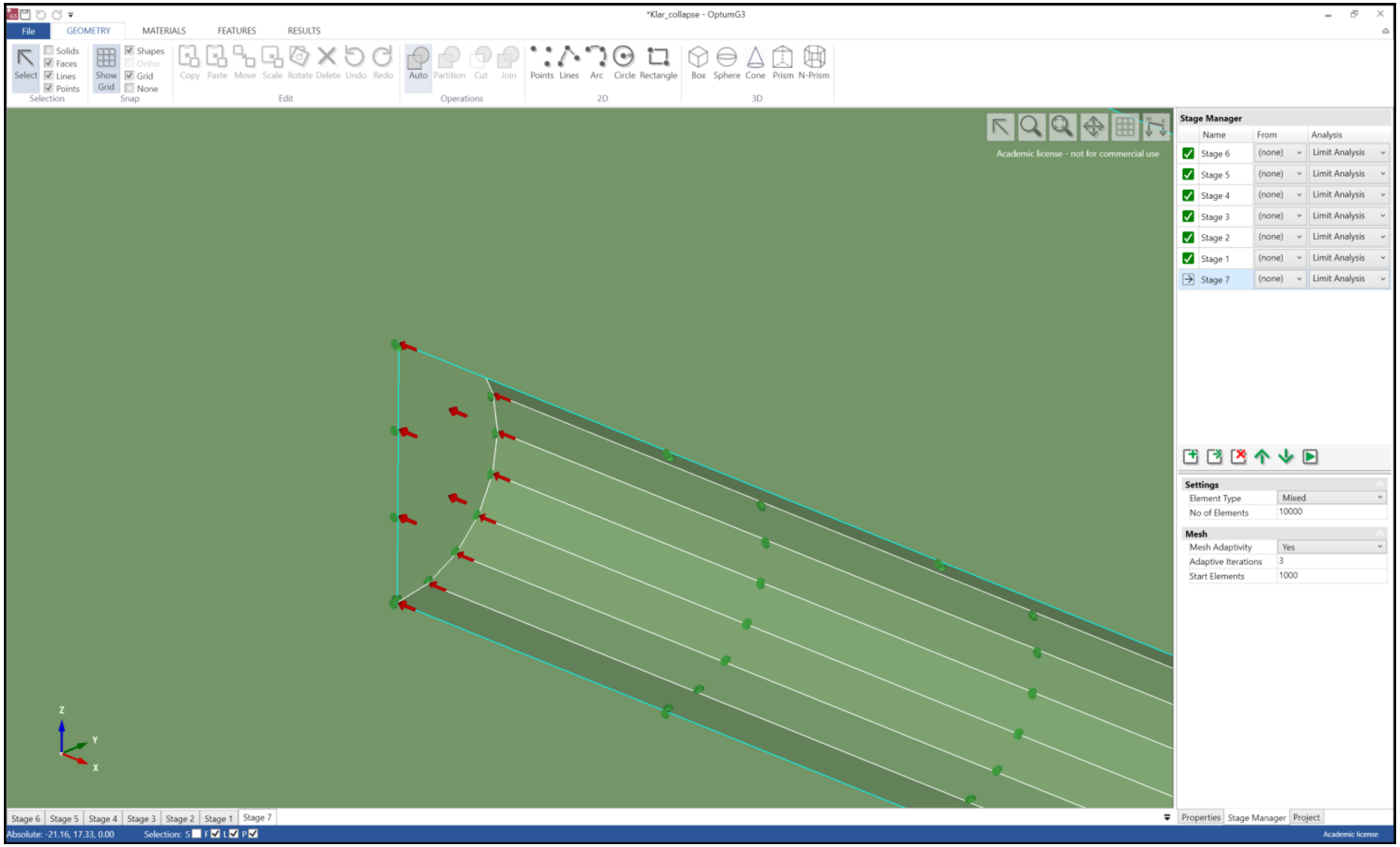


$s_{ue}/s_{uc} = 0.7$

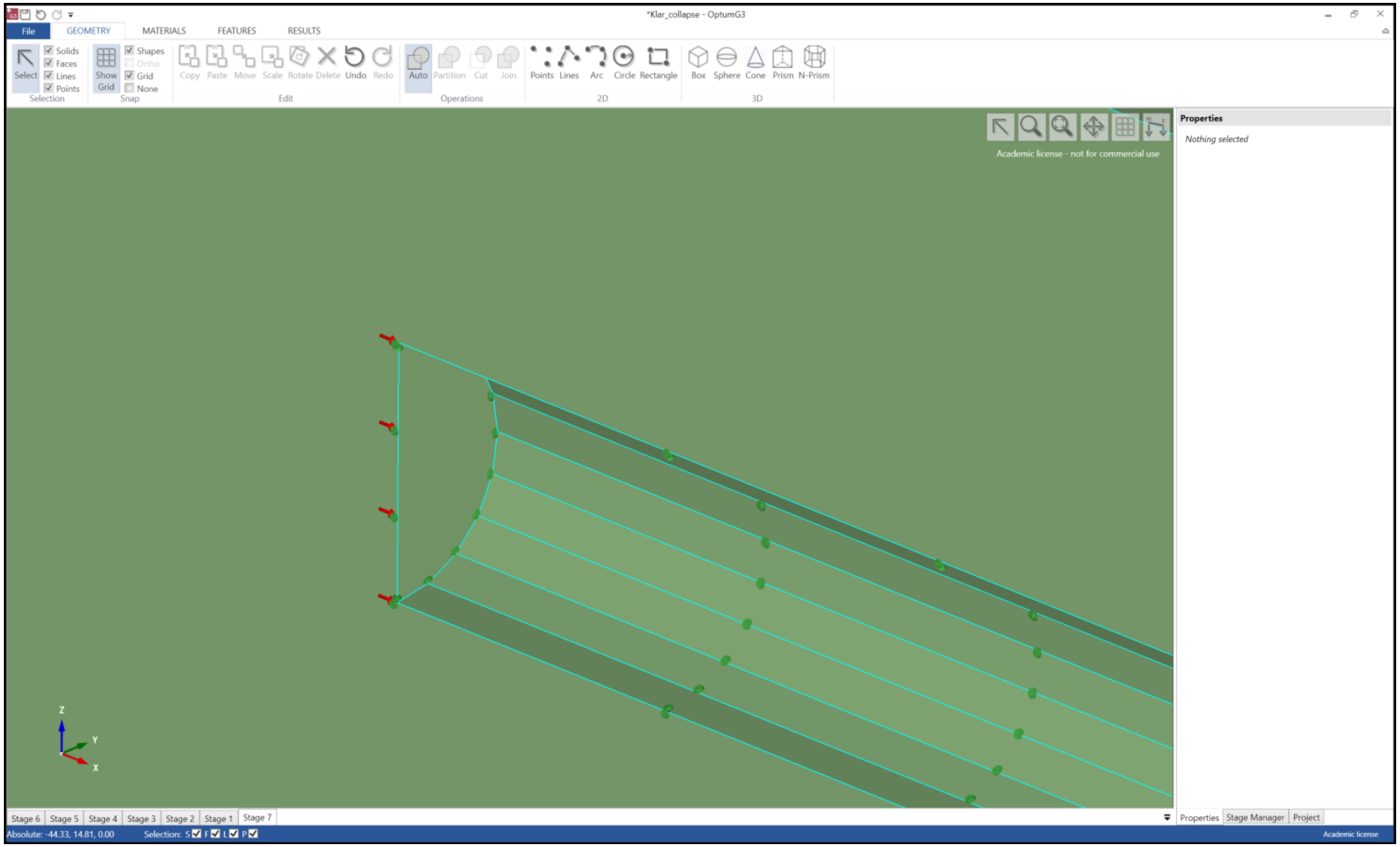
Tunnel face stability



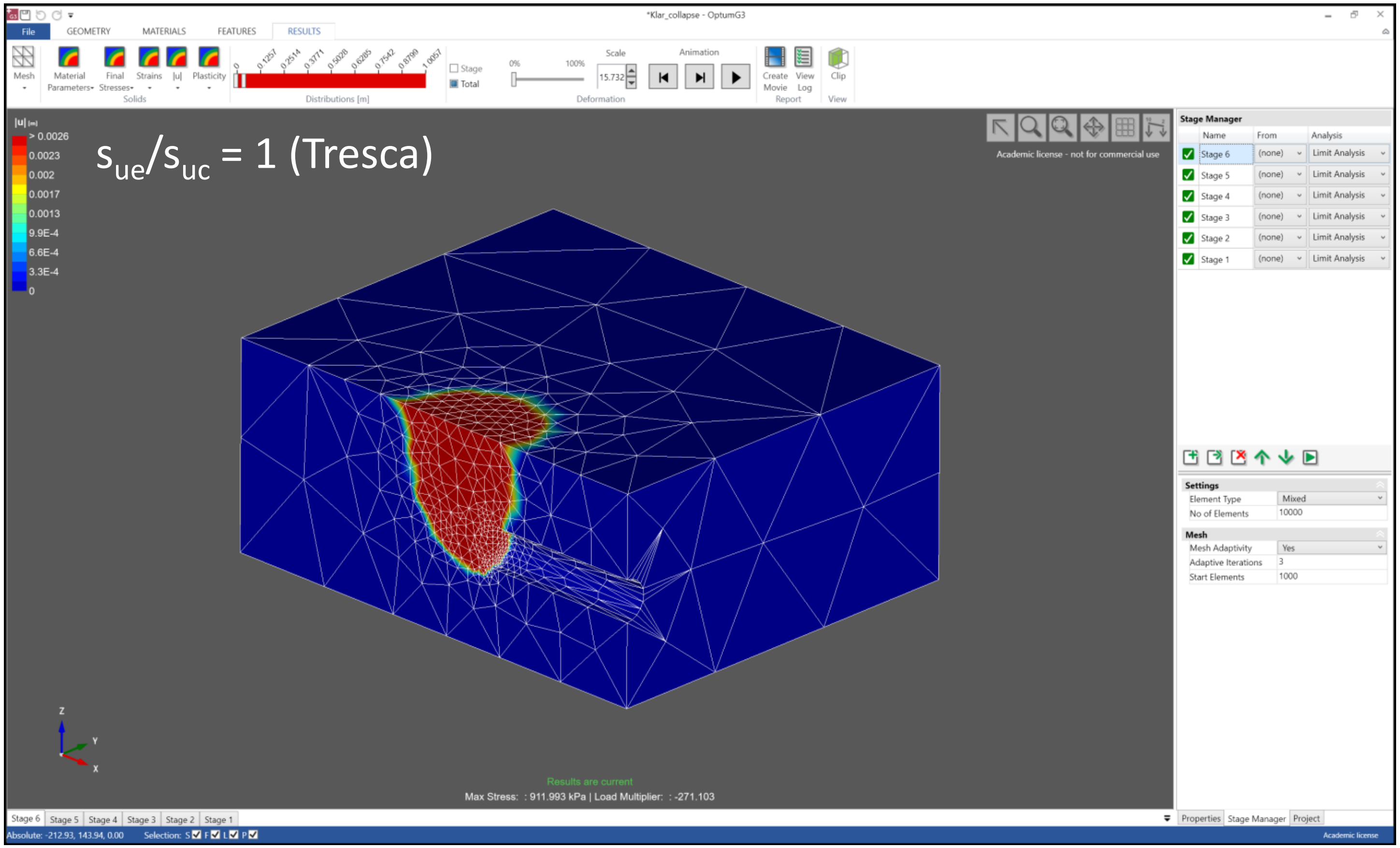
Tunnel face stability – blowout



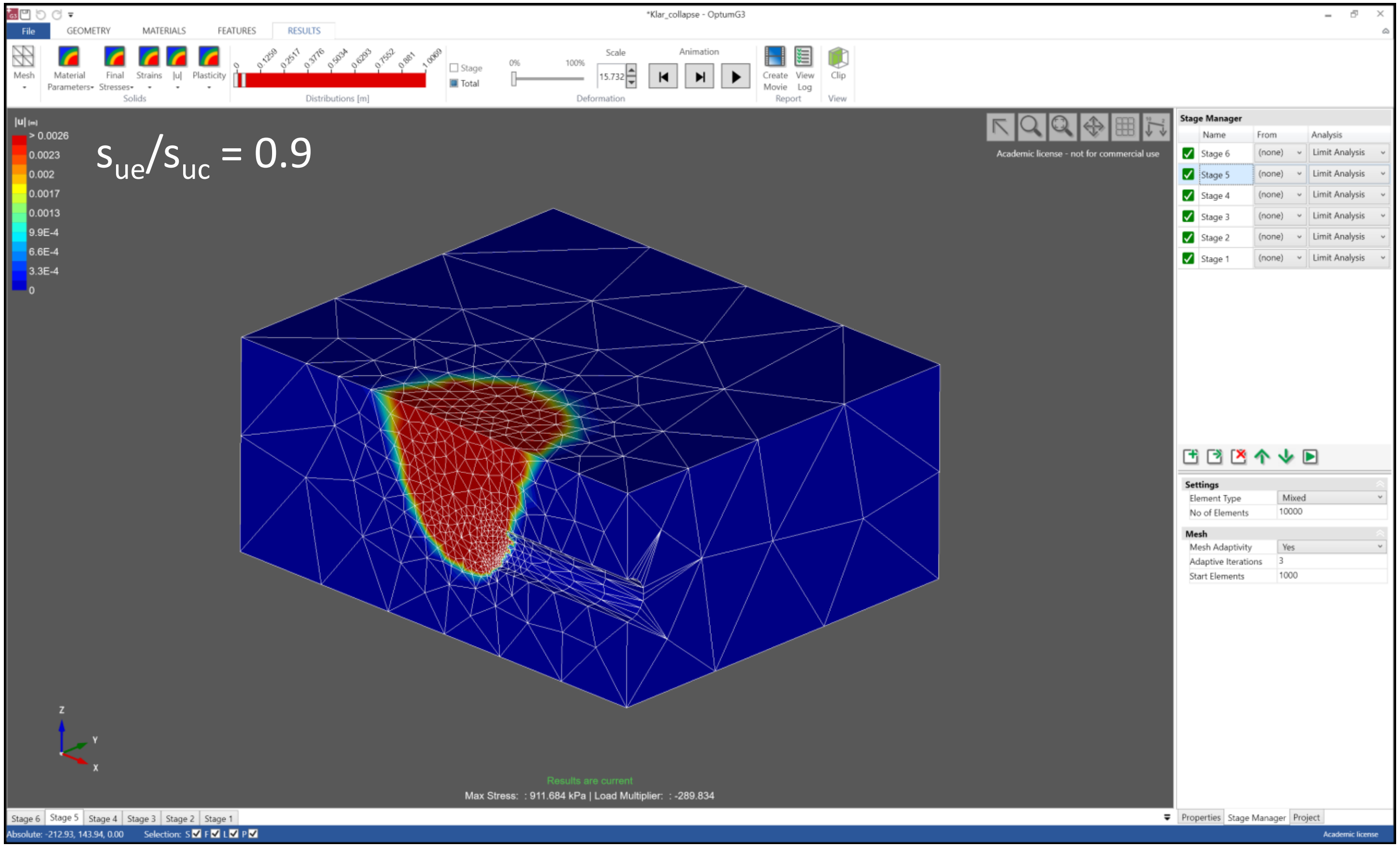
Tunnel face stability – collapse



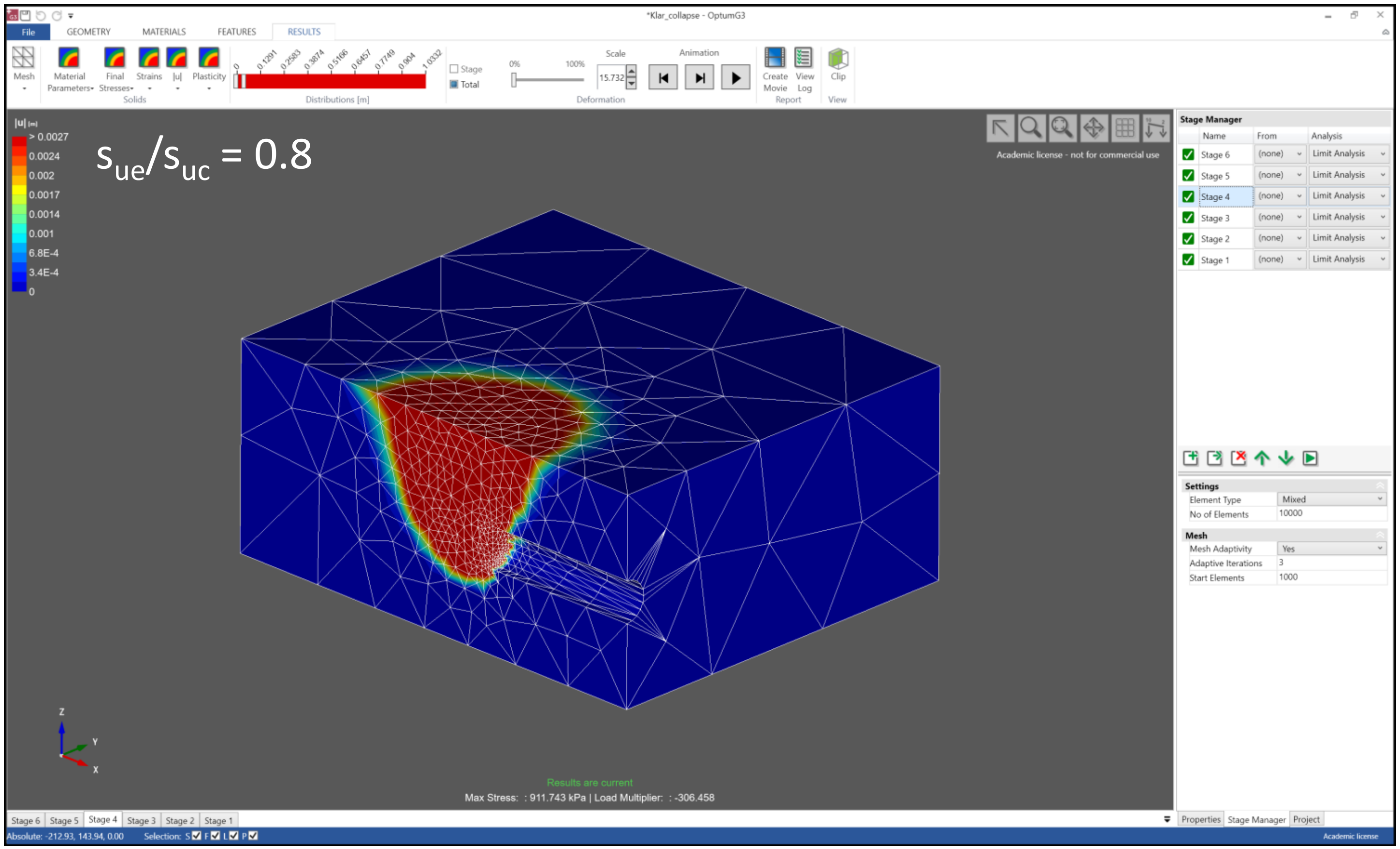
Tunnel face stability – collapse



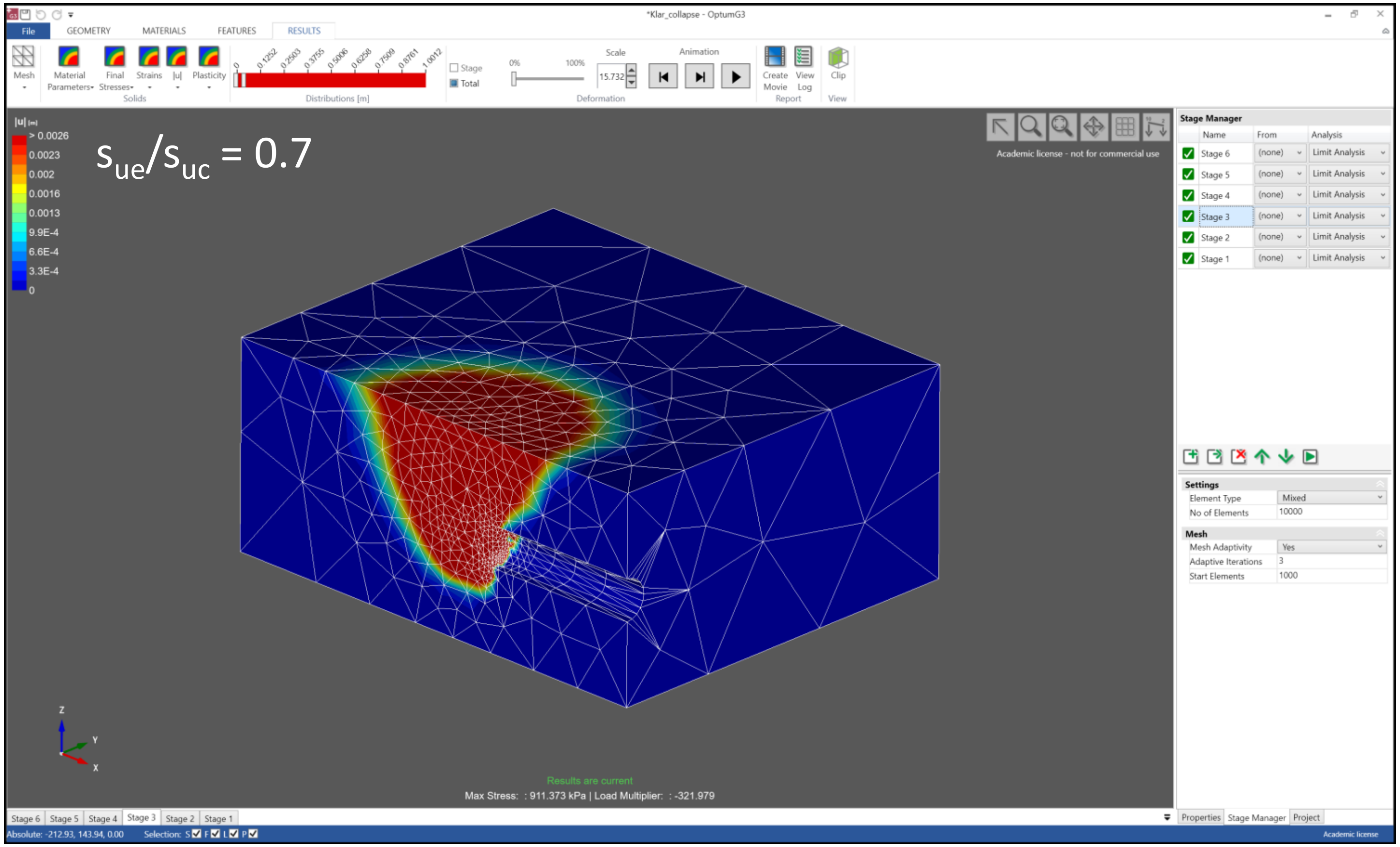
Tunnel face stability – collapse



Tunnel face stability – collapse

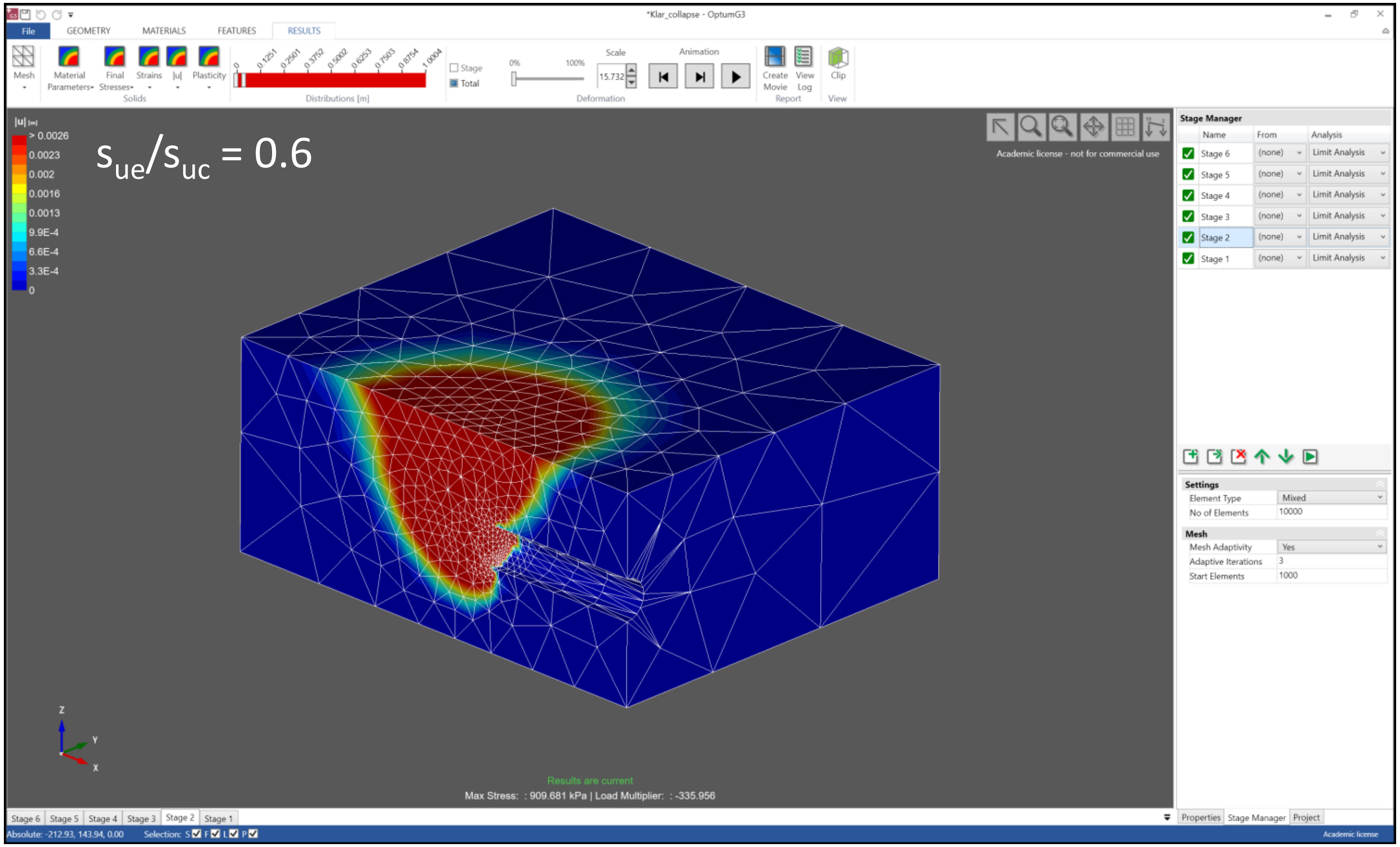


Tunnel face stability – collapse

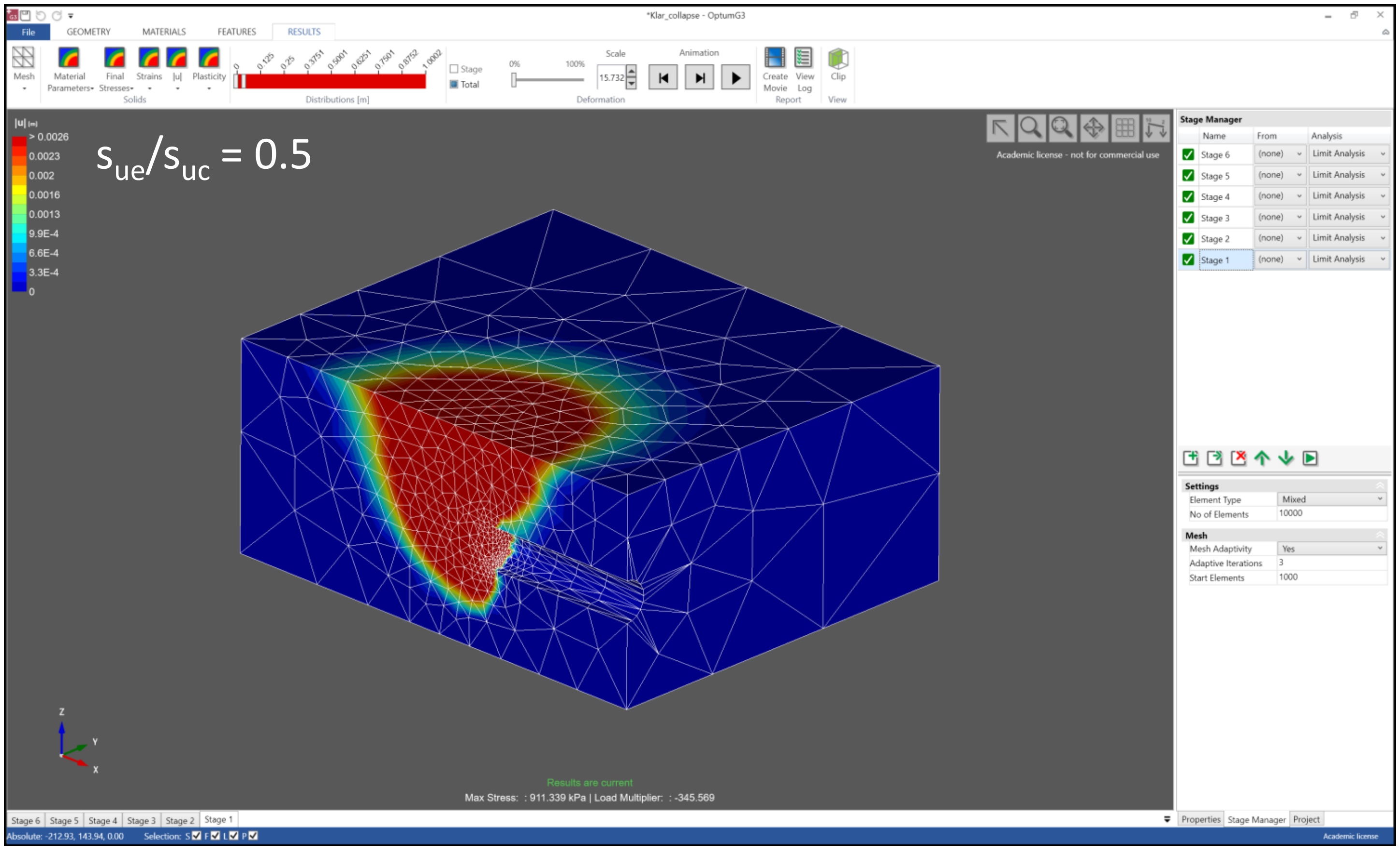


Optum^CE

Tunnel face stability – collapse

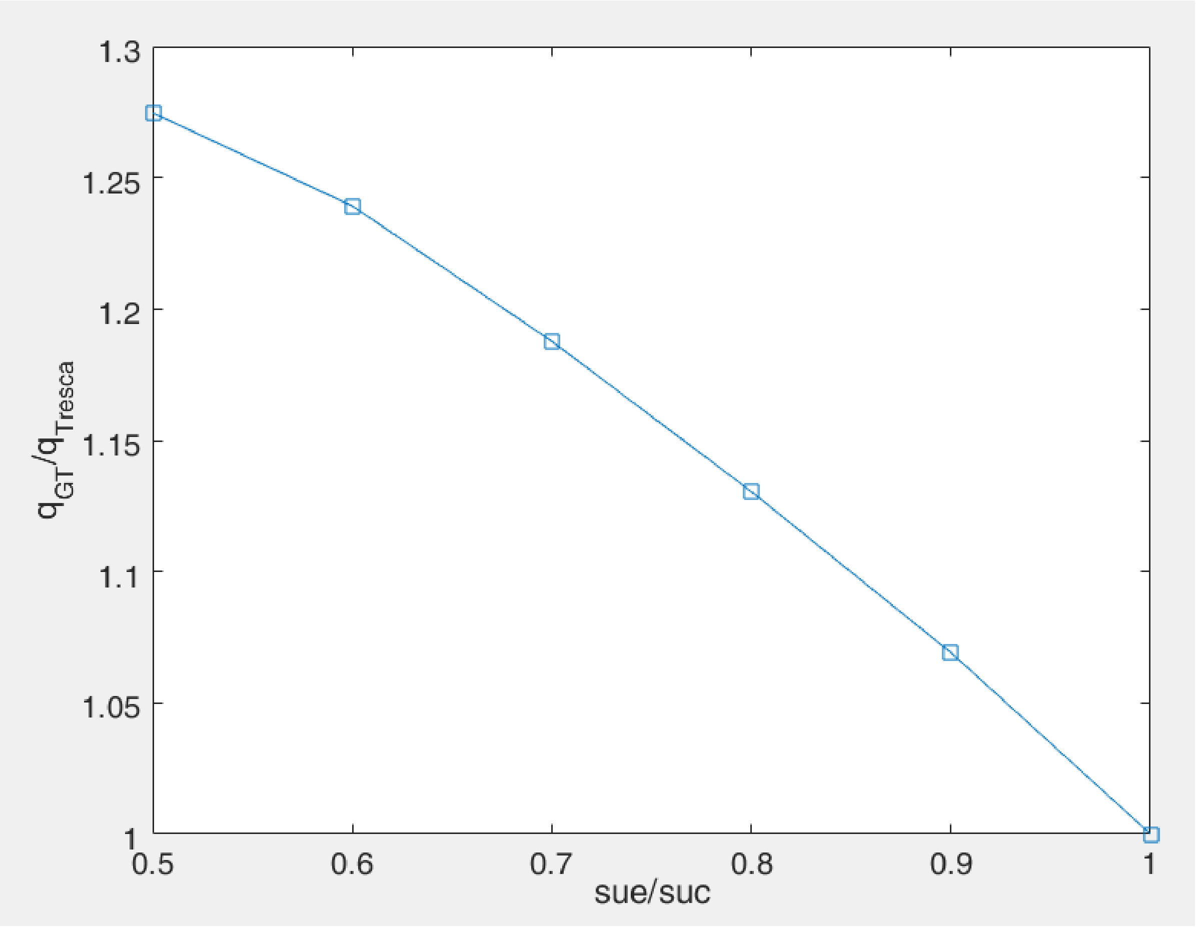


Tunnel face stability – collapse

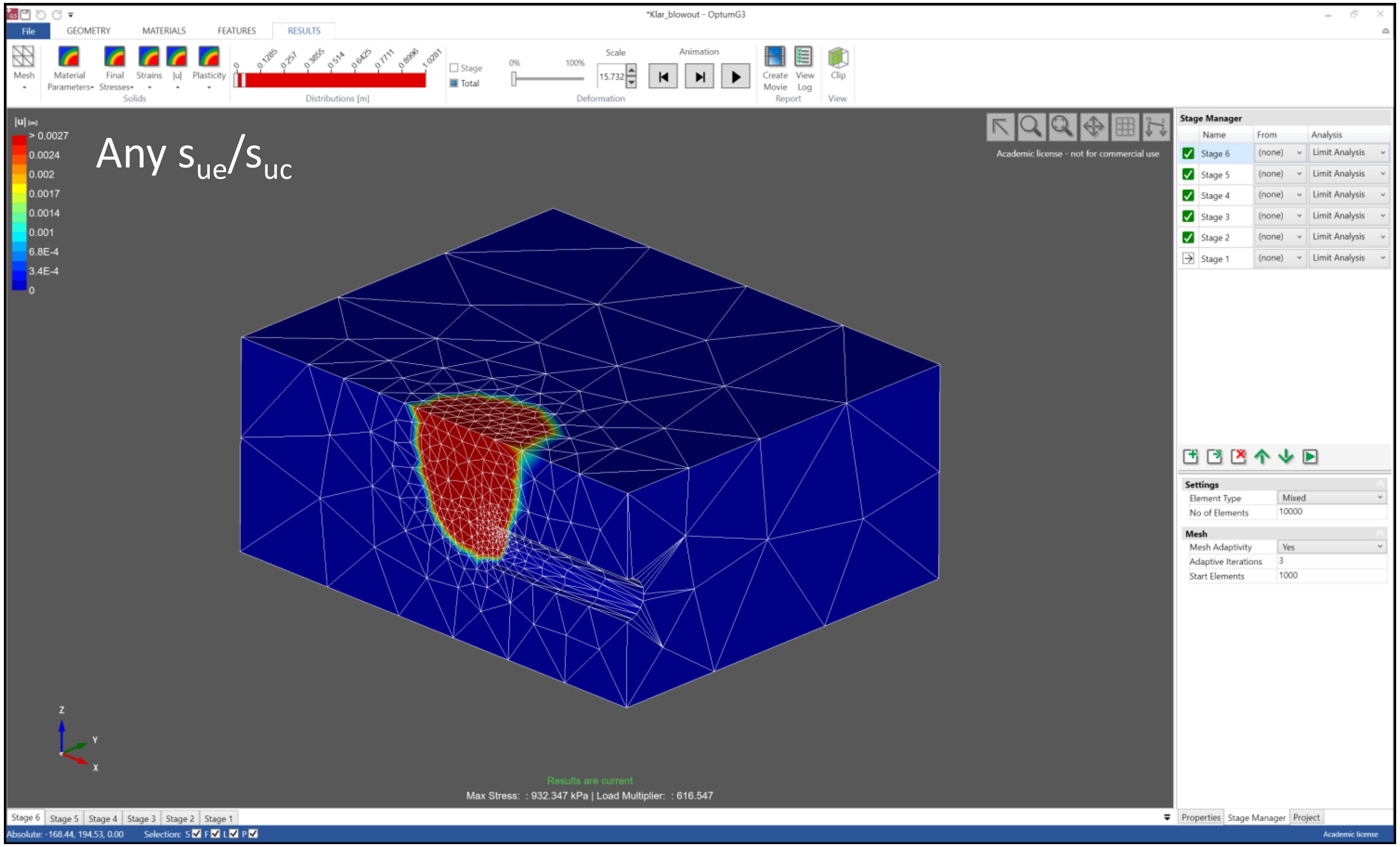


Optum CE

Tunnel face stability – collapse



Tunnel face stability – blowout



Optum CE

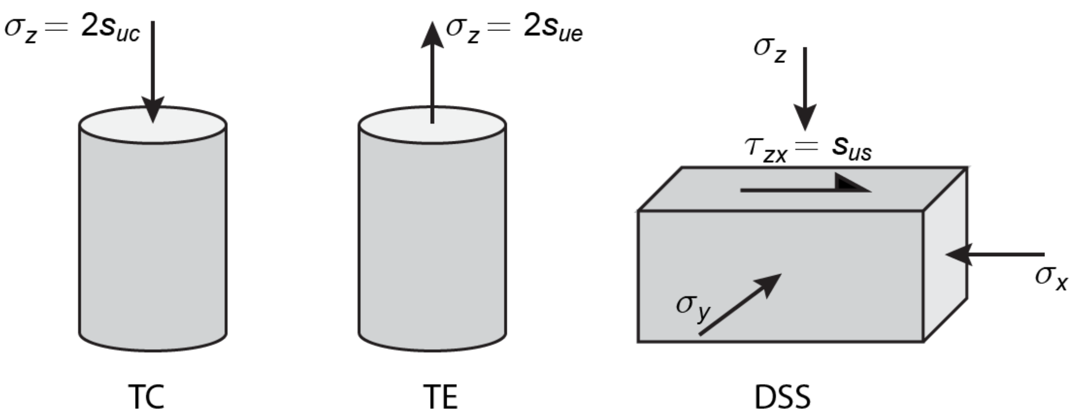
Strength predictions

Generalized Tresca:

s_{uc}: any value ≥ 0

s_{ue} : any value between $0.5s_{uc}$ and s_{uc}

$s_{us} = (0.5/s_{uc} + 0.5/s_{ue})^{-1}$



For example:

s_{uc} = 100 kPa

s_{ue} = 60 kPa

s_{us} = 75 kPa

Optum CE

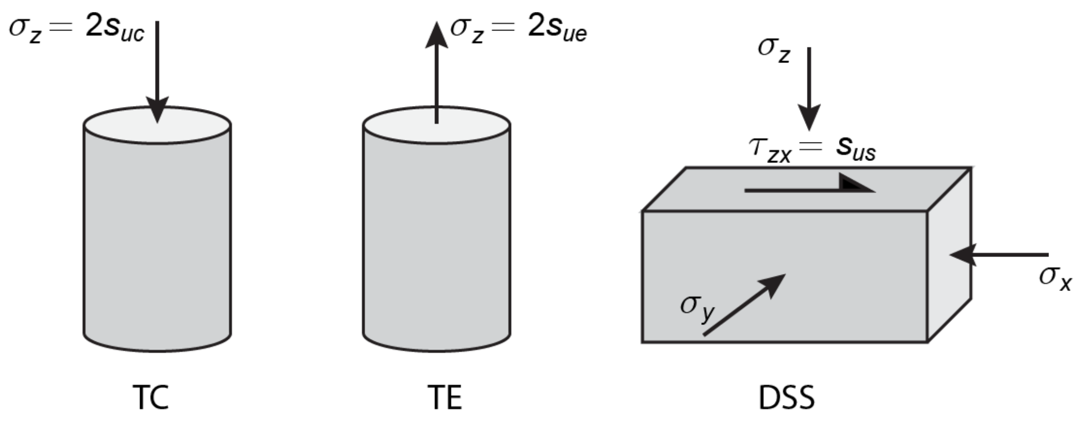
Strength predictions

Generalized Tresca:

s_{uc}: any value ≥ 0

s_{ue} : any value between $0.5s_{uc}$ and s_{uc}

$$\mathbf{s_{us} = (0.5/s_{uc} + 0.5/s_{ue})^{-1}}$$



For example:

s_{uc} = 100 kPa

s_{ue} = 60 kPa

$\mathbf{s_{us}}$ **= 75 kPa**

AUS

Strength predictions

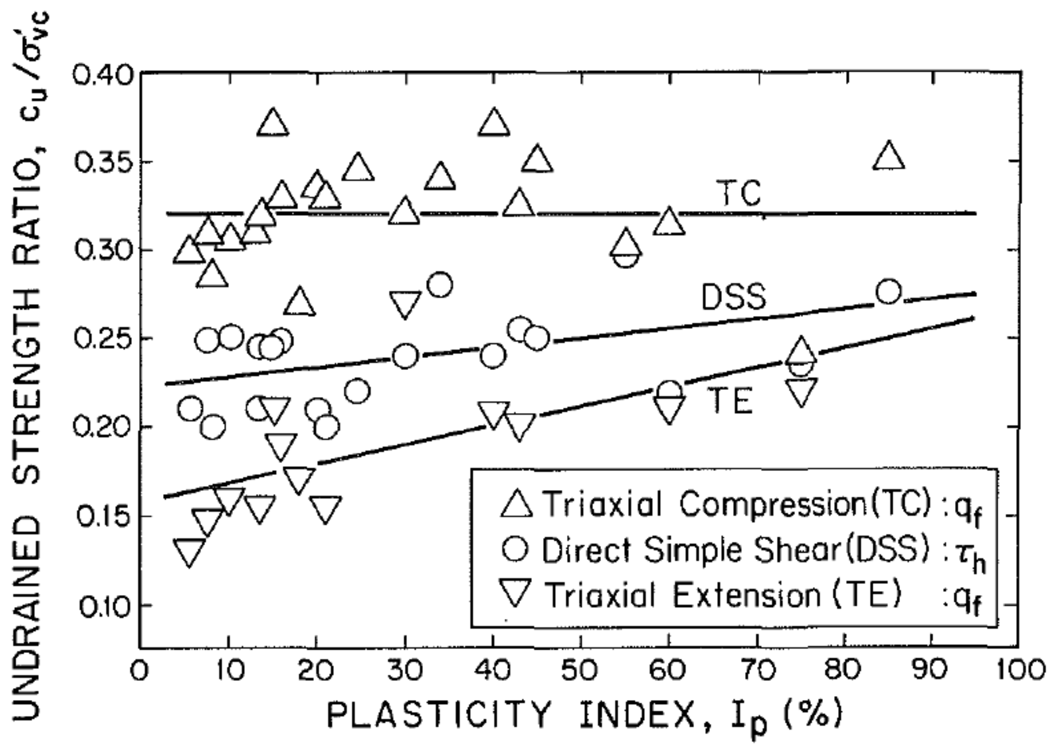


FIG. 15. Undrained Strength Anisotropy from CK_0U Tests on Normally Consolidated Clays and Silts [Data from Lefebvre et al. (1983); Vaid and Campanella (1974); and Various MIT and NGI Reports]

Ladd (1990)

Opt+um[CE]

Strength predictions

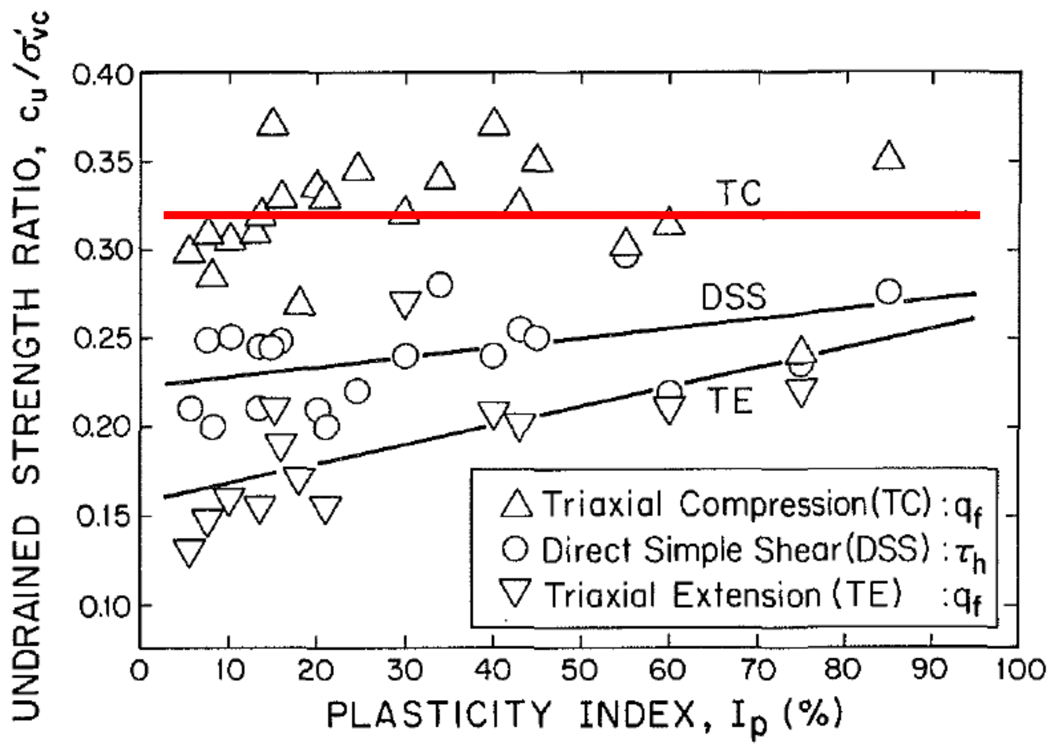


FIG. 15. Undrained Strength Anisotropy from CK_0U Tests on Normally Consolidated Clays and Silts [Data from Lefebvre et al. (1983); Vaid and Campanella (1974); and Various MIT and NGI Reports]

Ladd (1990)

Strength predictions

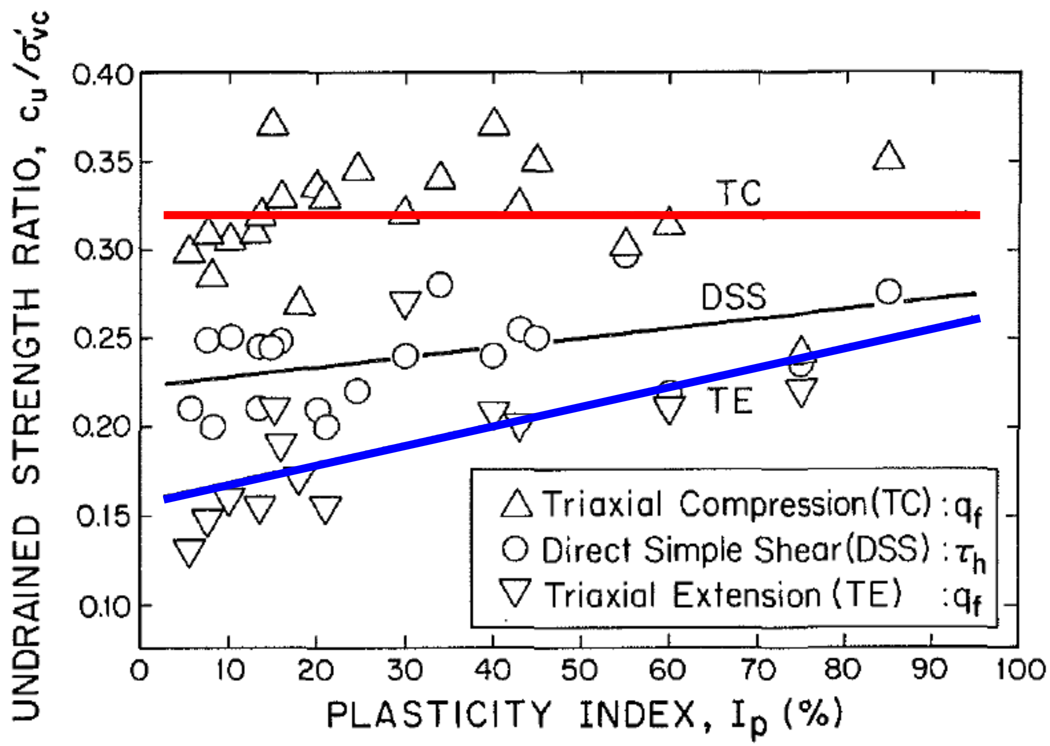


FIG. 15. Undrained Strength Anisotropy from CK_0U Tests on Normally Consolidated Clays and Silts [Data from Lefebvre et al. (1983); Vaid and Campanella (1974); and Various MIT and NGI Reports]

Ladd (1990)

Strength predictions

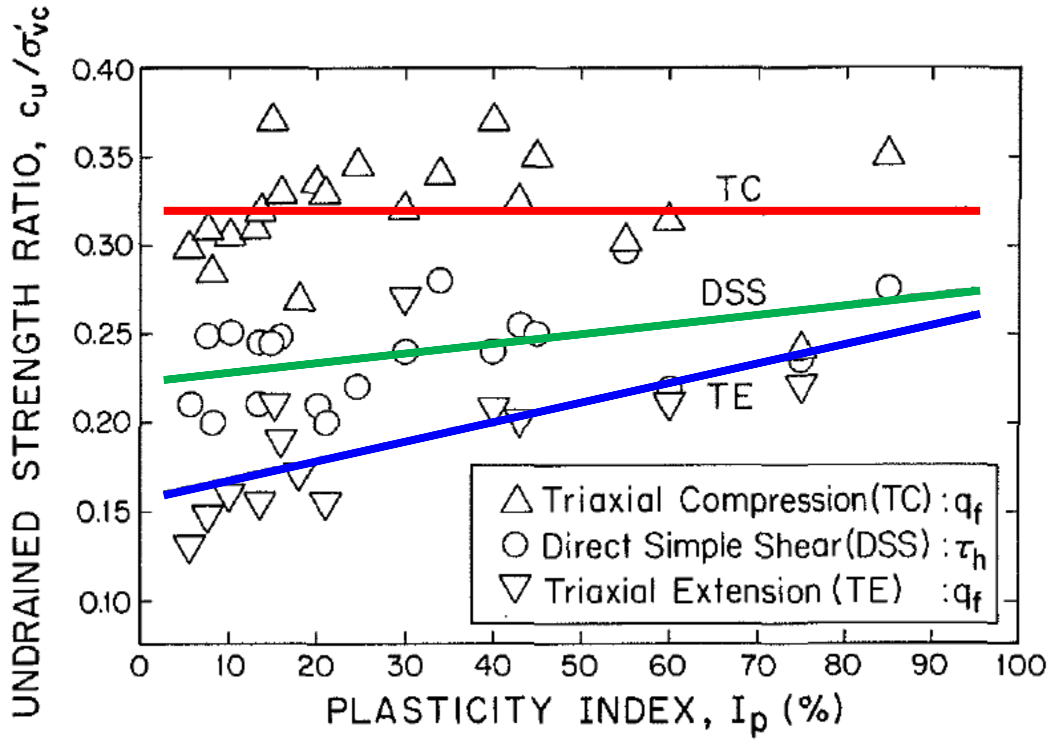


FIG. 15. Undrained Strength Anisotropy from CK_0U Tests on Normally Consolidated Clays and Silts [Data from Lefebvre et al. (1983); Vaid and Campanella (1974); and Various MIT and NGI Reports]

Ladd (1990)

Optum CE

Strength predictions

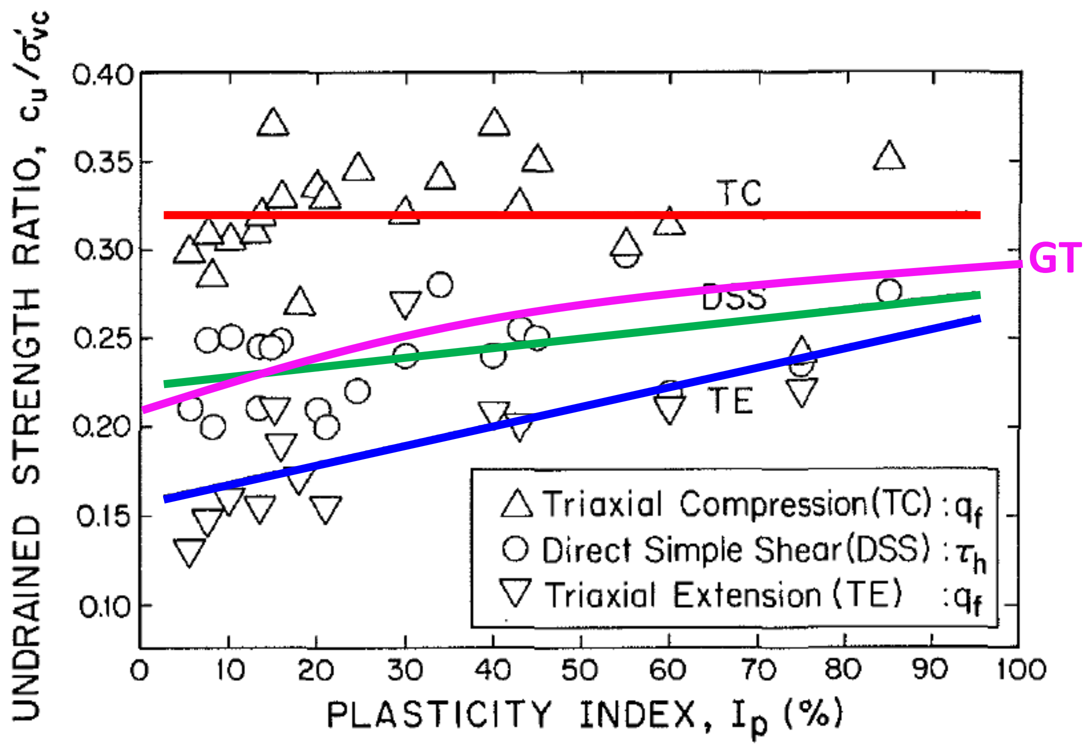
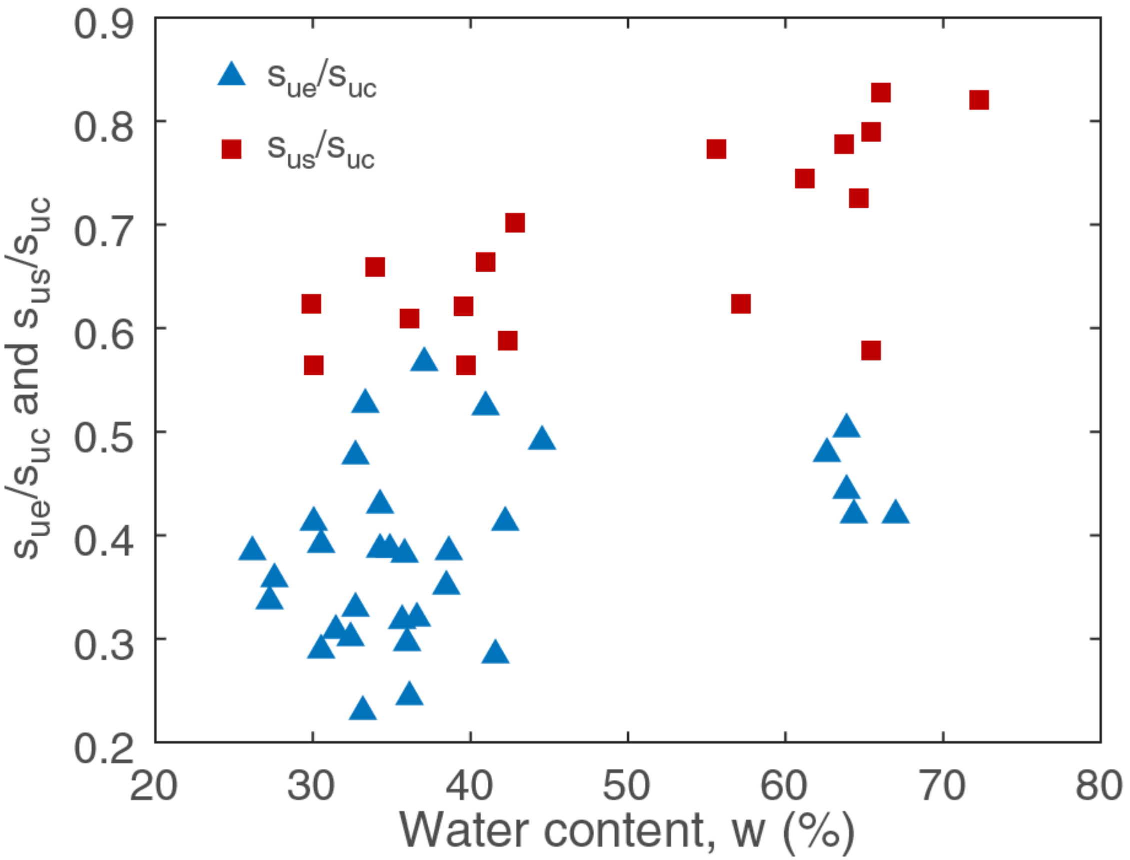


FIG. 15. Undrained Strength Anisotropy from CK_0U Tests on Normally Consolidated Clays and Silts [Data from Lefebvre et al. (1983); Vaid and Campanella (1974); and Various MIT and NGI Reports]

Ladd (1990)

Optum CE

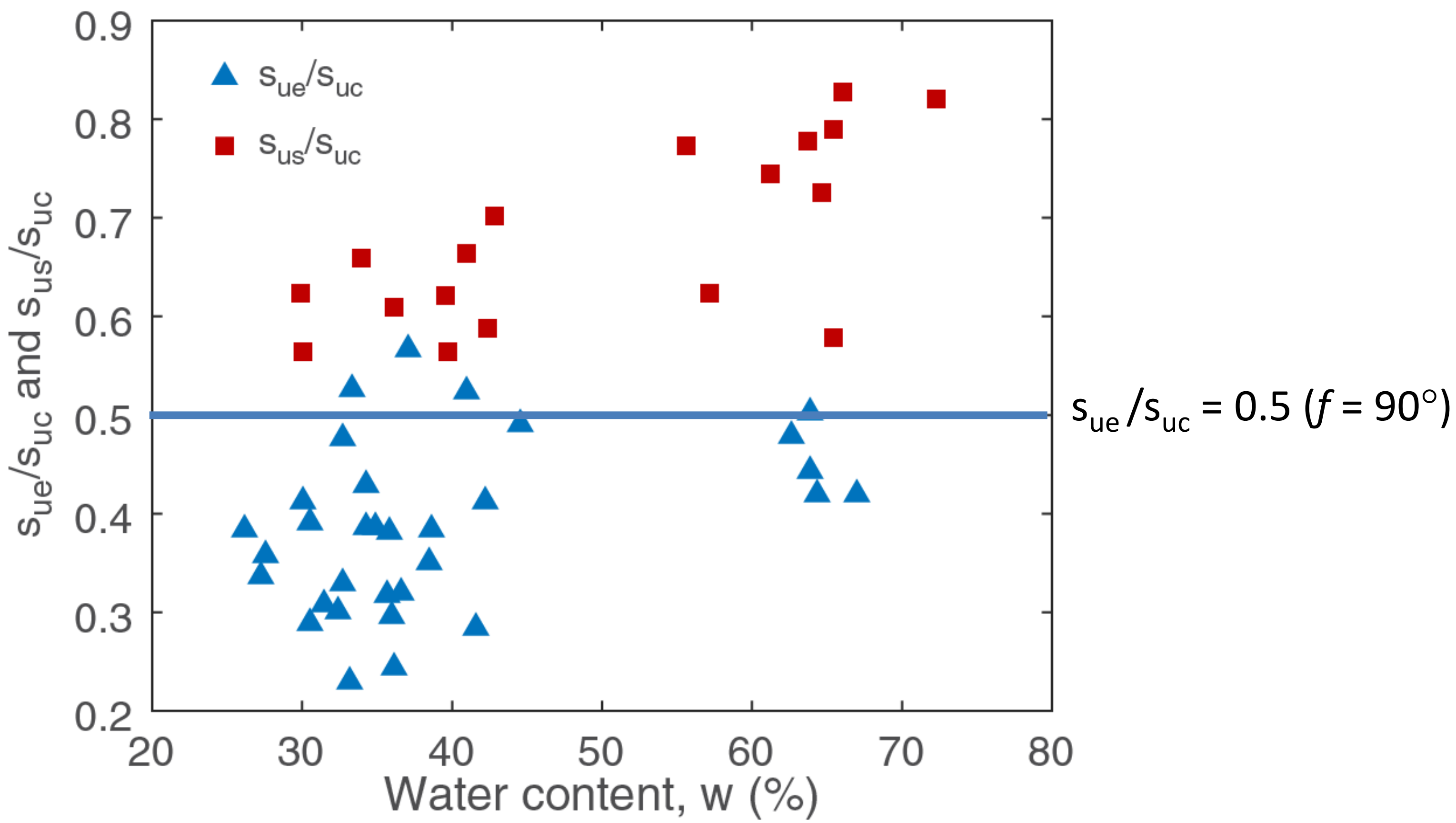
Strength predictions



Karlsrud & Hernandez-Martinez (2013)

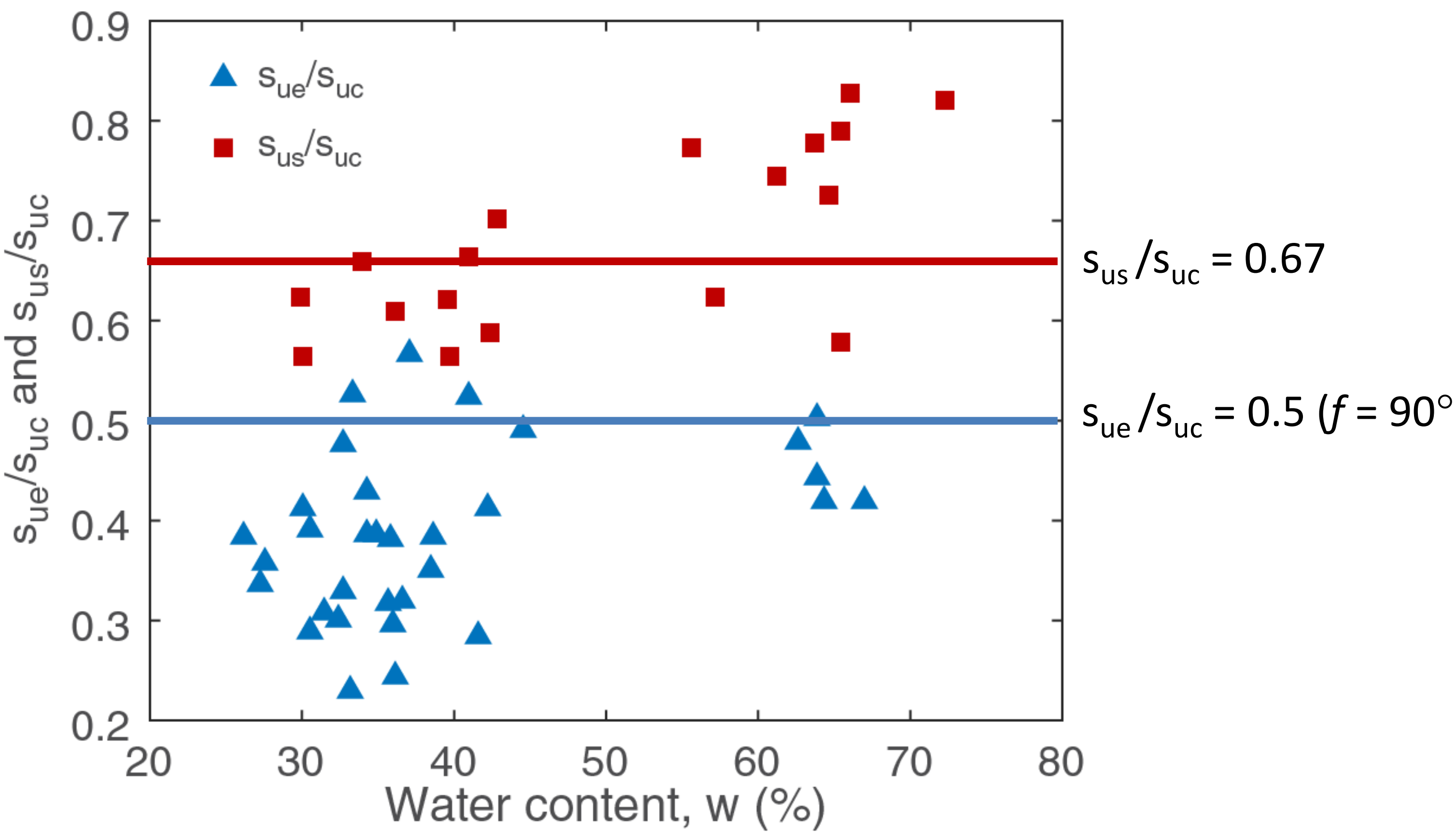
AUS

Strength predictions



Karlsrud & Hernandez-Martinez (2013)

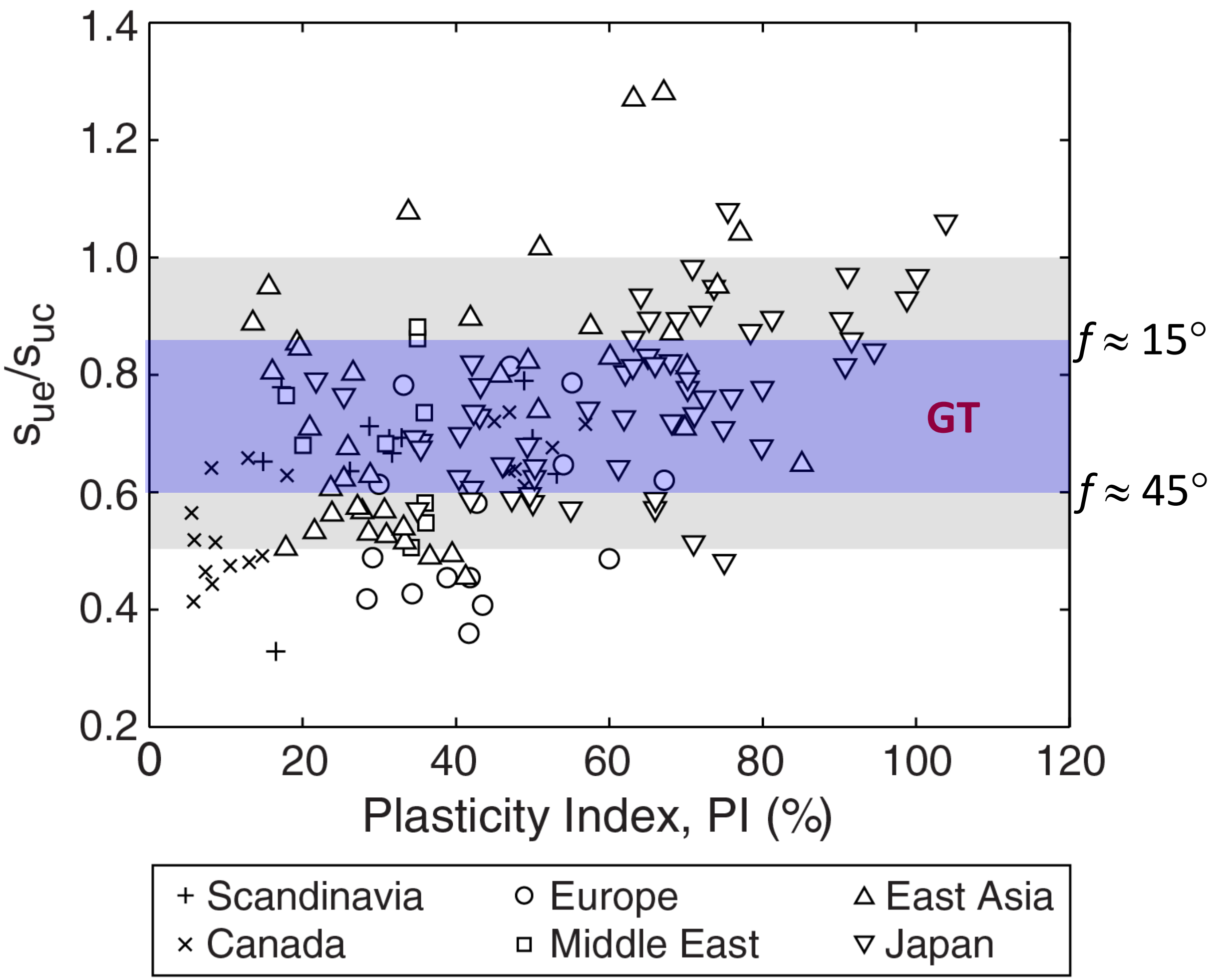
Strength predictions



Karlsrud & Hernandez-Martinez (2013)

Optum CE

Strength predictions



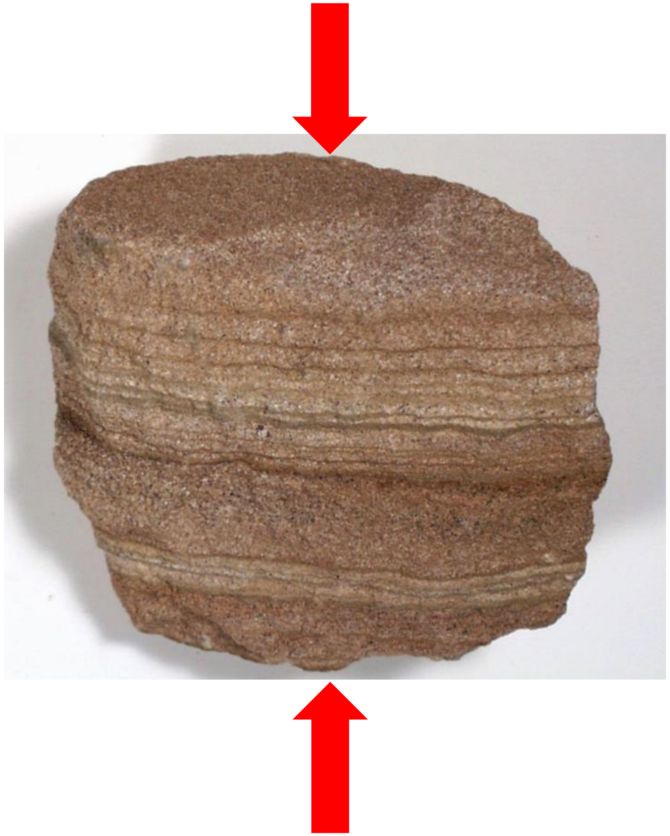
Anisotropy

- Considerable confusion in the soil mechanics literature
- Sometimes taken to mean different properties in different directions, e.g. vertically vs horizontally
- Sometimes taken to mean different strengths in extension and compression

Anisotropy

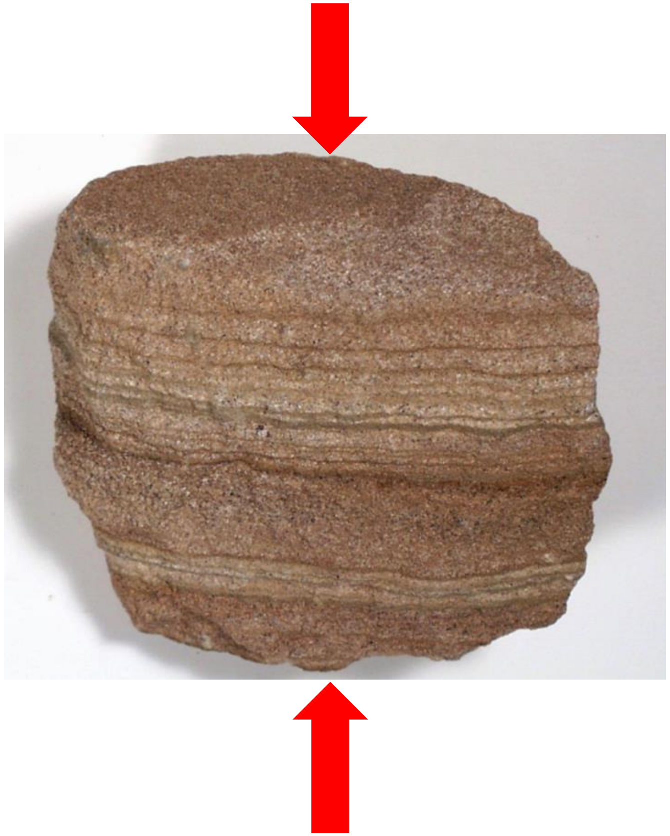
Anisotropy

Anisotropy



Anisotropy

Extension/compression



Extension/compression

Extension/compression

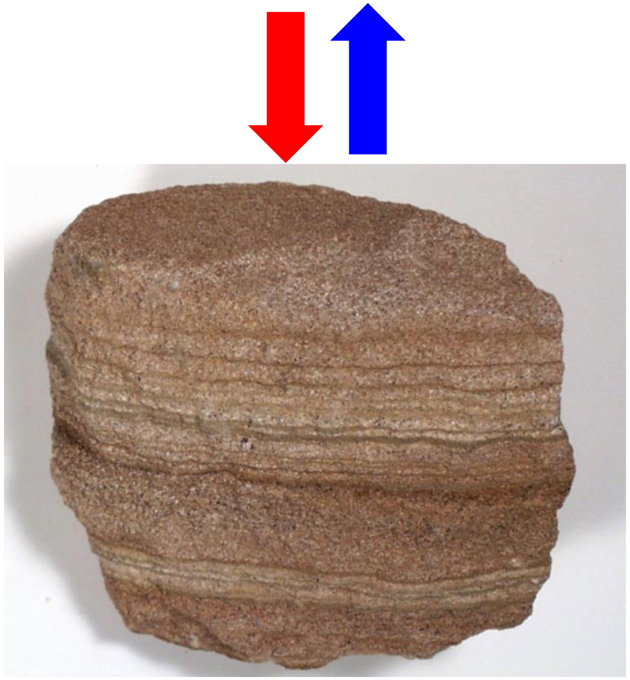
Lode angle: $\theta = -30°$

Extension/compression

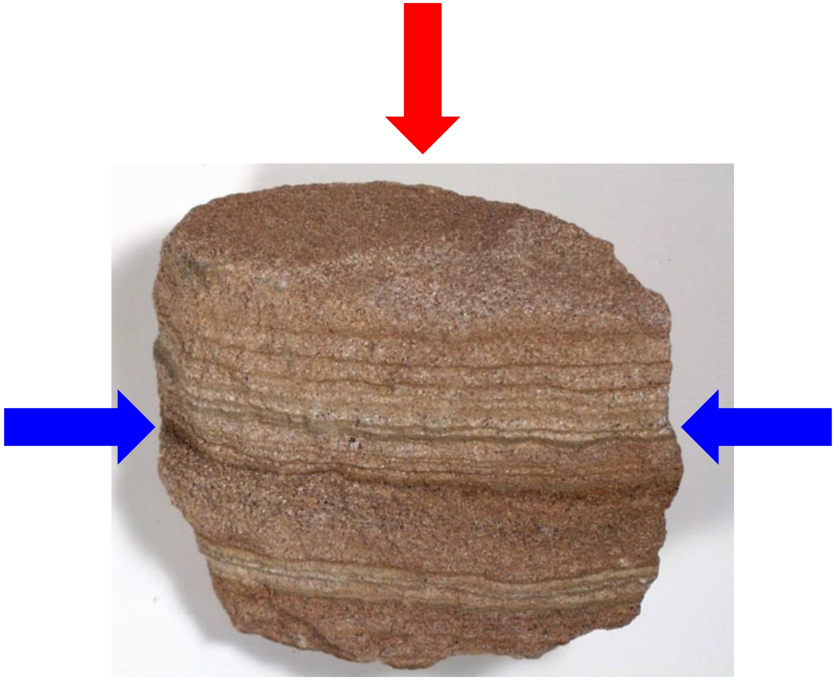
Lode angle: $\theta = +30°$

Summary

- Different strengths in extension and compression – standard feature of isotropic frictional materials
- Strength may depend on direction of load application, e.g. vertically vs horizontally – anisotropy



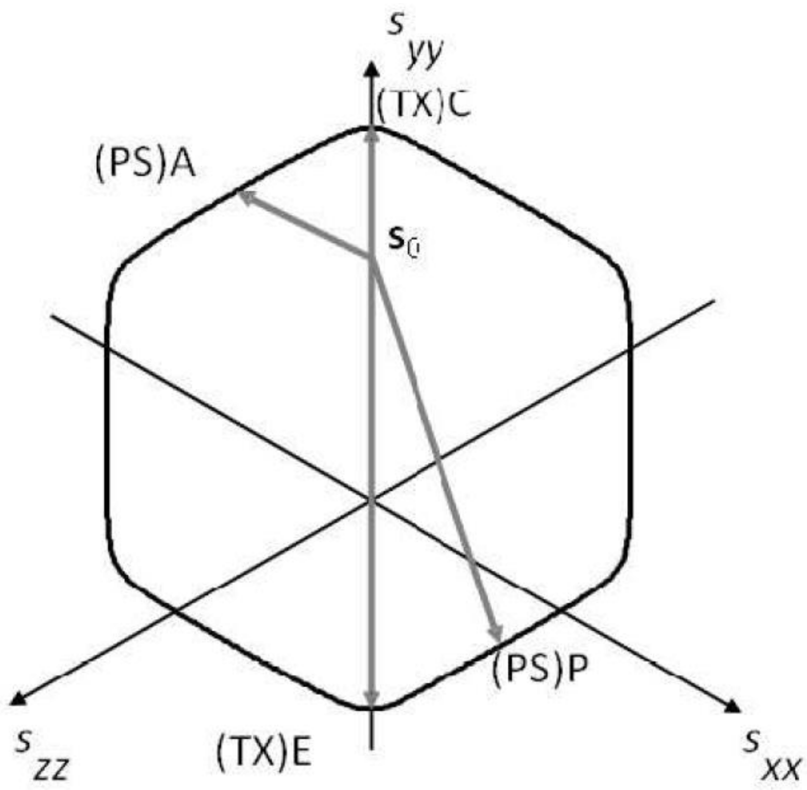
Extension/compression



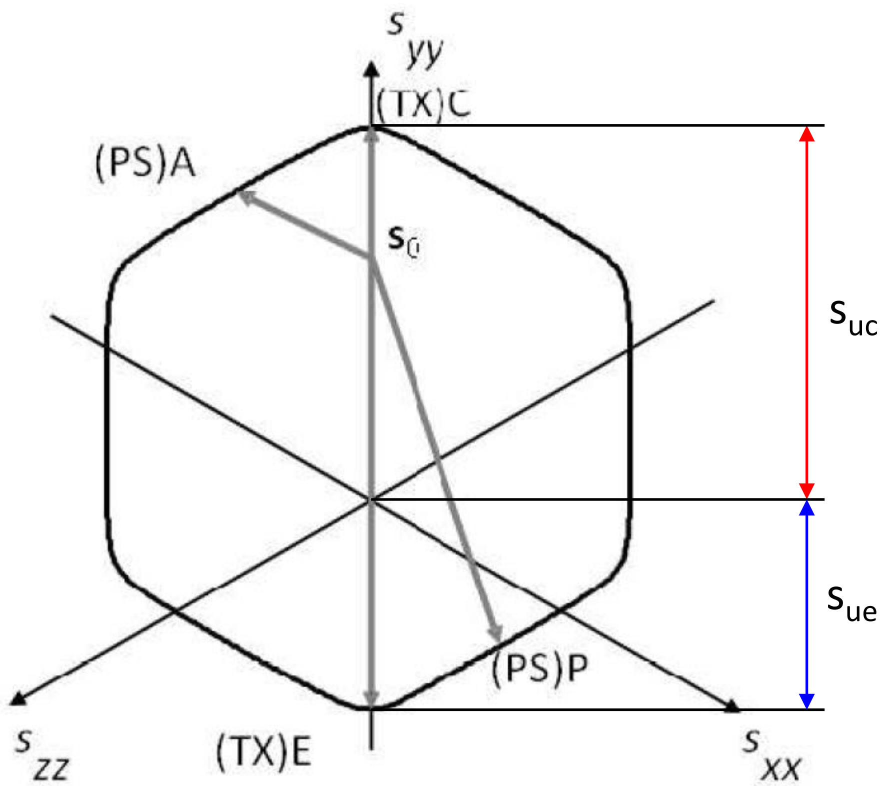
Anisotropy

Optum^CE

NGI-ADP (Plaxis)



NGI-ADP (Plaxis)

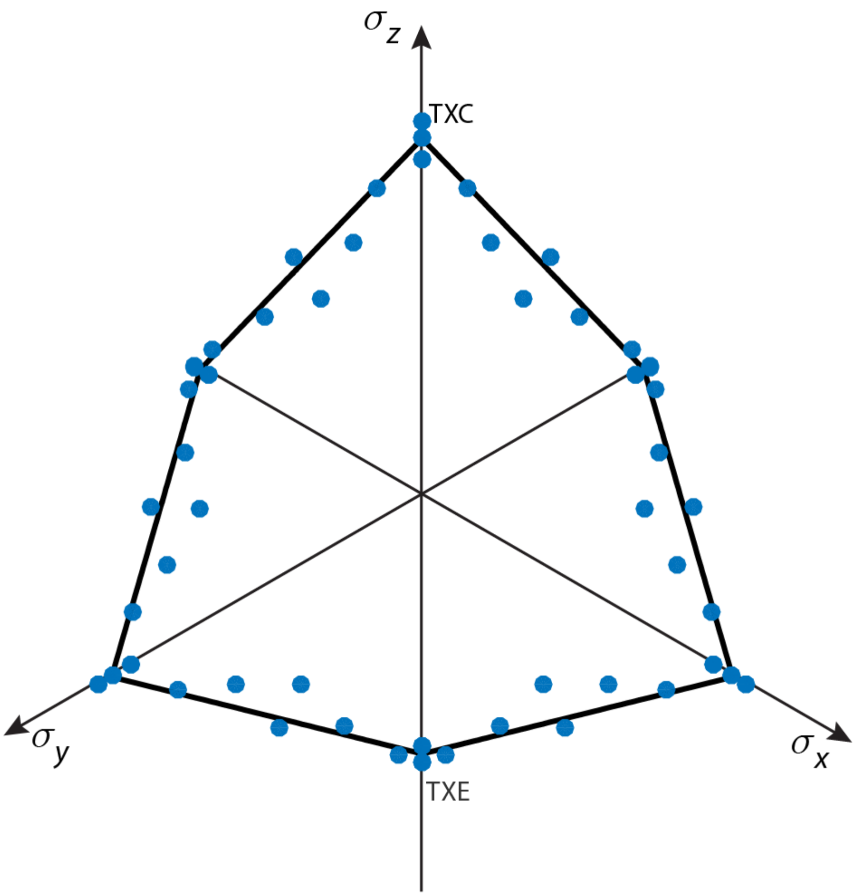


$s_{ue}/s_{uc} < 1$ for an ideal isotropic material!

Optum CE

Yield surface

Undrained true triaxial tests on NC Edgar Plastic Kaolinite (Lade 1990)

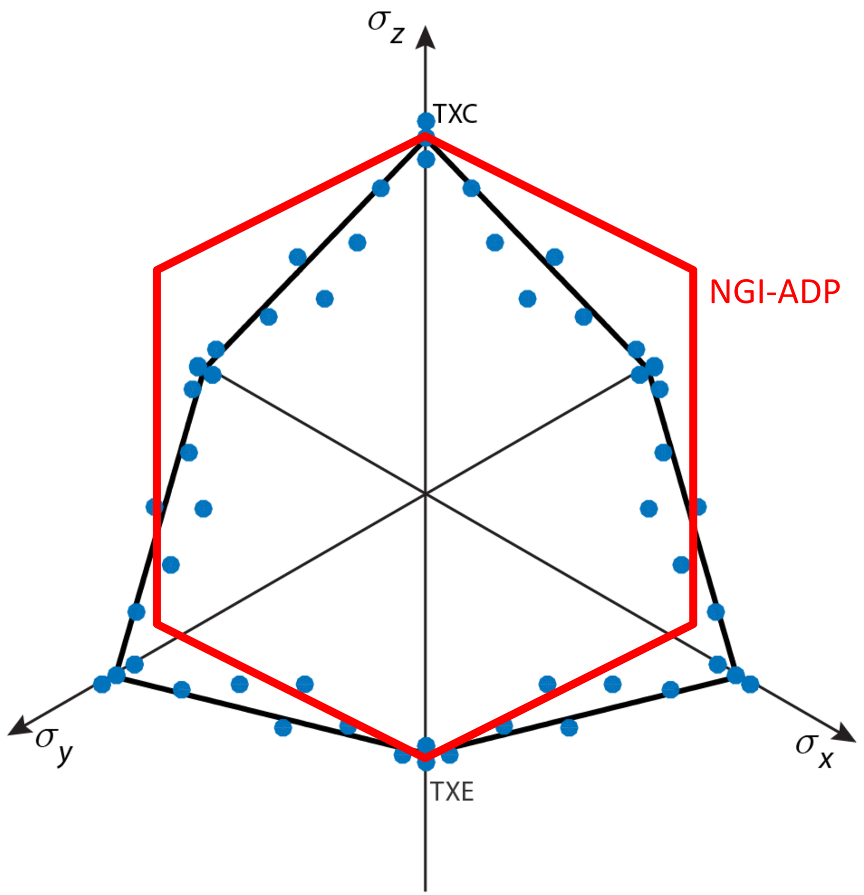


$s_{ue}/s_{uc} = 0.7$

Optum^CE

Yield surface

Undrained true triaxial tests on NC Edgar Plastic Kaolinite (Lade 1990)

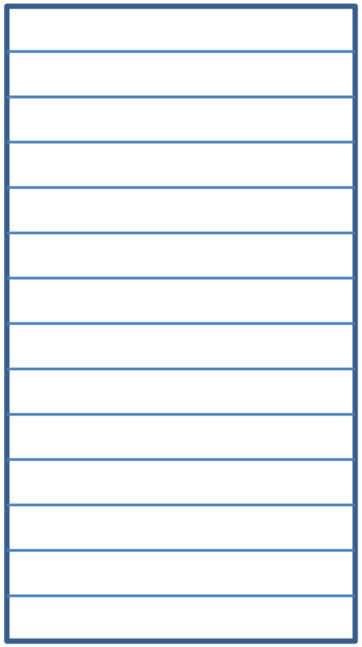


$s_{ue}/s_{uc} = 0.7$

AUS

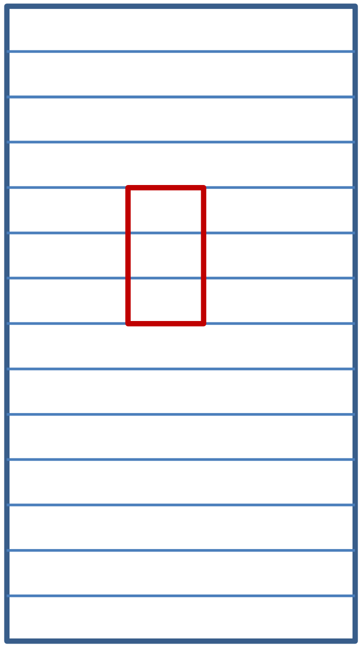
Anisotropy

Cross-anisotropy



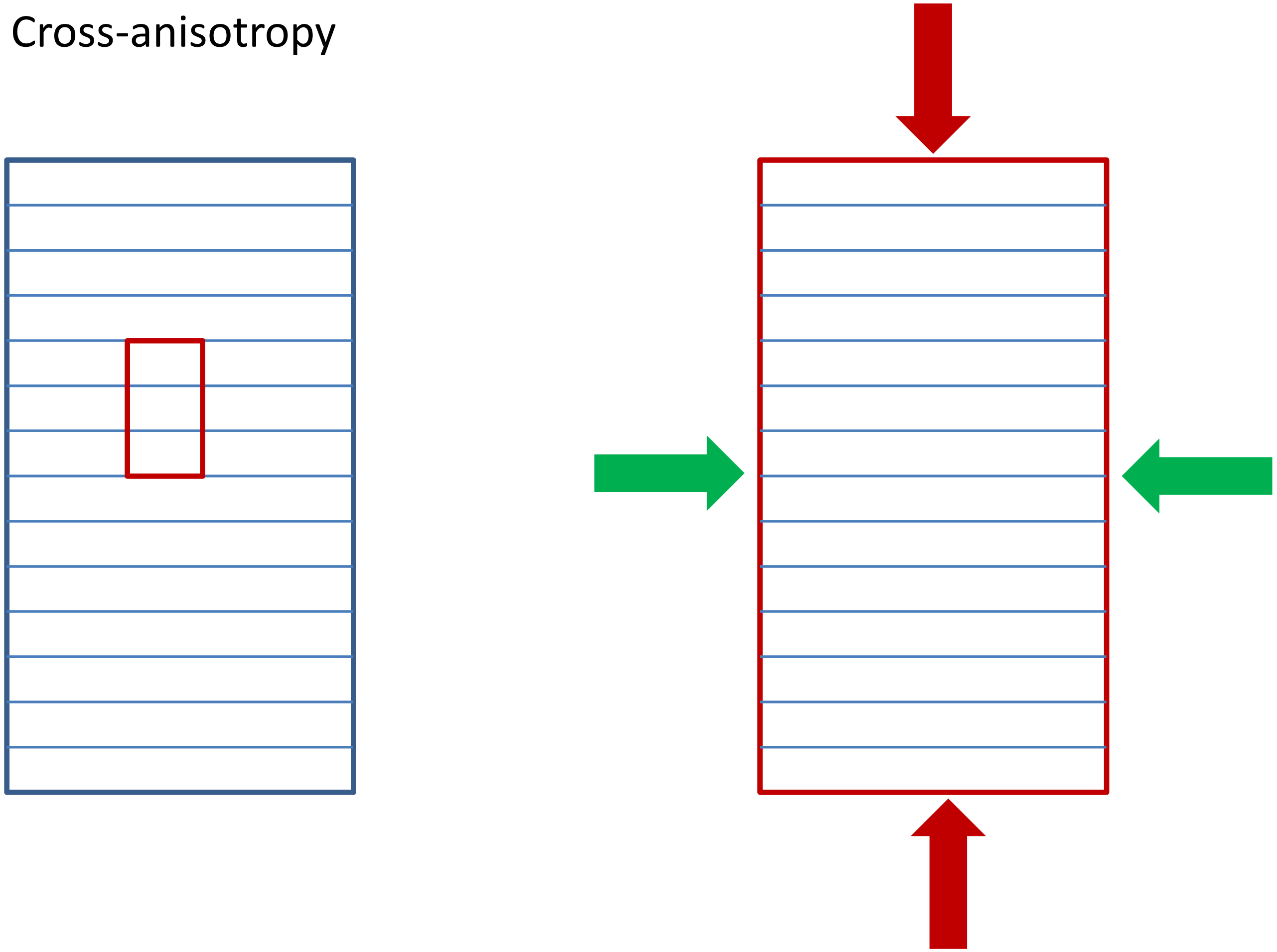
Anisotropy

Cross-anisotropy



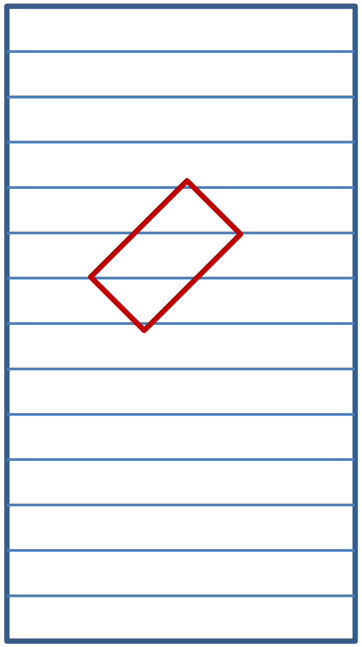
Anisotropy

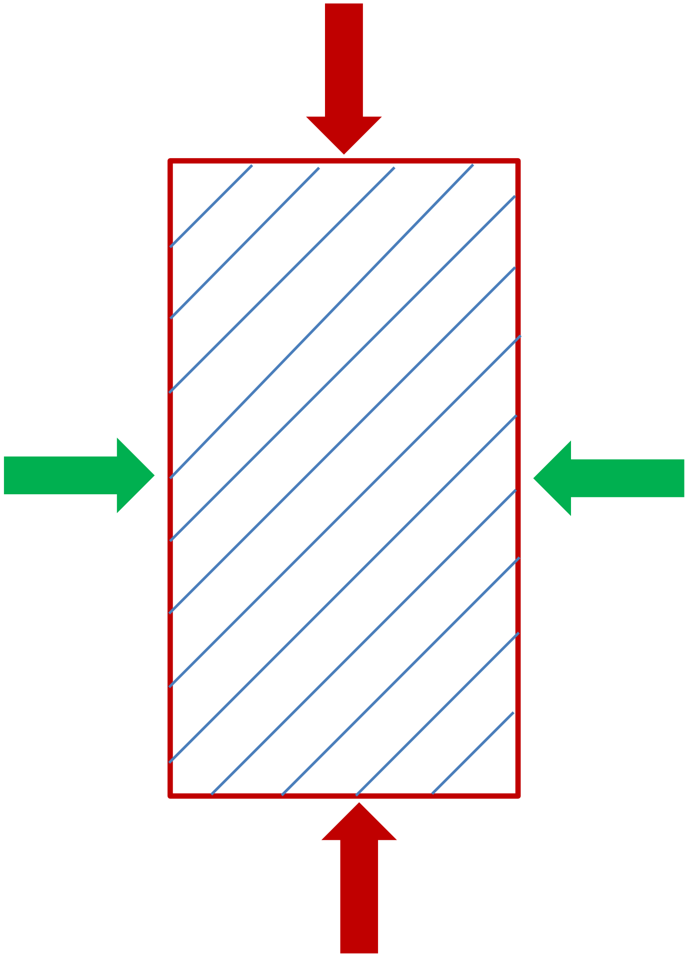
Cross-anisotropy



Anisotropy

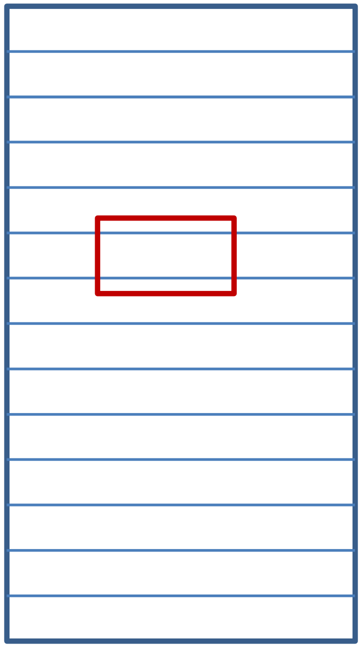
Cross-anisotropy

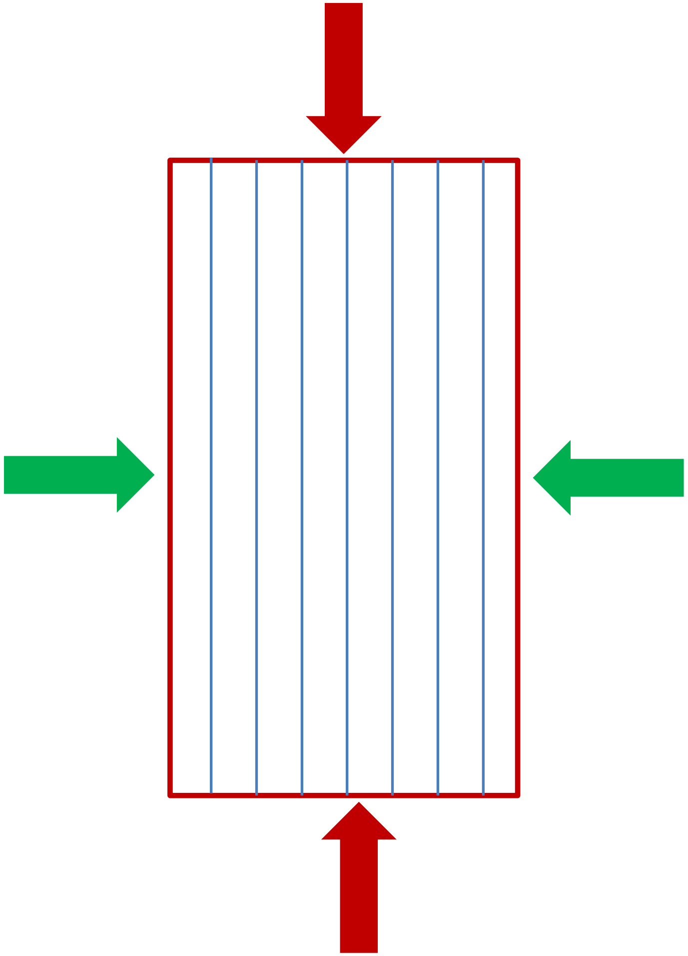




Anisotropy

Cross-anisotropy





Optum CE

Anisotropy

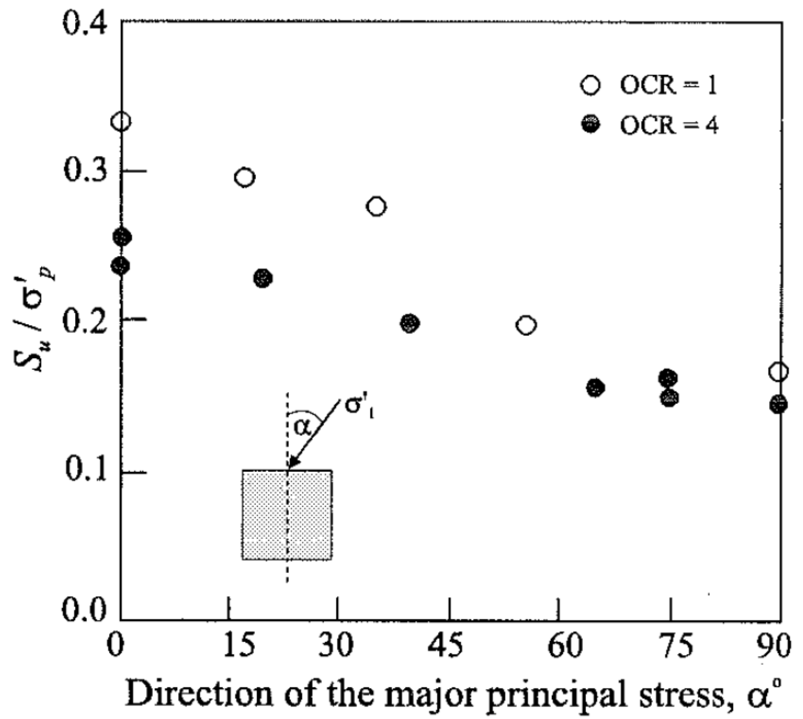
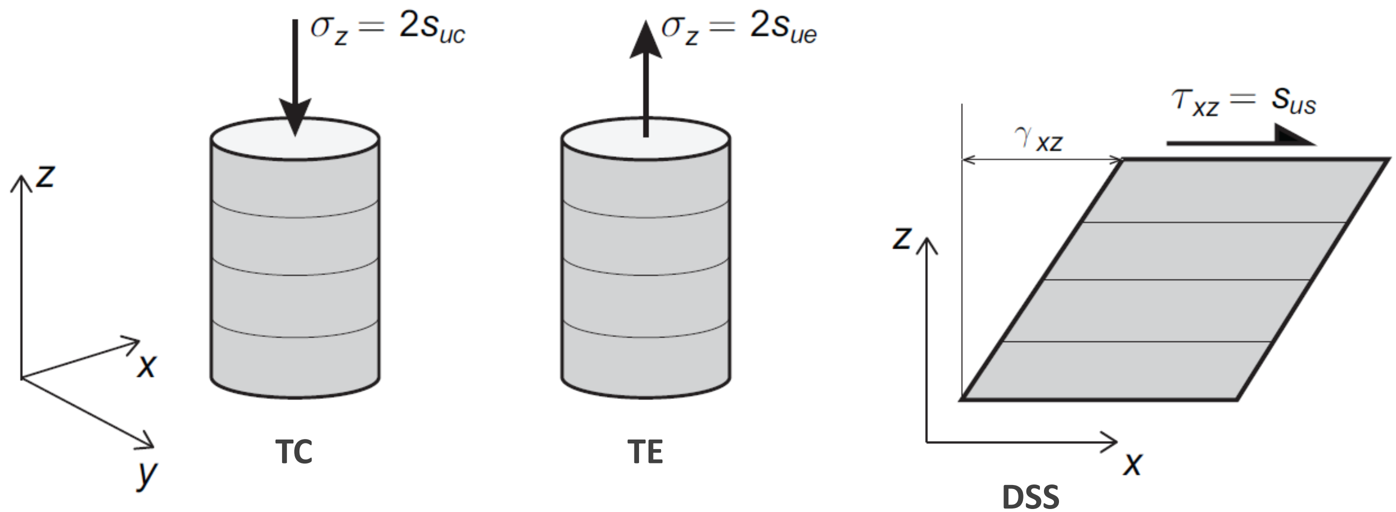


Figure 4.9: Undrained strength anisotropy of Boston Blue clay (Seah (1990))

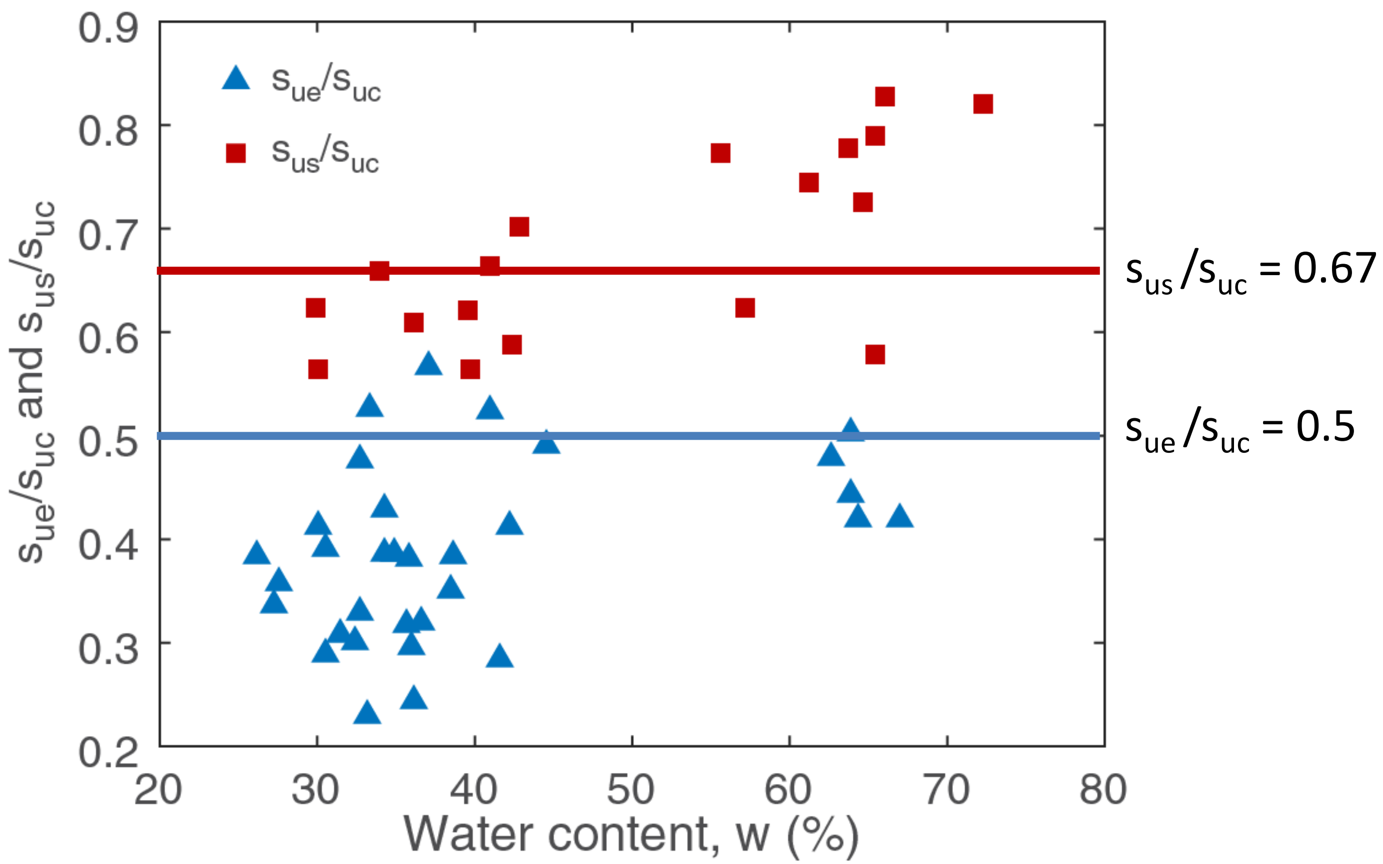
Anisotropy

Assume cross-anisotropy with plane of anisotropy being ortho to z-axis:



Optum CE

Strength predictions

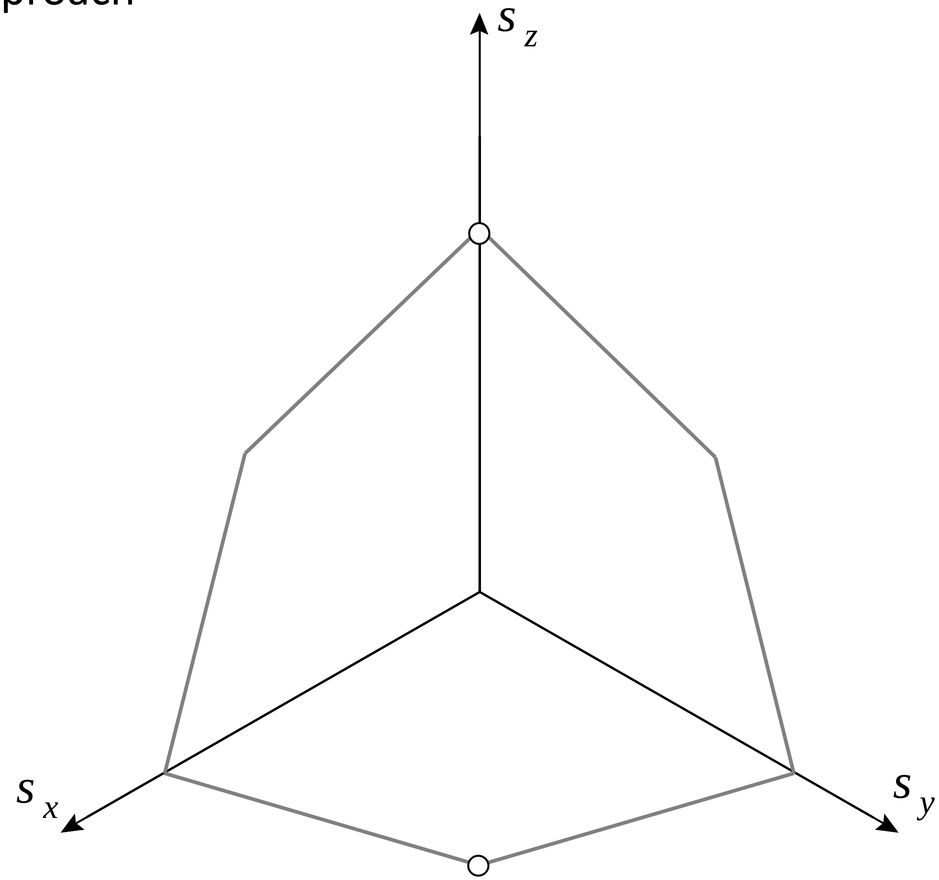


Karlsrud & Hernandez-Martinez (2013)

Optum CE

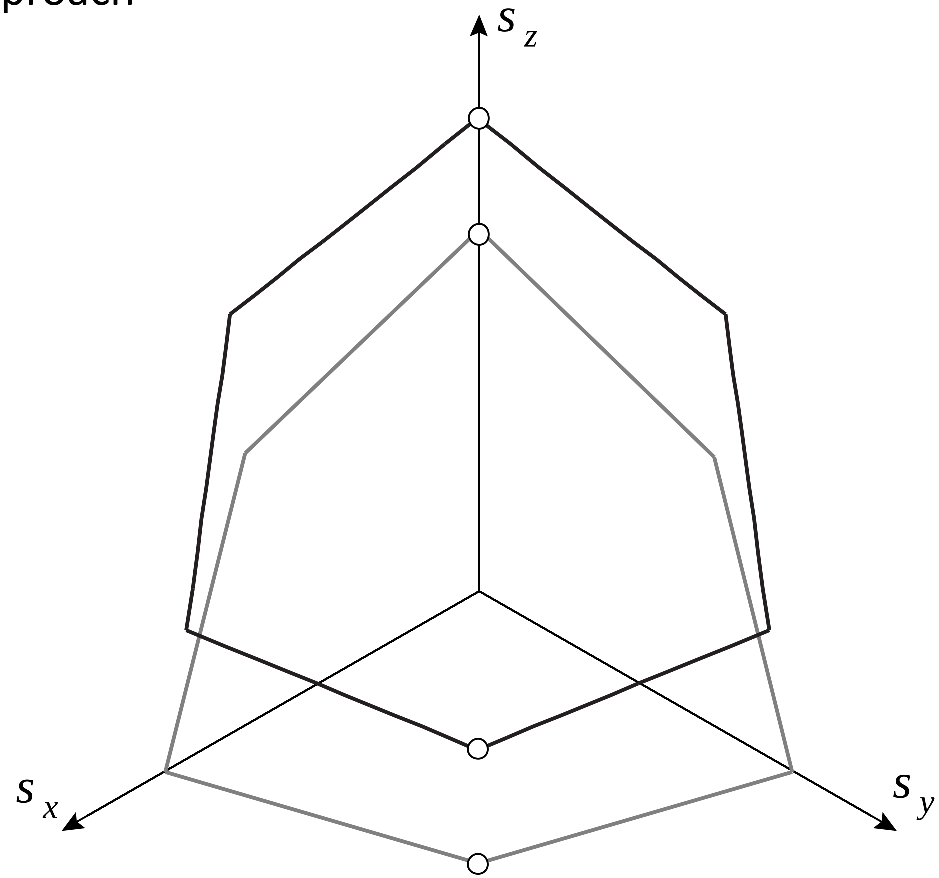
Anisotropy – AUS

NGI-ADP approach



Anisotropy – AUS

NGI-ADP approach

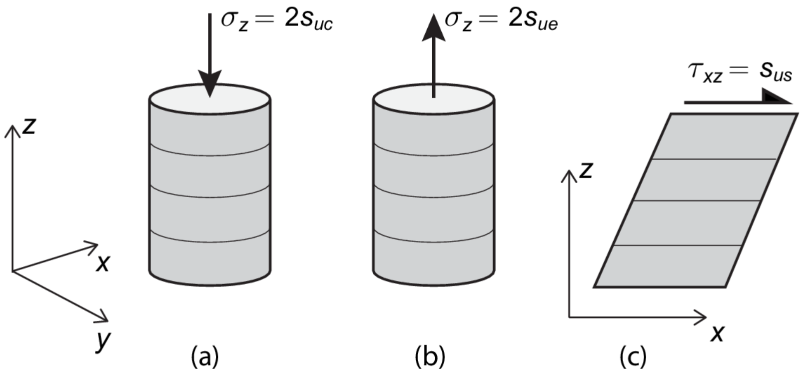


Optum^CE

Anisotropy – AUS

$$F_u = \hat{q} - \frac{6k}{\sqrt{3}(1+1/\rho)\cos\hat{\theta} - 3(1-1/\rho)\sin\hat{\theta}}$$

$$\hat{q} = \sqrt{3\hat{J}_2}$$
$$\hat{J}_2 = \tfrac{1}{2}\hat{\boldsymbol{s}}^{\mathsf{T}}\boldsymbol{D}\hat{\boldsymbol{s}}$$
$$\hat{\boldsymbol{s}} = \boldsymbol{\sigma} - \boldsymbol{m}p - ak\boldsymbol{r}$$
$$\boldsymbol{m} = (1,1,1,0,0,0)^{\mathsf{T}}$$
$$\boldsymbol{D} = \mathrm{diag}(1,1,1,2,2,2)^{\mathsf{T}}$$
$$p = \tfrac{1}{3}\boldsymbol{m}^{\mathsf{T}}\boldsymbol{\sigma}$$
$$\boldsymbol{r} = (\tfrac{1}{2},\tfrac{1}{2},-1,0,0,0)^{\mathsf{T}}$$
$$\hat{\theta} = \tfrac{1}{3}\arcsin\left(\frac{3\sqrt{3}}{2}\frac{\hat{J}_3}{\hat{J}_2^{3/2}}\right)$$
$$\hat{J}_3 = \hat{s}_{11}\hat{s}_{22}\hat{s}_{33} + 2\hat{s}_{12}\hat{s}_{23}\hat{s}_{31} - \hat{s}_{12}^2\hat{s}_{33} - \hat{s}_{23}^2\hat{s}_{11} - \hat{s}_{31}^2\hat{s}_{22}$$



Three parameters:

- k – size
- a - shift
- ρ – shape

Can be related uniquely to three strengths:

- s_{uc} – triaxial compression
- s_{ue} – triaxial extension
- s_{us} – simple shear

Optum^CE

Anisotropy – AUS

$$F_u = \hat{q} - \frac{6k}{\sqrt{3}(1+1/\rho)\cos\hat{\theta} - 3(1-1/\rho)\sin\hat{\theta}}$$

$$\hat{q} = \sqrt{3\hat{J}_2}$$

$$\hat{J}_2 = \tfrac{1}{2}\hat{\boldsymbol{s}}^{\mathsf{T}}\boldsymbol{D}\hat{\boldsymbol{s}}$$

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$$\boldsymbol{m} = (1,1,1,0,0,0)^{\mathsf{T}}$$

$$\boldsymbol{D} = \mathrm{diag}(1,1,1,2,2,2)^{\mathsf{T}}$$

$$p = \tfrac{1}{3}\boldsymbol{m}^{\mathsf{T}}\boldsymbol{\sigma}$$

$$\boldsymbol{r} = (\tfrac{1}{2}, \tfrac{1}{2}, -1, 0, 0, 0)^{\mathsf{T}}$$

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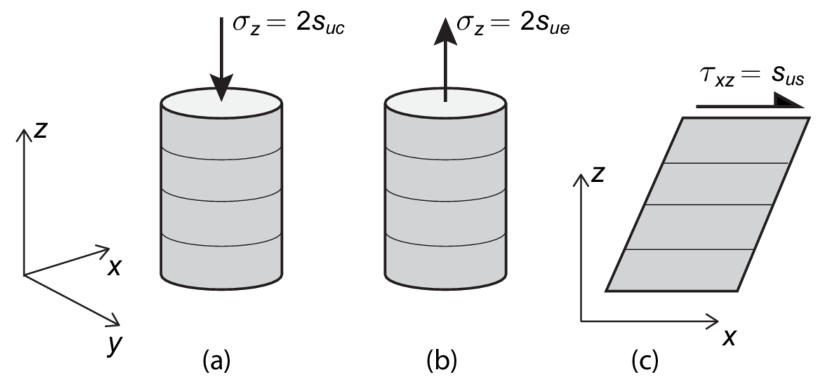

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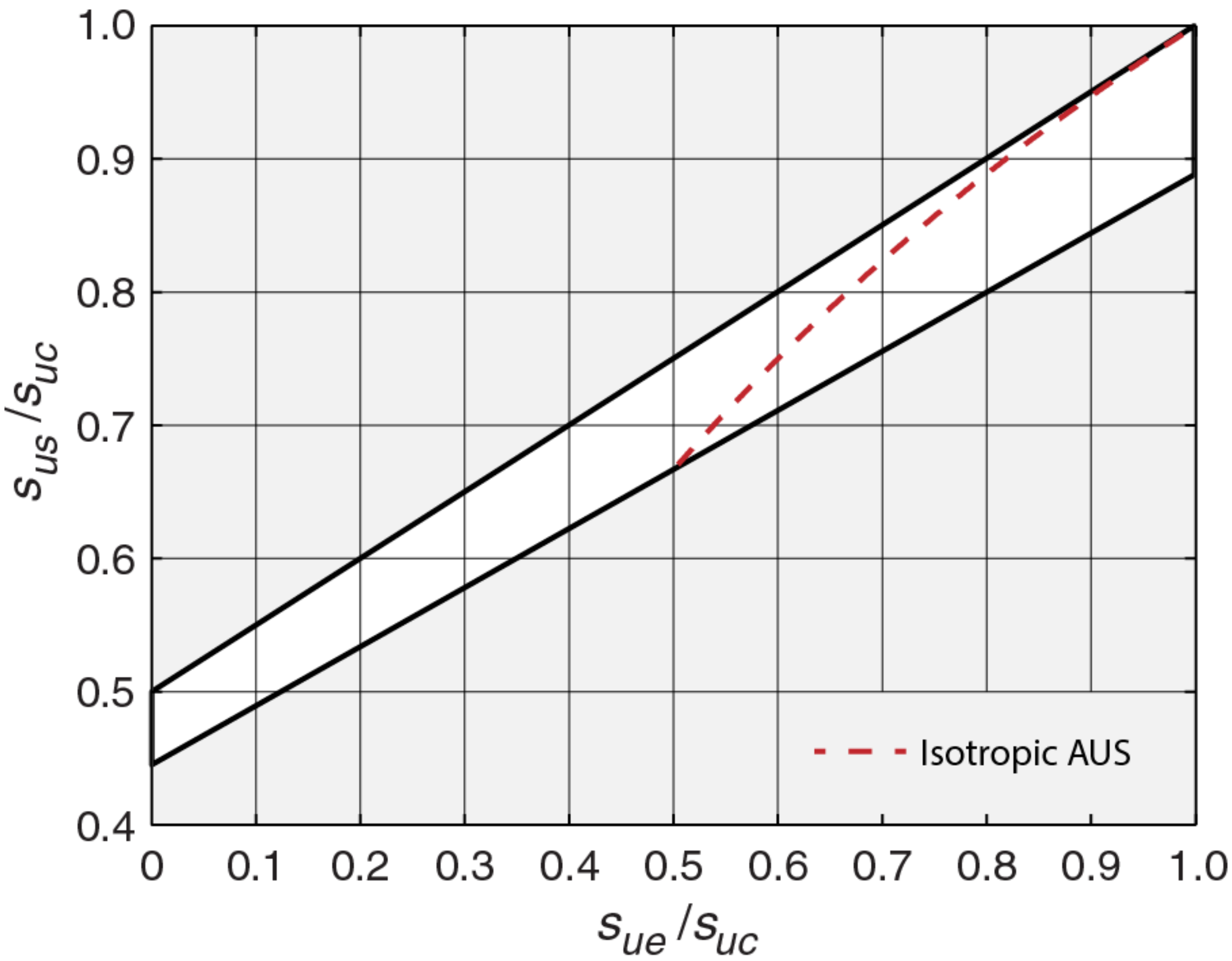
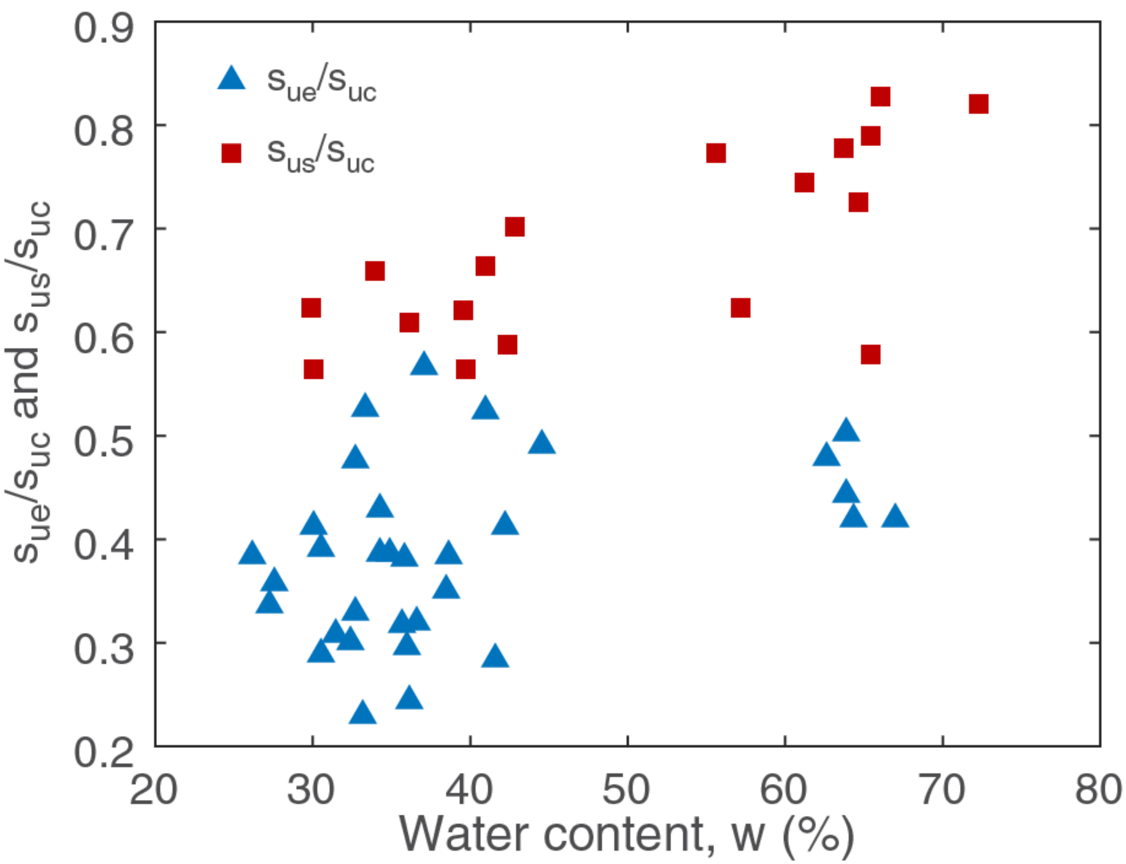
Optum CE

AUS admissibility



Optum^CE

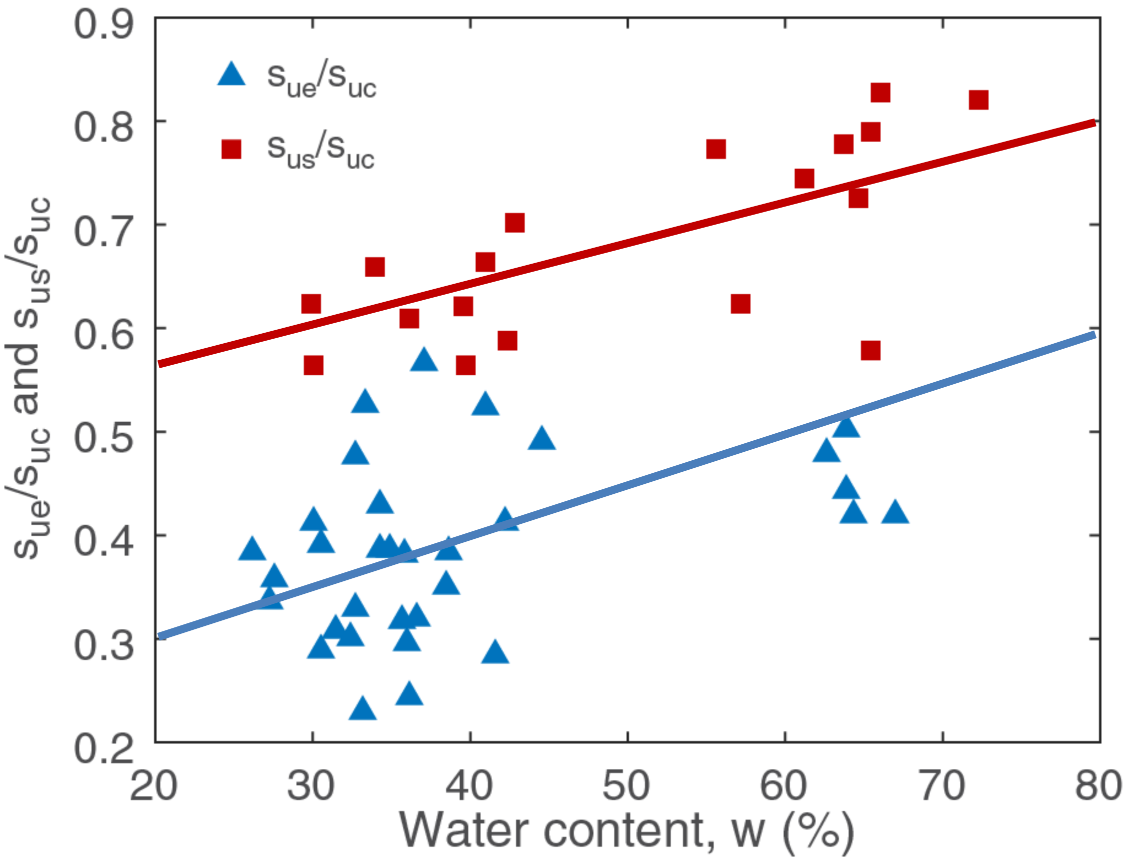
Strength predictions



Karlsrud & Hernandez-Martinez (2013)

Optum CE

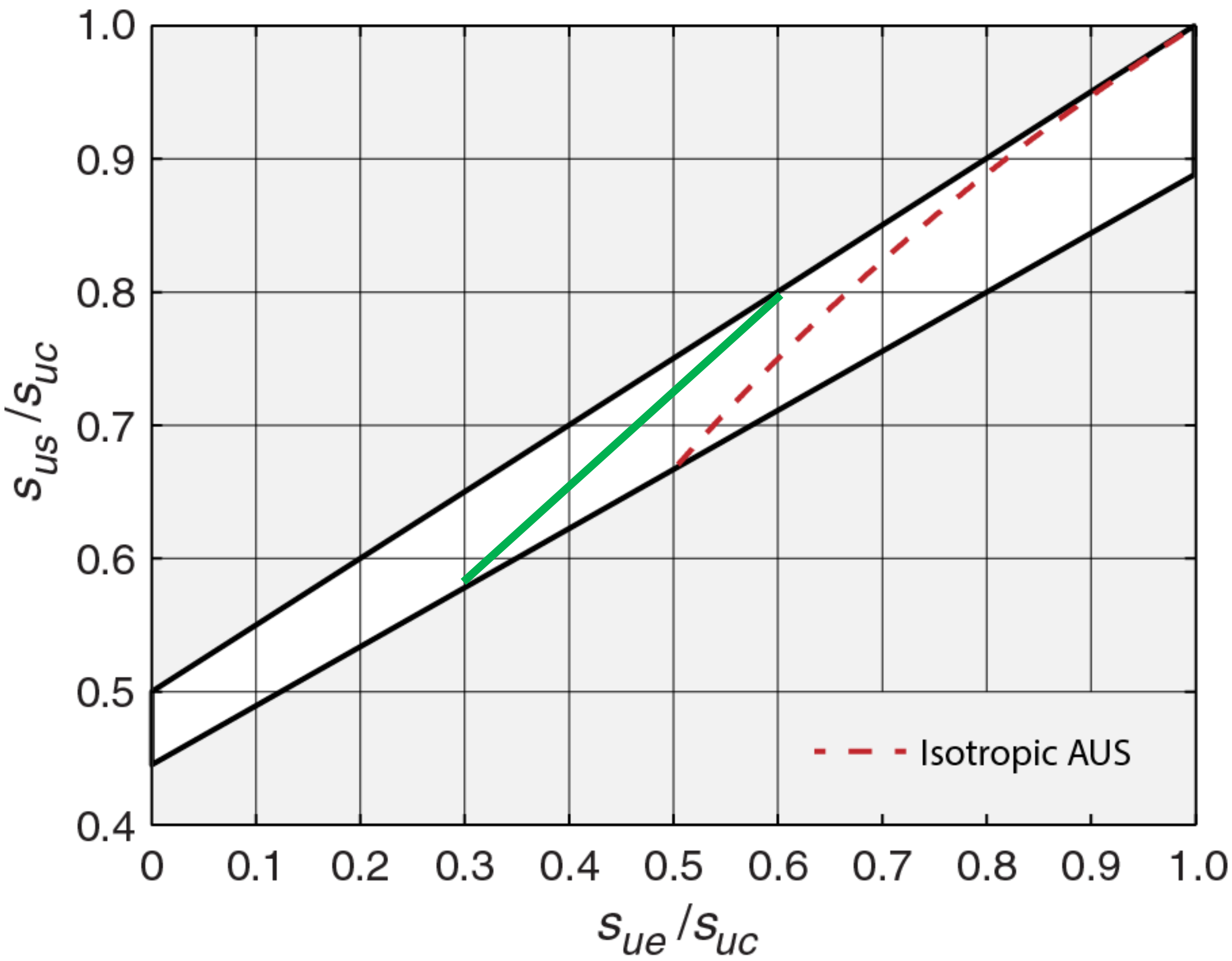
Strength predictions



Karlsrud & Hernandez-Martinez (2013)

AUS

AUS admissibility



AUS

Isotropic: s_{uc} and s_{ue}

Material	
Name	AUS Basic
Material Model	AUS
Color	*click to change*
Reducible Strength	Yes

Strength	
Option	Isotropic
s_{uc} (kPa)	30
s_{ue}/s_{uc}	0.6

$$s_{us} = (0.5/s_{uc} + 0.5/s_{ue})^{-1}$$

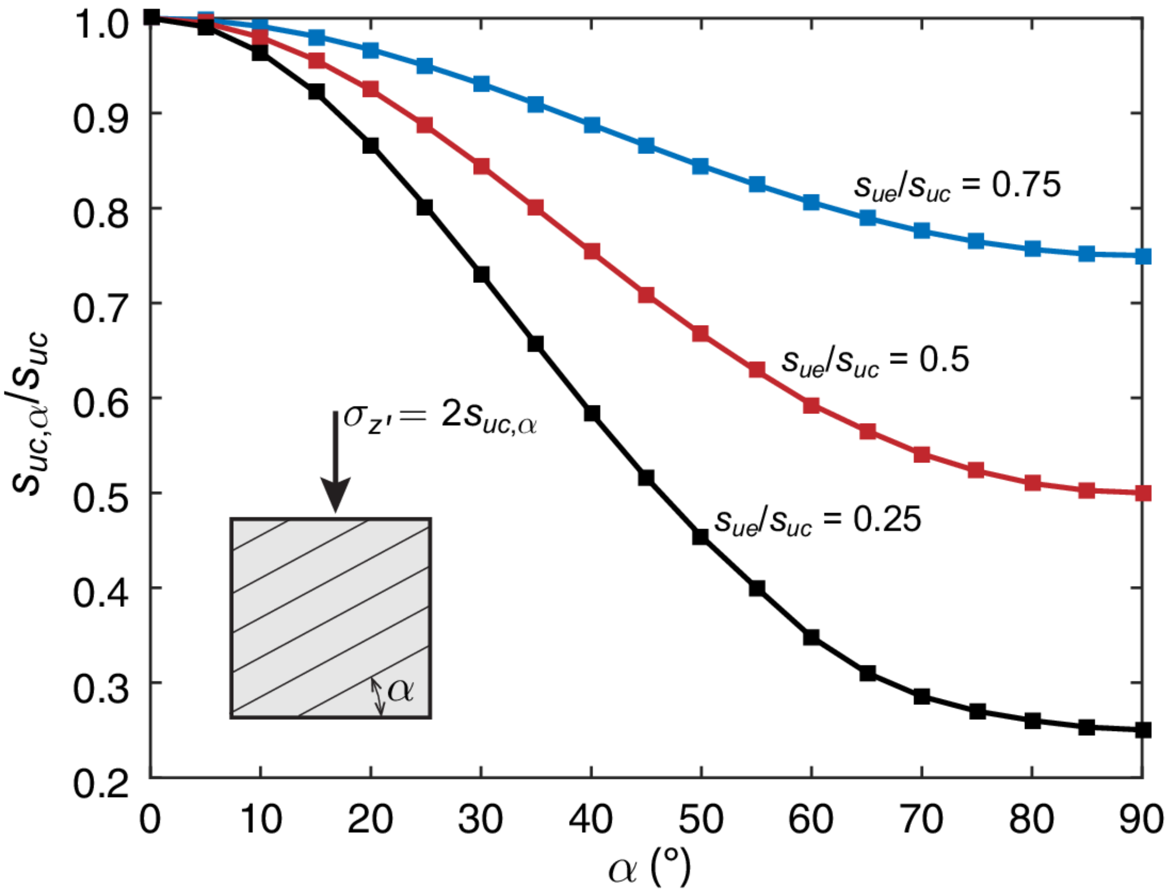
Anisotropic: s_{uc}, s_{ue}, and s_{us}

Material	
Name	AUS Basic
Material Model	AUS
Color	*click to change*
Reducible Strength	Yes

Strength	
Option	Anisotropic
s_{uc} (kPa)	30
s_{ue}/s_{uc}	0.6
s_{us}/s_{uc}	0.75

Anisotropy – AUS

$s_{us} = 0.5(s_{uc} + s_{ue})$



Anisotropy – AUS

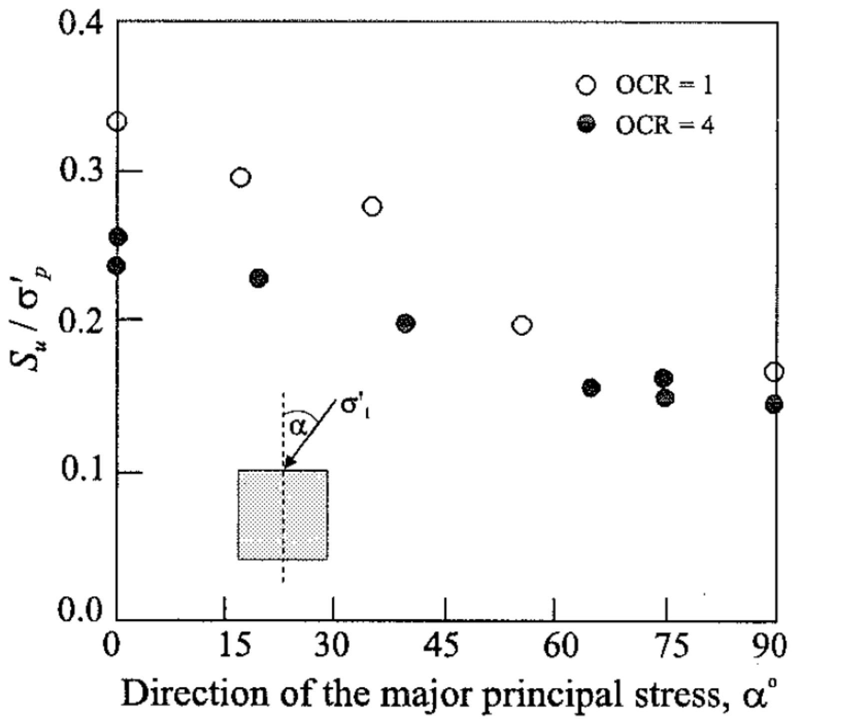
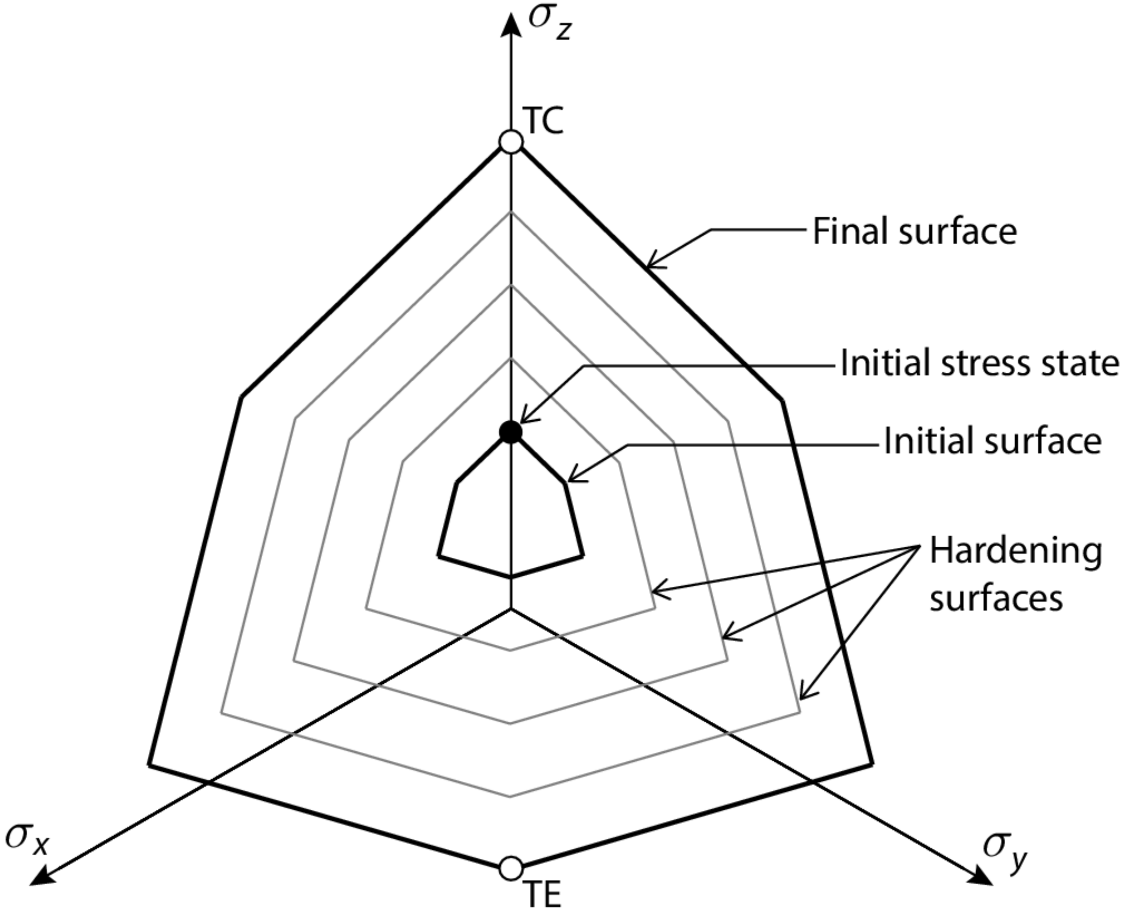


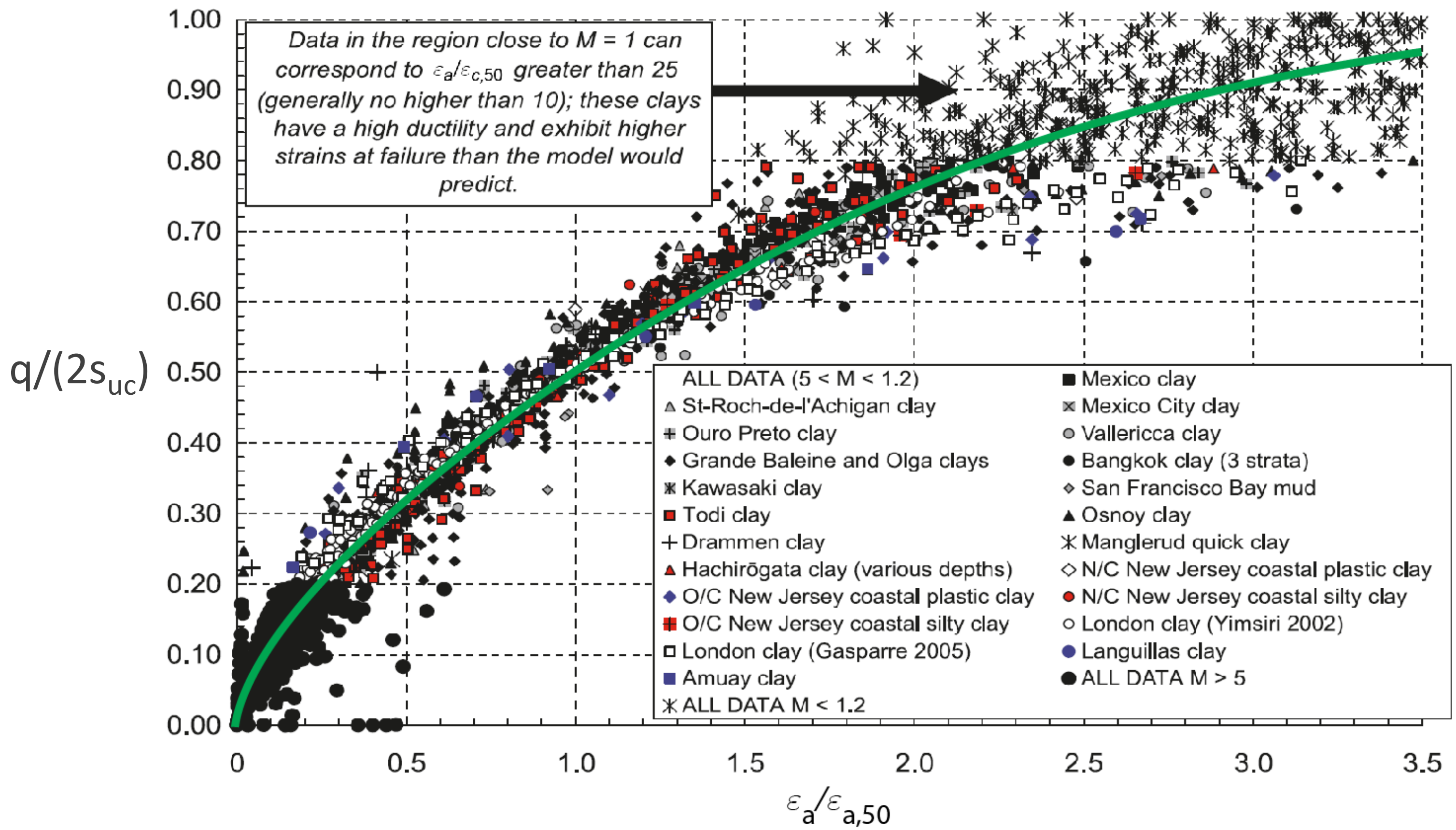
Figure 4.9: Undrained strength anisotropy of Boston Blue clay (Seah (1990))

Hardening – AUS



Hardening – AUS

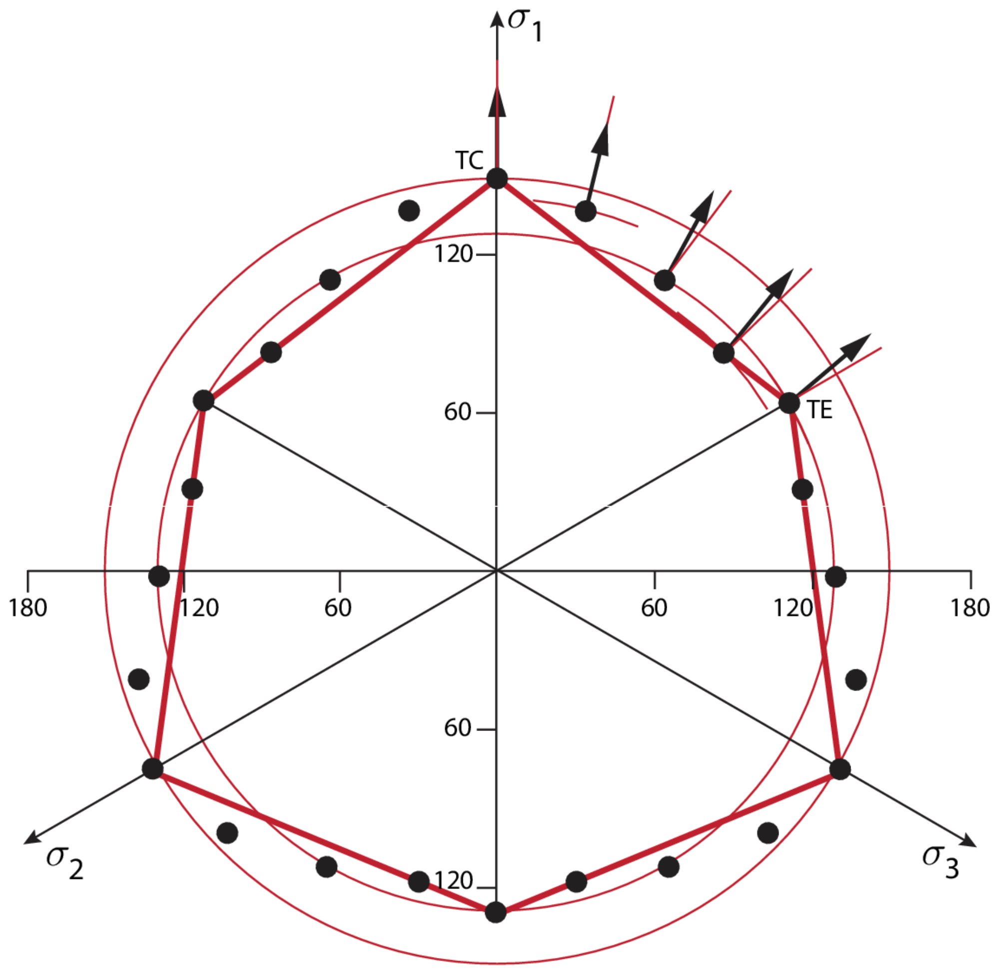
Vardanega & Bolton approach: strain at half the failure load as a hardening param.



Optum CE

Flow rule – AUS

Mises



Elasticity – Hooke

Shear modulus:

$$G \approx 10 G_{50} = 10 \frac{s_{uc}}{3 e_{c,50}}$$

Optum[CE]

Parameters

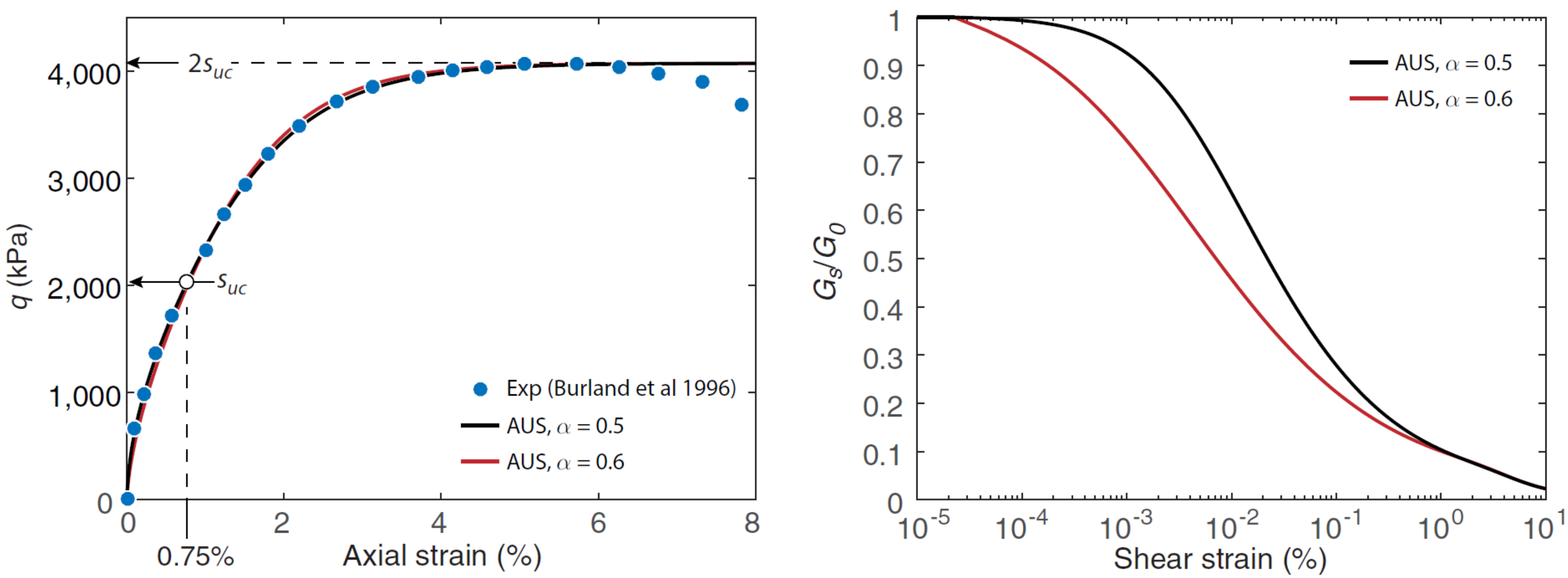
Material	
Name	AUS Basic
Material Model	AUS
Color	click to change
Reducible Strength	Yes
Strength	
Option	Isotropic
s_{uc} (kPa)	30
s_{ue}/s_{uc}	0.6
Unit Weight	
γ (kN/m^3)	18
Tension Cut-Off	
Tension Cut-Off	No
Stiffness	
Parameter Set	A
E_u (MPa)	30
$\varepsilon_{c,50}$ (%)	0.5
$\varepsilon_{e,50}$ (%)	2
Initial Conditions	
K_0	0.5

Material	
Name	AUS Basic
Material Model	AUS
Color	click to change
Reducible Strength	Yes
Strength	
Option	Anisotropic
s_{uc} (kPa)	30
s_{ue}/s_{uc}	0.6
s_{us}/s_{uc}	0.75
Unit Weight	
γ (kN/m^3)	18
Tension Cut-Off	
Tension Cut-Off	No
Stiffness	
Parameter Set	A
E_u (MPa)	30
$\varepsilon_{c,50}$ (%)	0.5
$\varepsilon_{e,50}$ (%)	2
Initial Conditions	
K_0	0.5

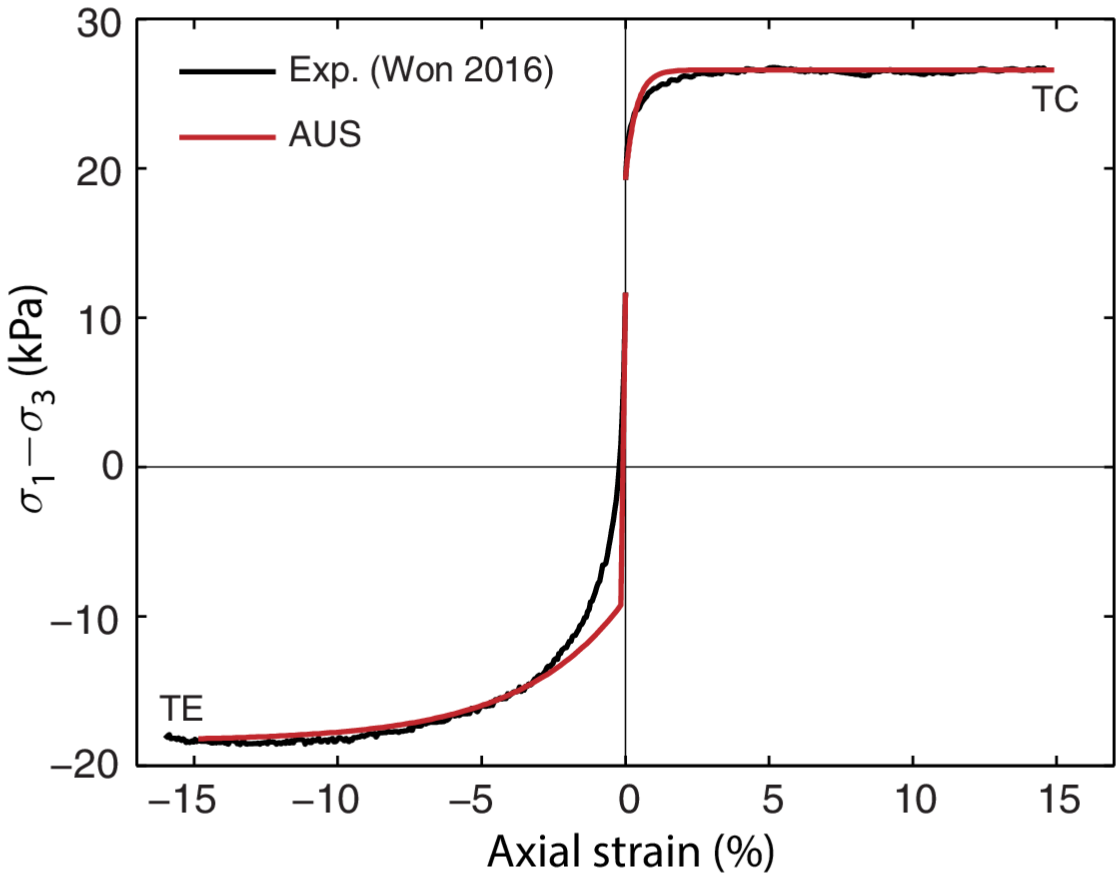
AUS

Optum CE

Fit to data

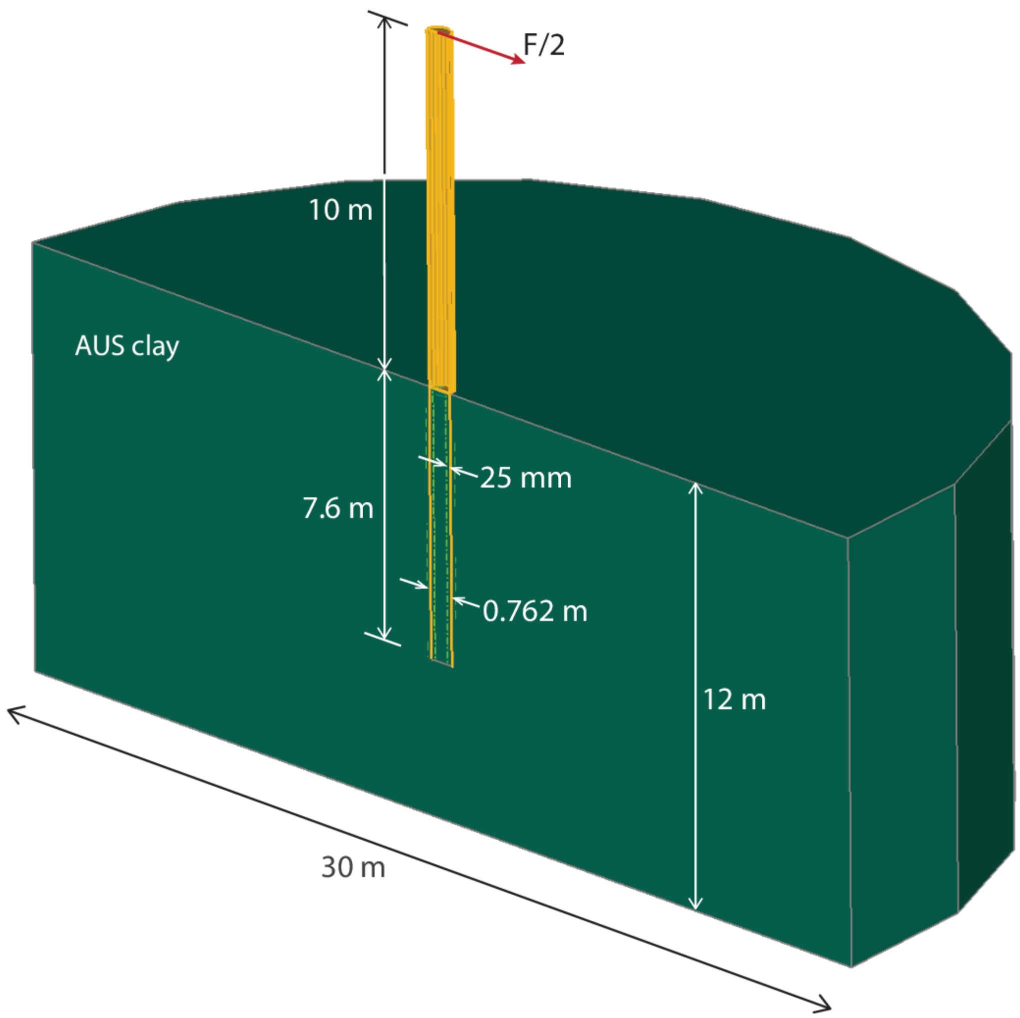


Fit to data


30
20
10
0
−10
−20
$\sigma_1 - \sigma_3$ (kPa)
Exp. (Won 2016)
AUS
TC
TE
−15
−10
−5
0
5
10
15
Axial strain (%)

Optum^CE

Monopile (PISA Project)



Optum CE

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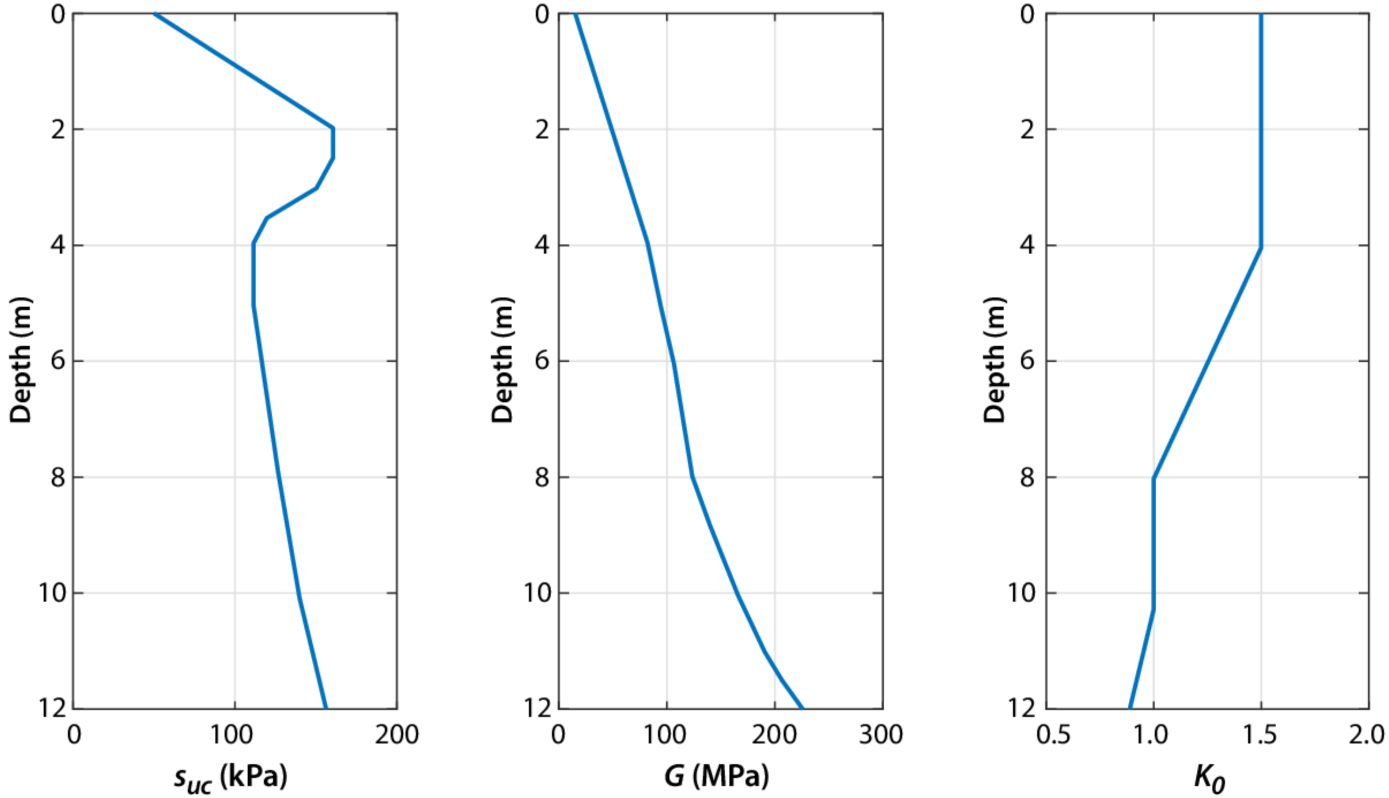

Optum CE

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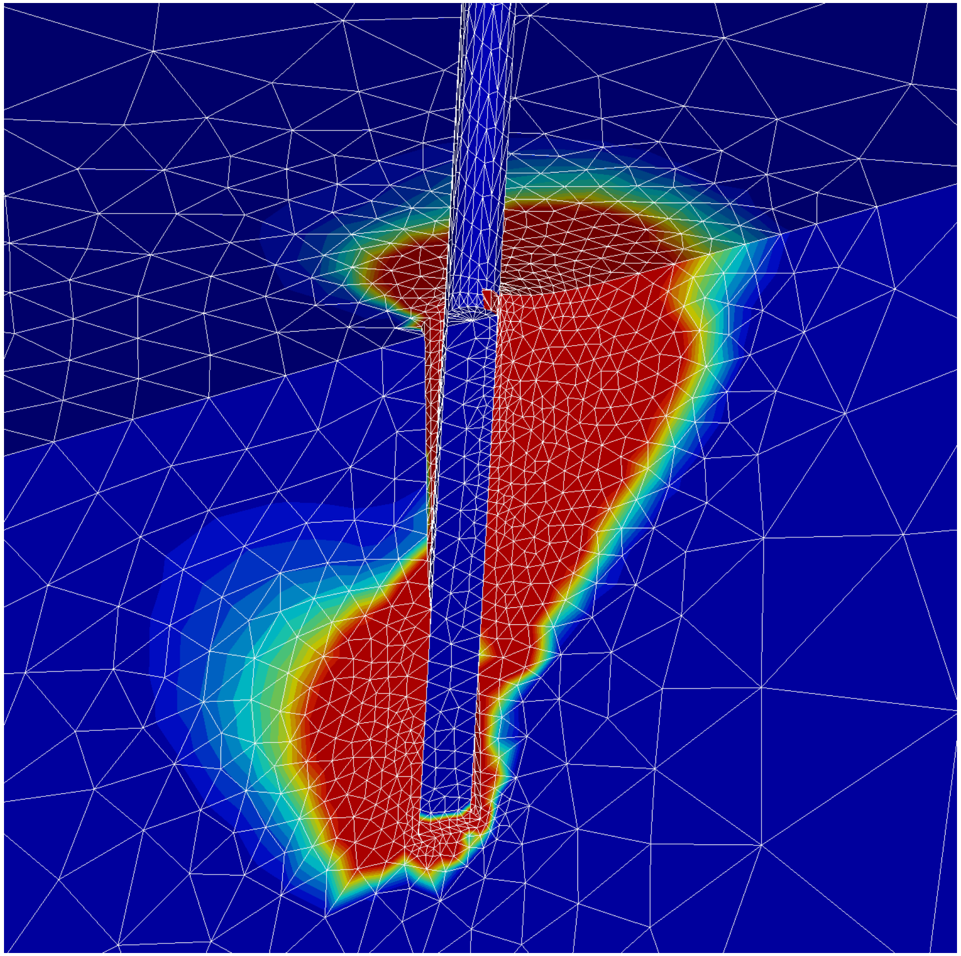

Optum CE

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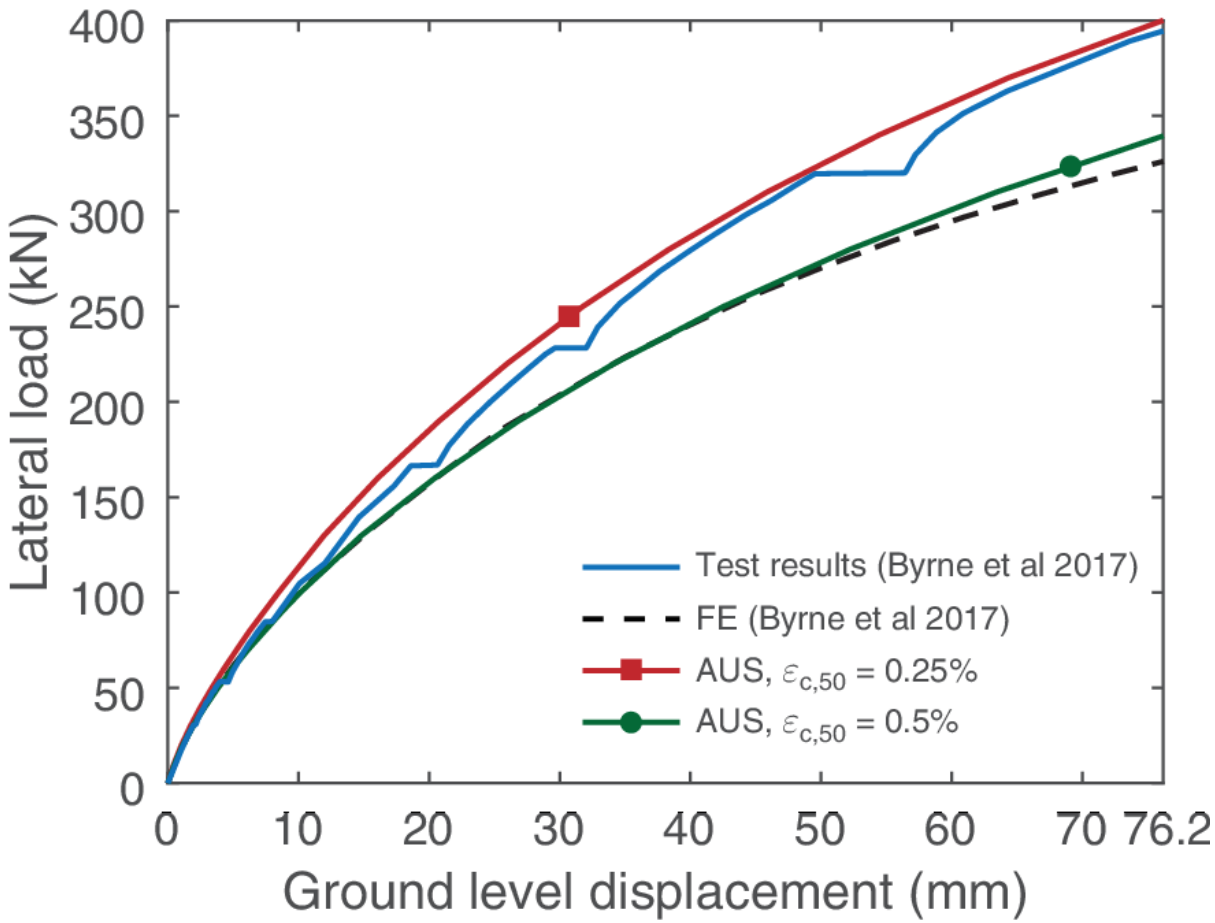

Optum^CE

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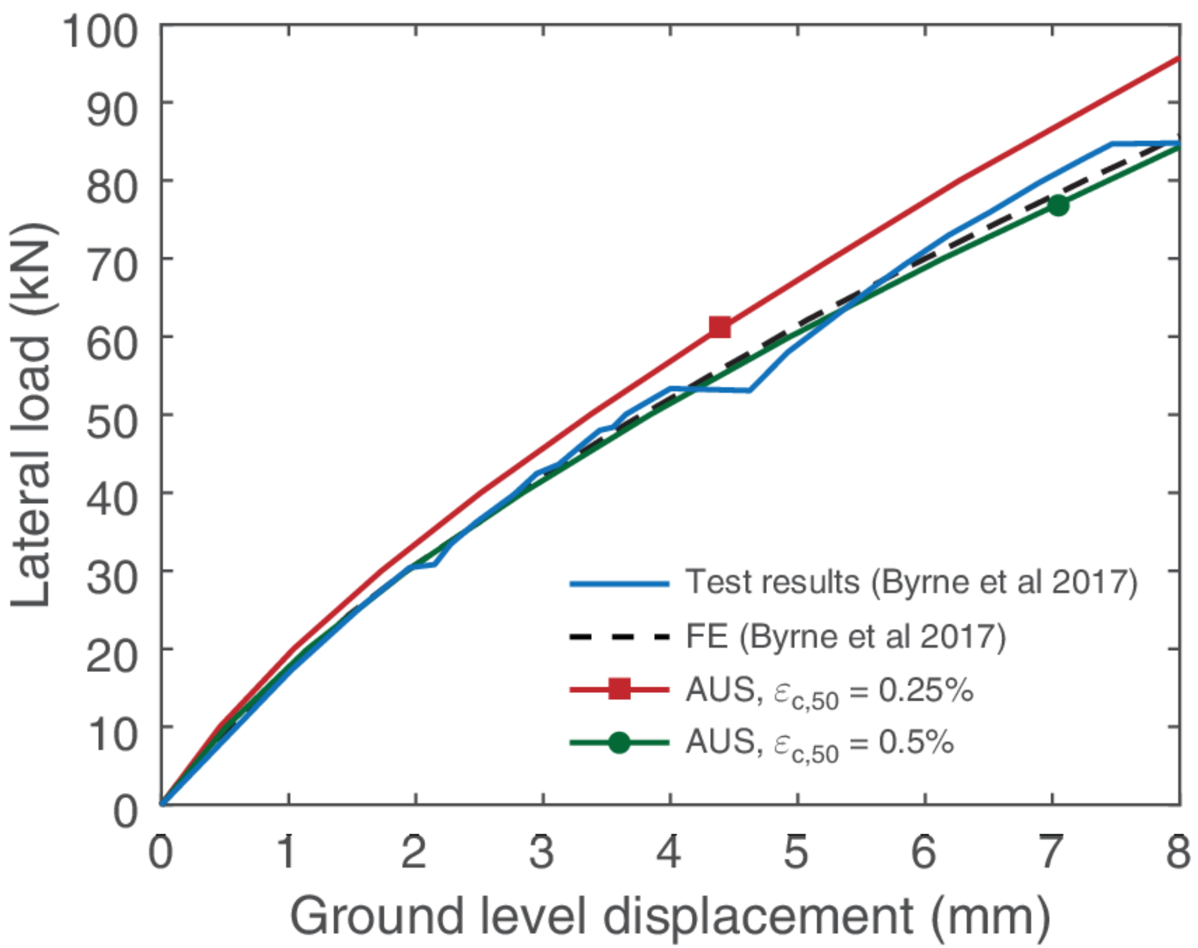
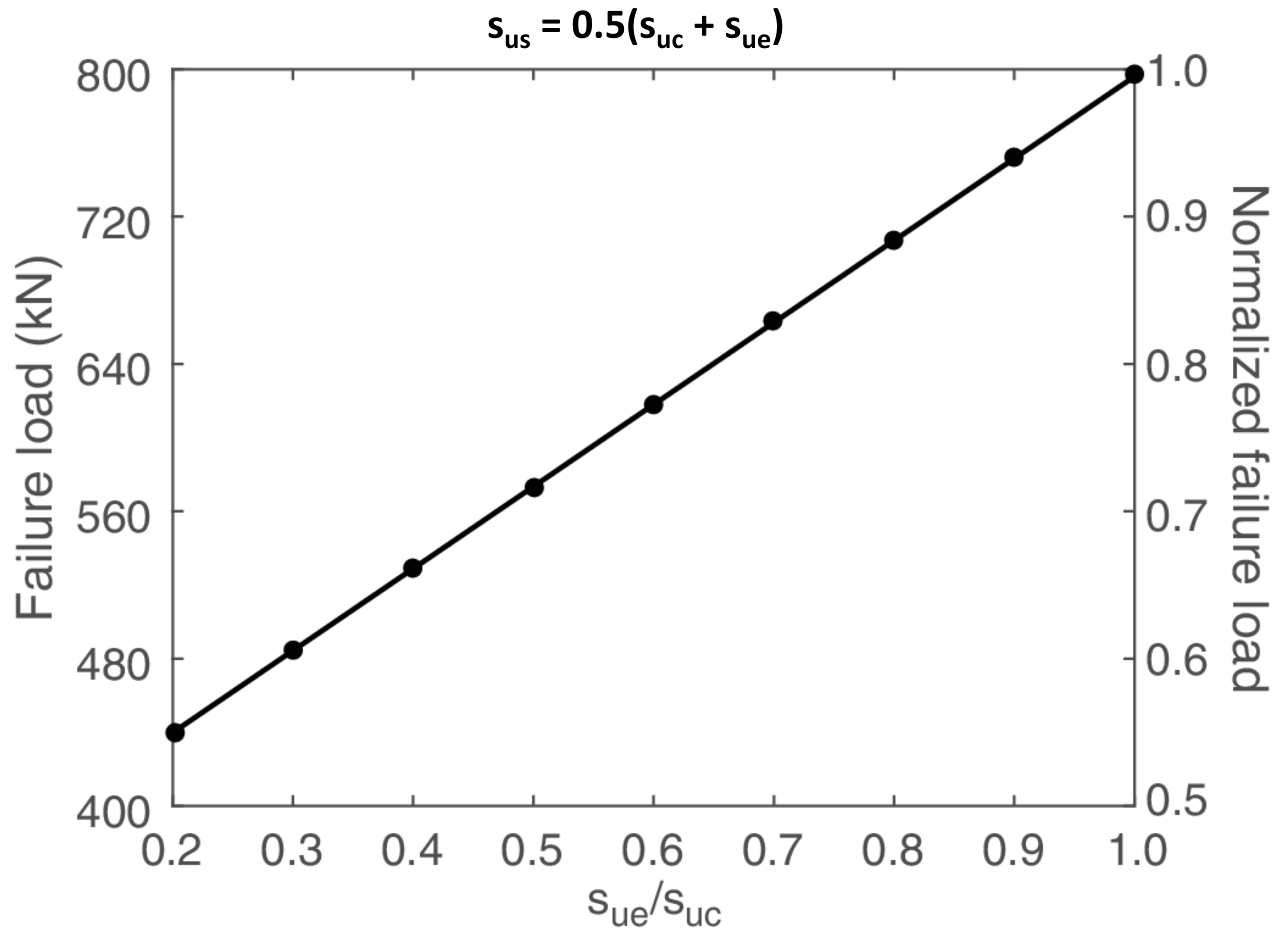


Monopile (PISA Project)



Conclusions

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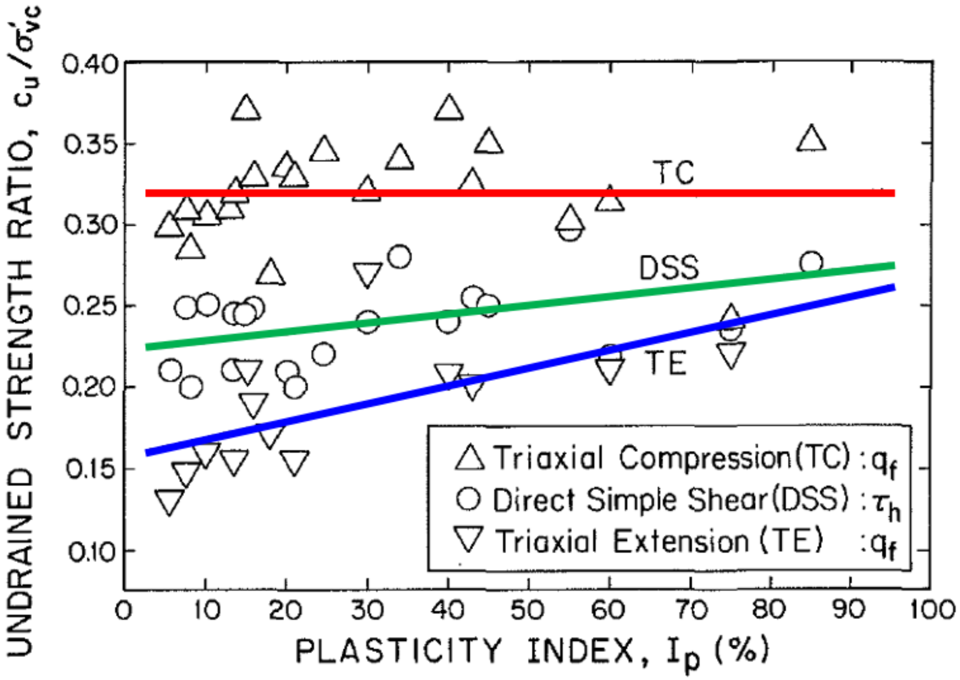

AUS

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Optum CE

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RESEARCH ARTICLE

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AUS: Anisotropic undrained shear strength model for clays

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