TORSIONAL LOADING OF SHALLOW FOUNDATIONS

KRISTIAN KRABBENHOFT
Optum Computational Engineering
University of Liverpool

Outline

+ Effect of torsion – “torsion factor” in the bearing capacity equation?
+ V-T diagrams
+ G3 scripting interface
+ Drained vs undrained solutions for limiting torsional capacity

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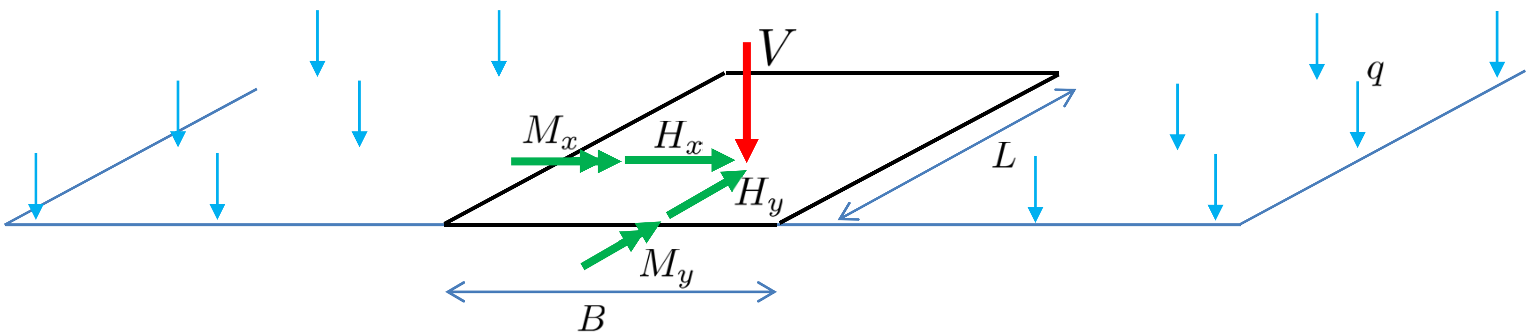
Rectangular foundations



Bearing capacity equation:

$$\frac{V_u}{A'} = cN_c i_c s_c + qN_q i_q s_q + \frac{1}{2}\gamma B' N_\gamma i_\gamma s_\gamma$$

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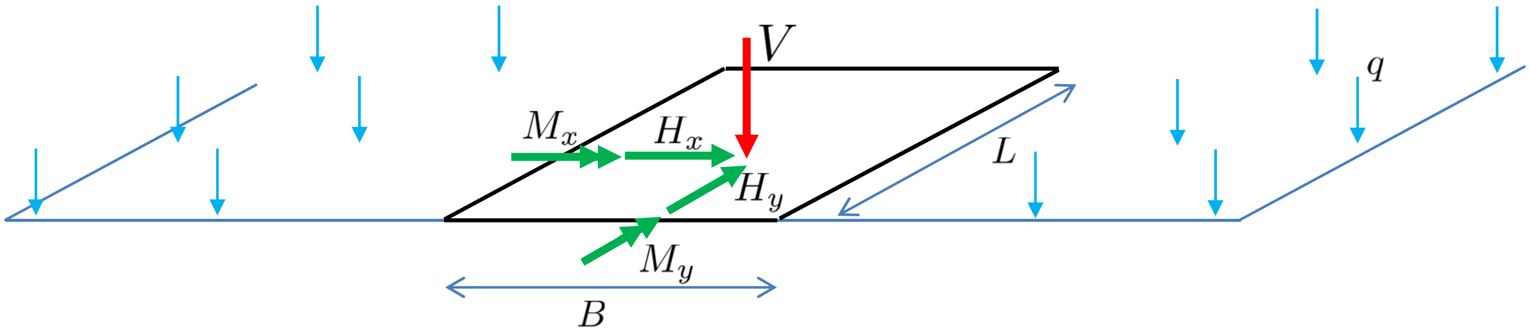

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Some issues with:

- Bearing capacity factor N_γ
- Superposition
- Inclined loading
- Eccentricity
- Shape – 2D to 3D

11th International Conference of IACMAG, Torino
21 Giugno 2005

Exact bearing capacity calculations using the method of characteristics

Dr C.M. Martin
Department of Engineering Science
University of Oxford

http://www2.eng.ox.ac.uk/civil/people/cmm/download/iacmag05_cmm.ppt

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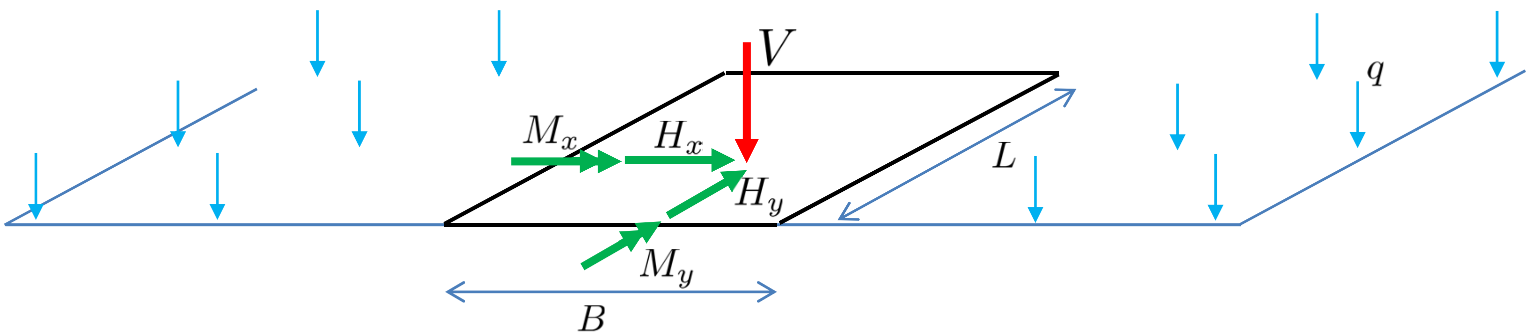

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TORSION

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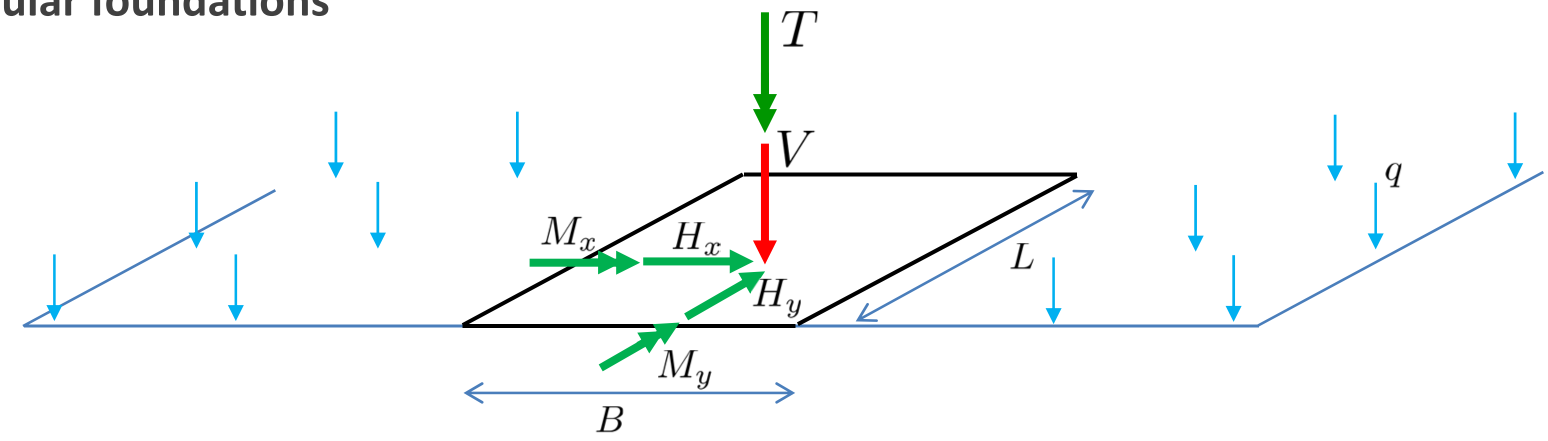

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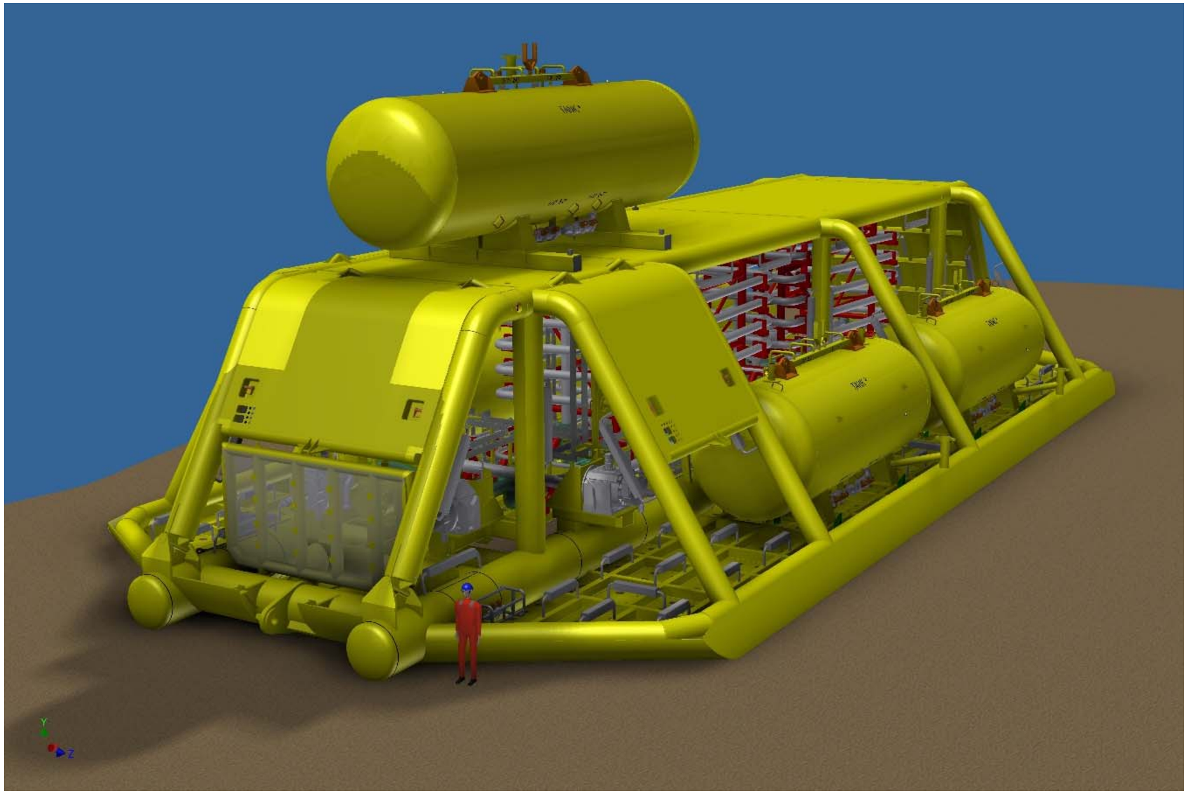
Missing: torsion

Offshore foundations



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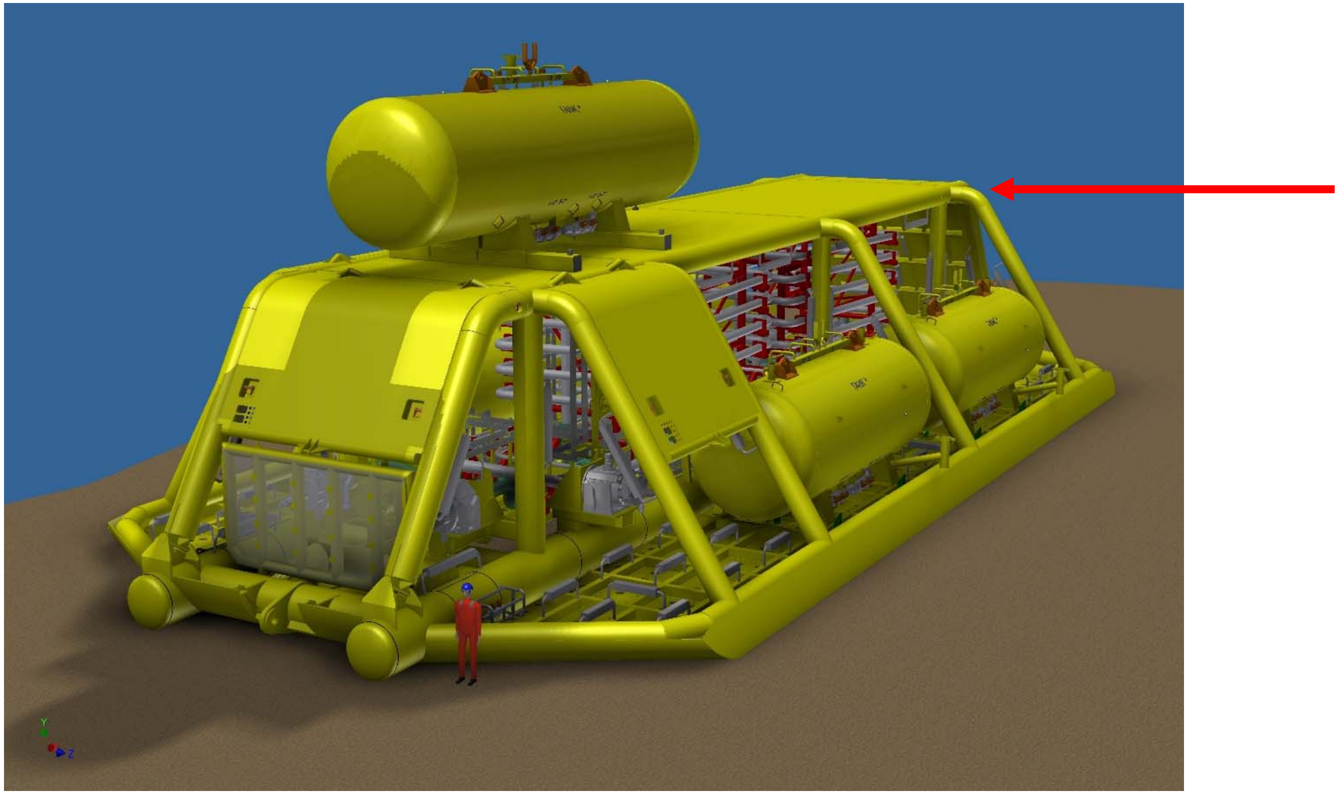

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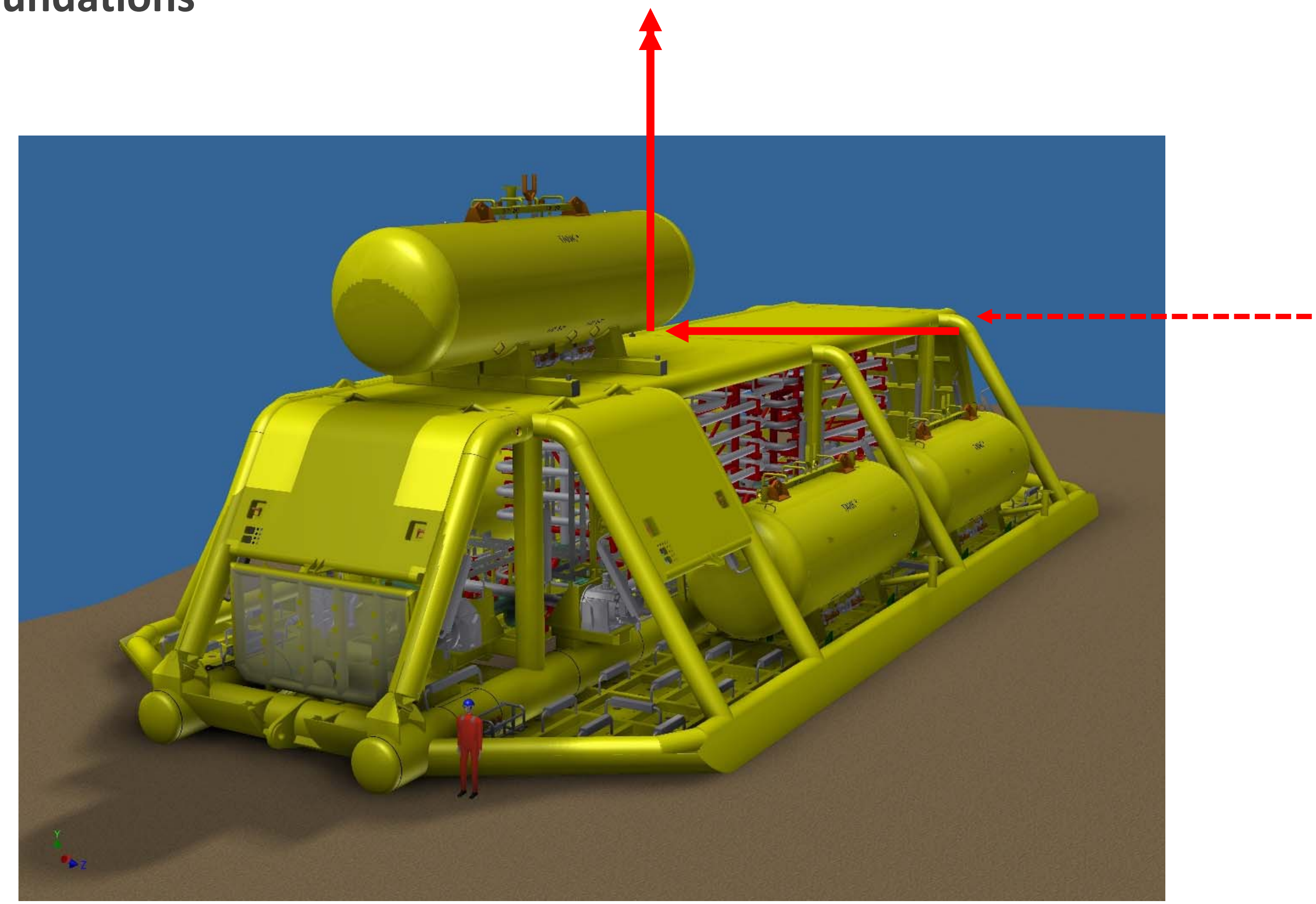

TORSION

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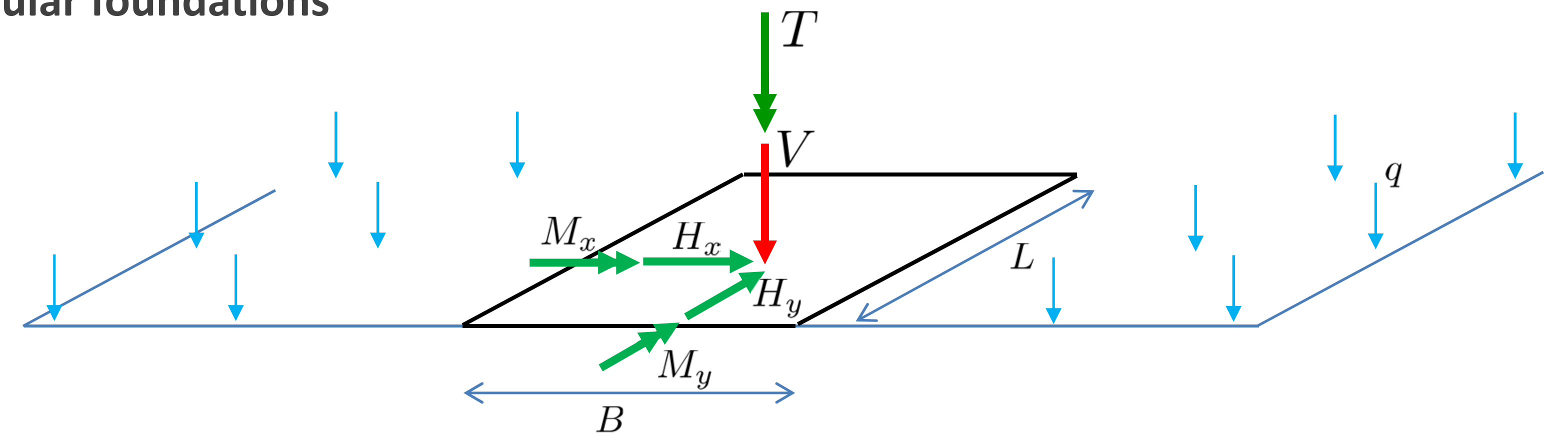

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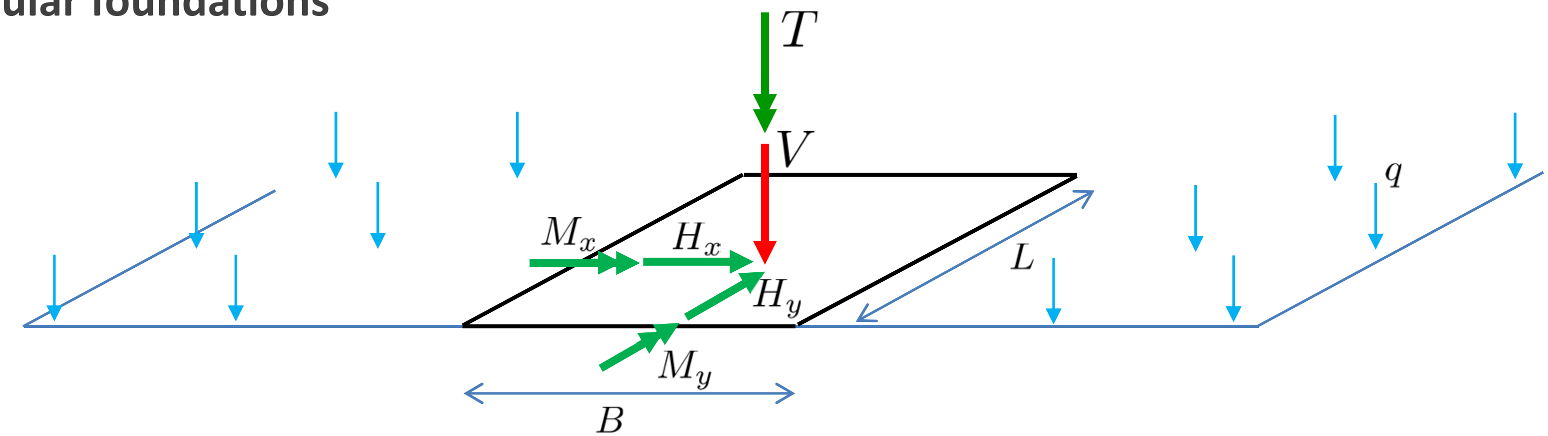

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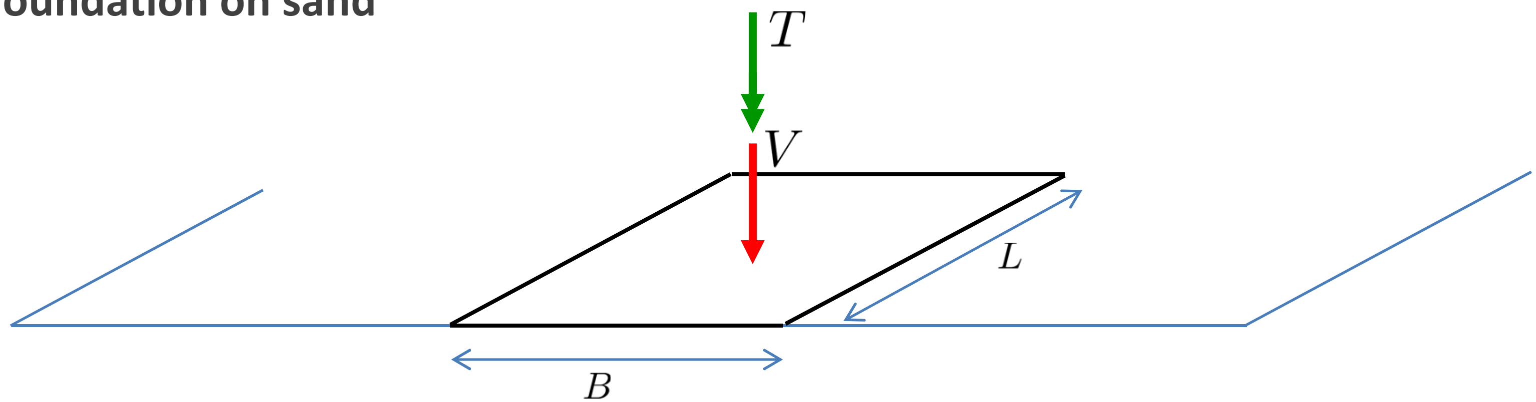
Missing: torsion

Possible modification:

$$\frac{V_u}{A'} = cN_c i_c s_c t_c + qN_q i_q s_q t_q + \tfrac{1}{2}\gamma B' N_\gamma i_\gamma s_\gamma t_\gamma$$

TORSION

Surface foundation on sand



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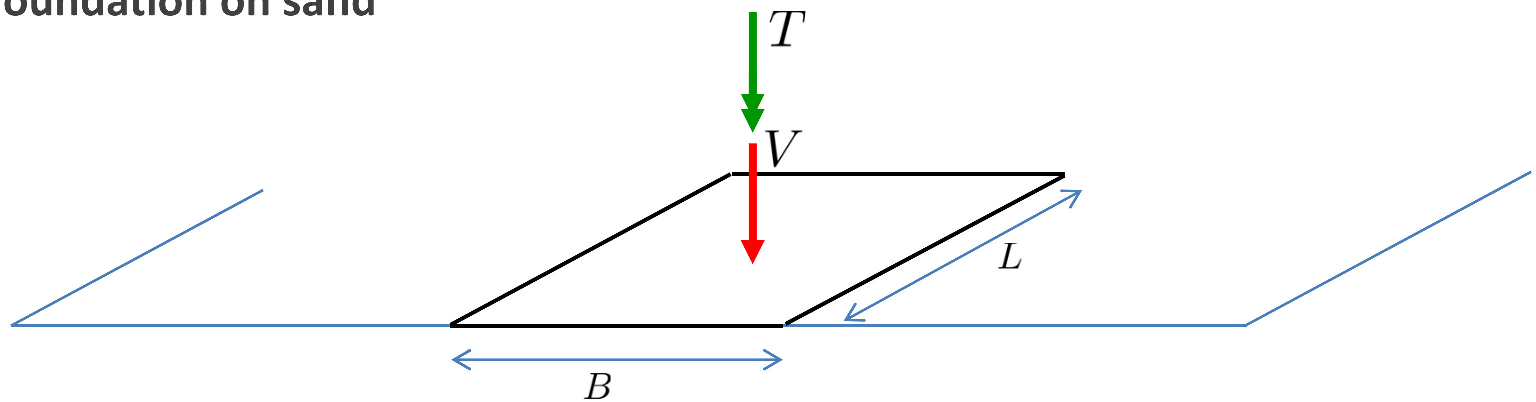


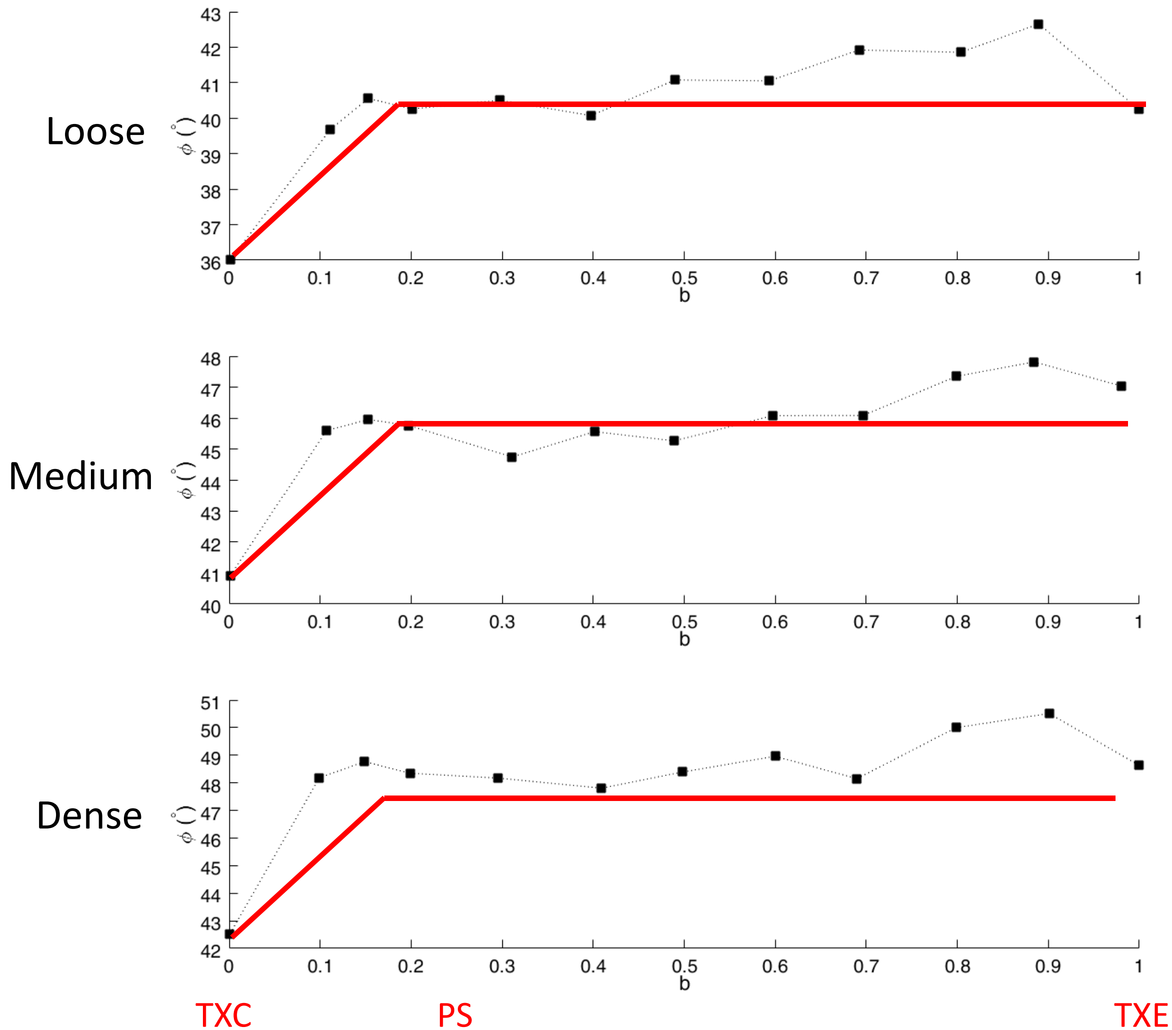
Modified bearing capacity equation:

$$\frac{V_u}{A} = \frac{1}{2}\gamma B N_\gamma s_\gamma t_\gamma$$

Redefinition of N_γ:

$$N_\gamma^* = N_\gamma s_\gamma = f(B/L, \phi_{ps}, \phi_{tc})$$

FASD



Matched in TXC and:

$$\phi_{ps} = 1.12\phi_{tc}$$

(Kulhawy & Mayne 1990)

Old Danish:

$$\phi_{ps} = 1.1\phi_{tc}$$

New Danish:

$$\phi_{ps} = (1 + 0.1 I_D)\phi_{tc}$$

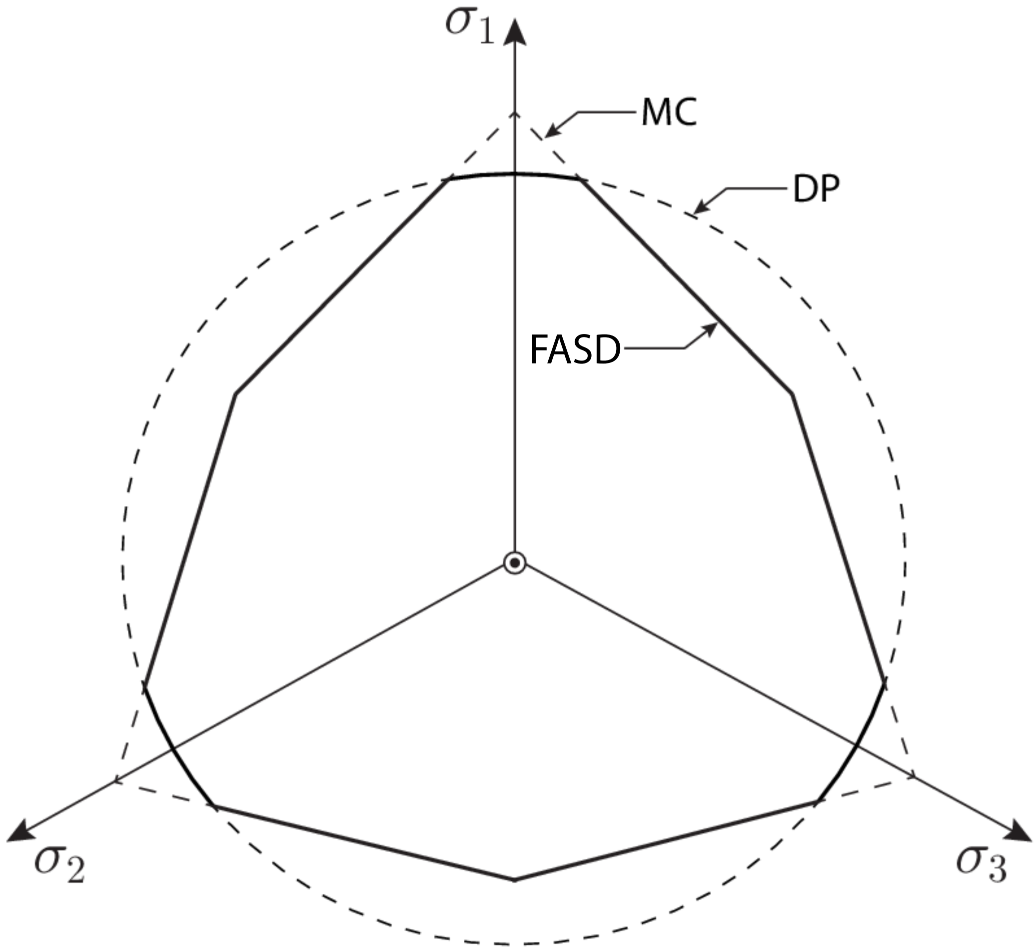
Stakemann (1976):

$$\phi_{ps} = (1 + 0.163 I_D)\phi_{tc}$$

Data: Lade & Wang (2001)

BEARING CAPACITY EQUATION

FASD Sand



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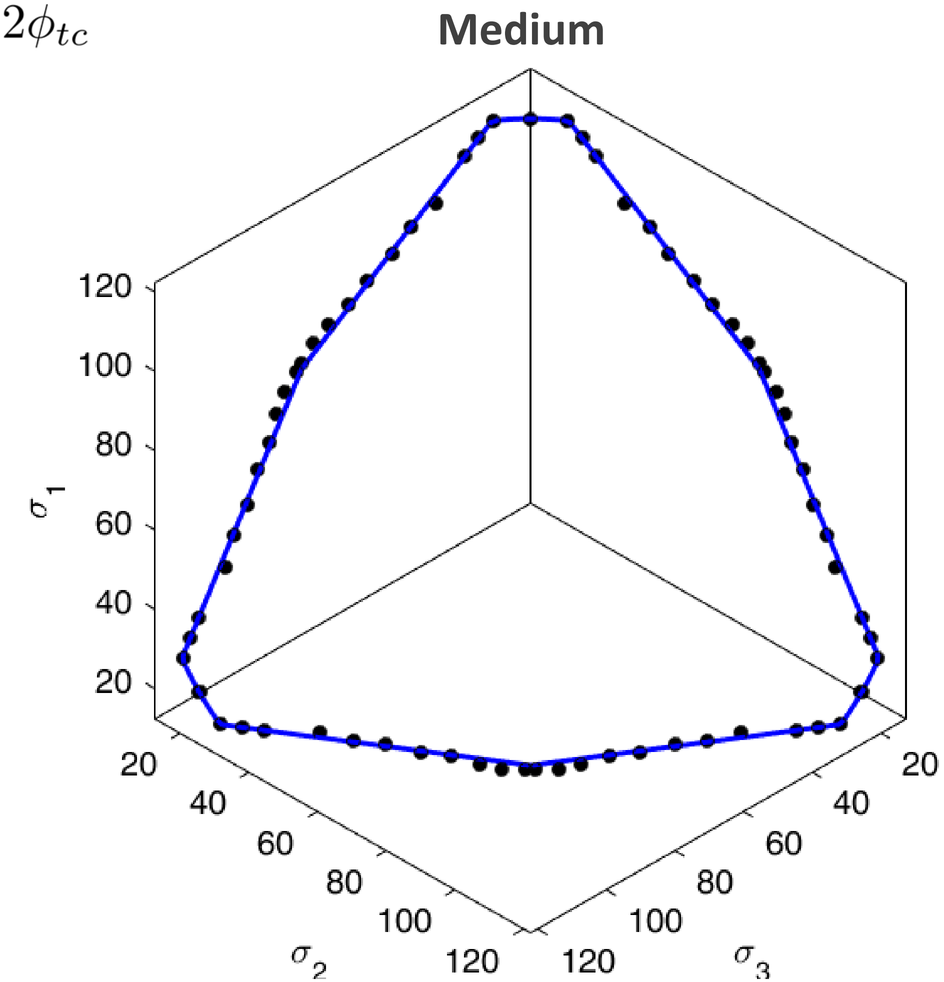

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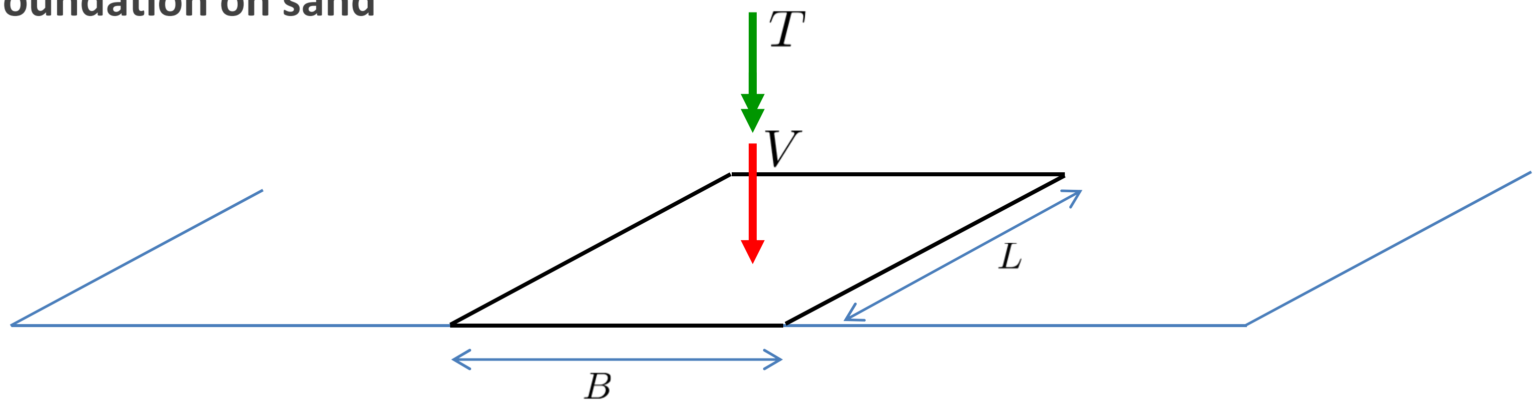

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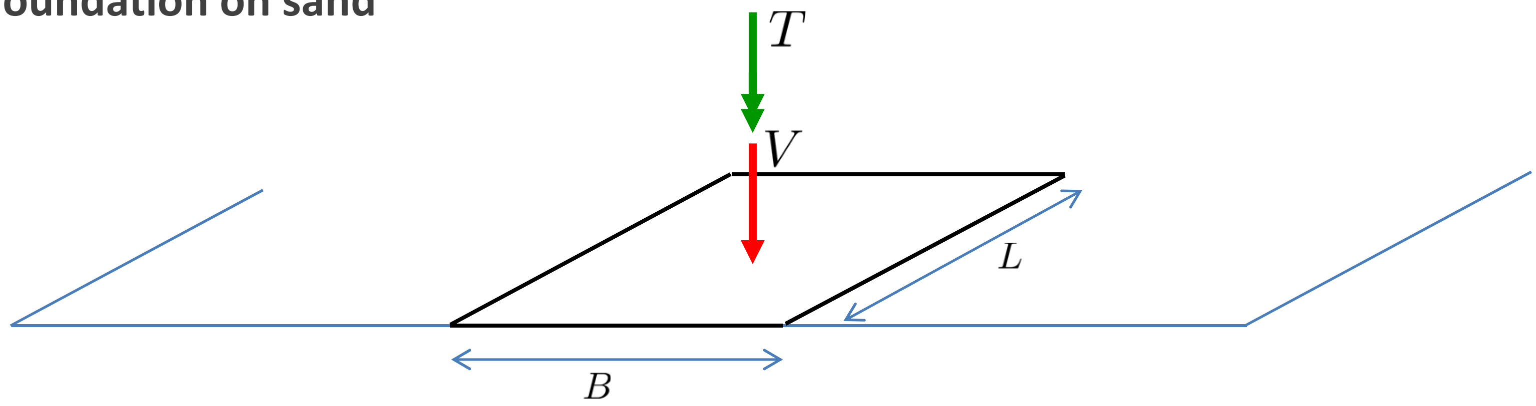

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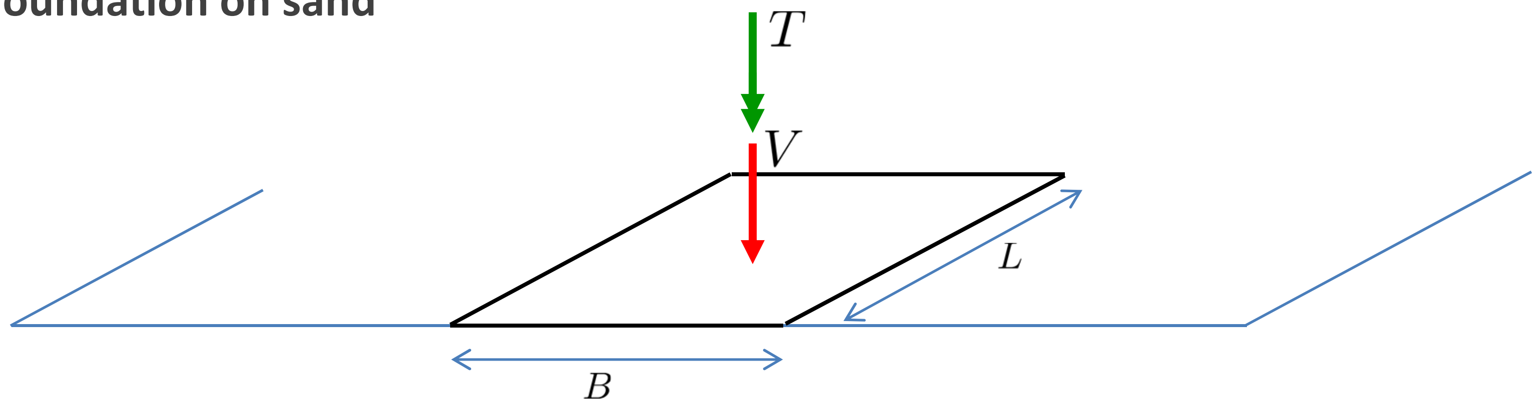

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V-T diagram:

$$F(V,T) = 0$$

V-T diagram

TORSION

V-T diagram

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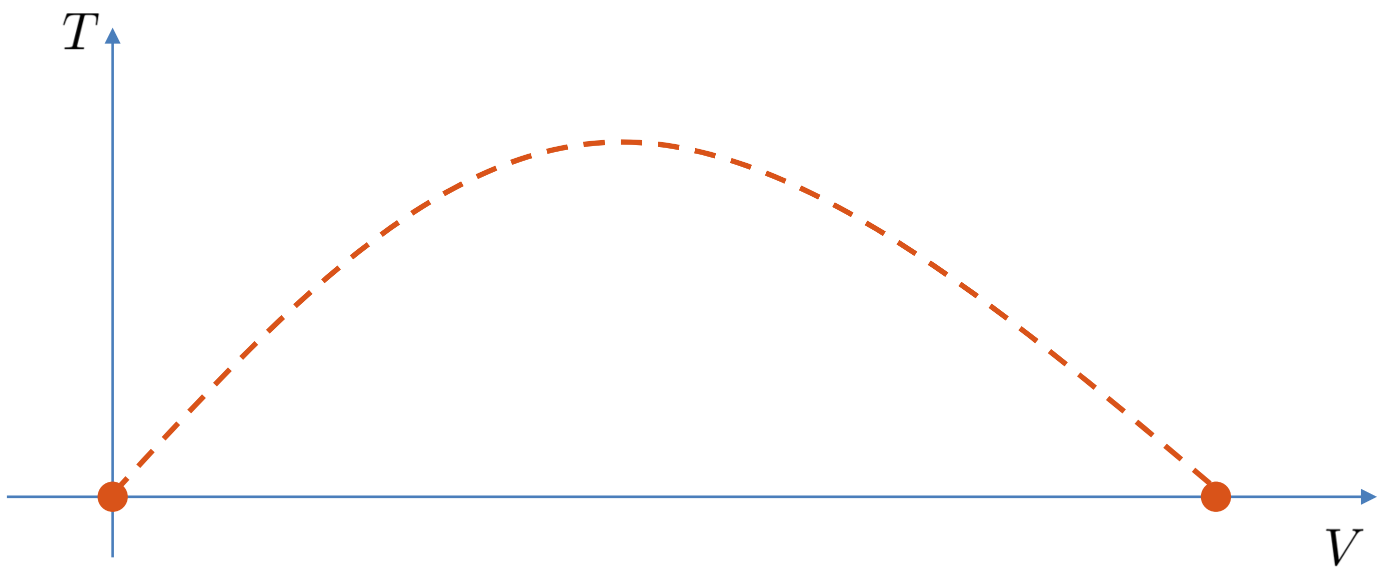
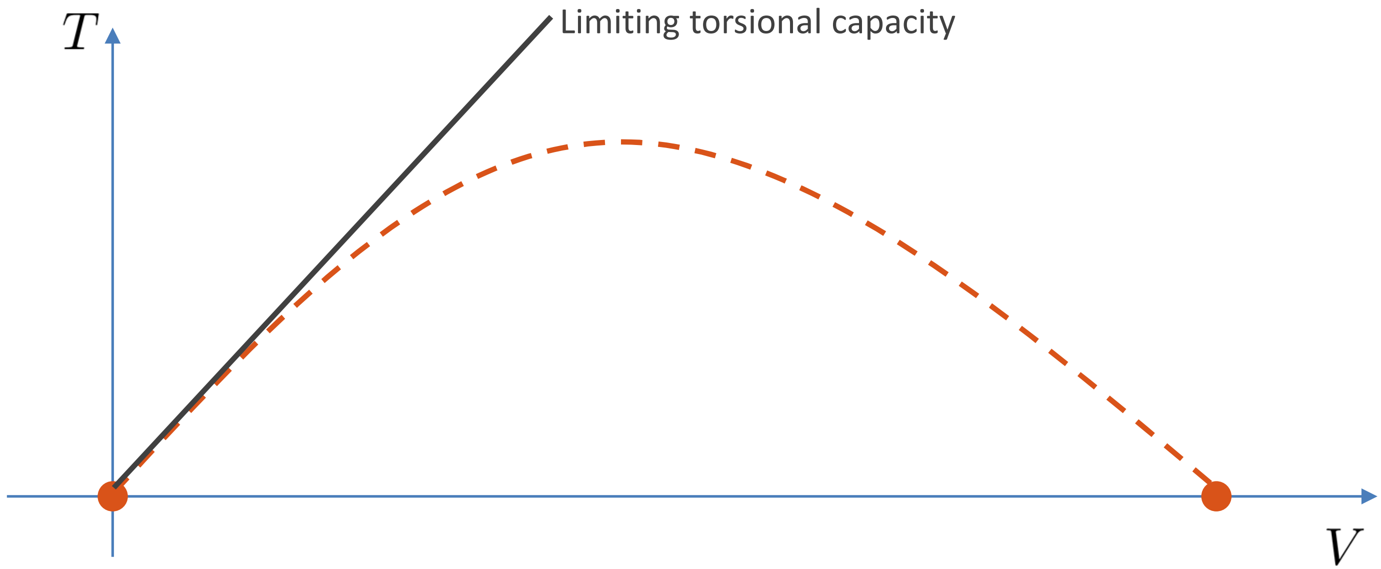
TORSION

V-T diagram

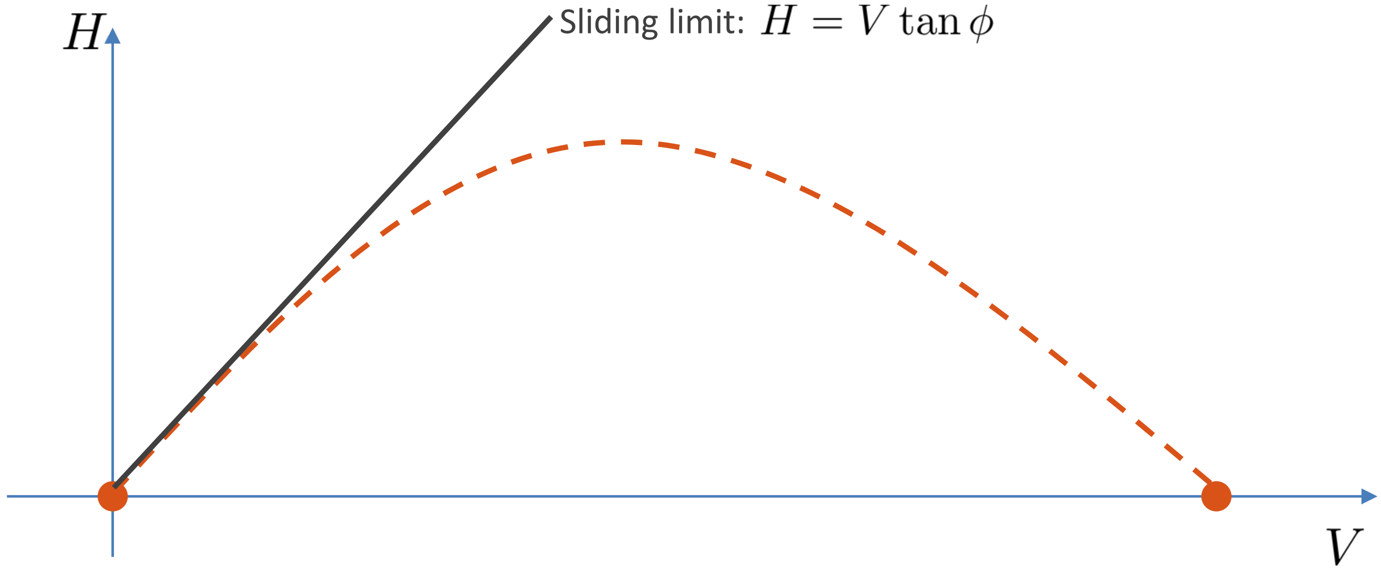


Optum CE

V-T diagram



V-H diagram



Optum CE

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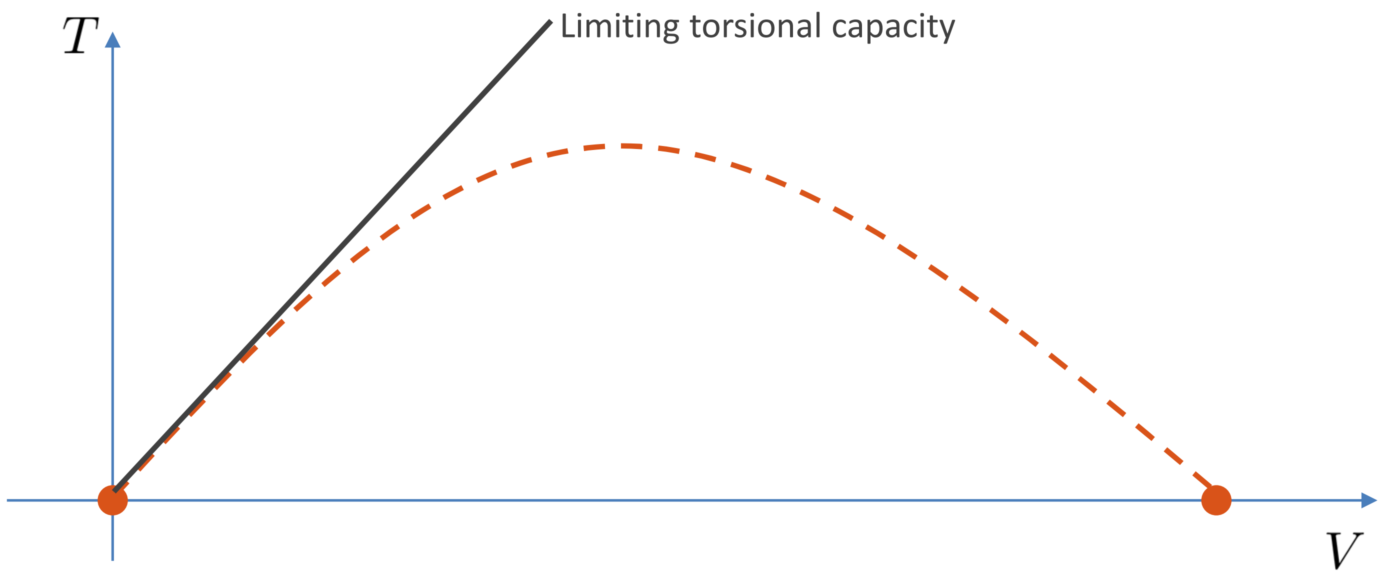
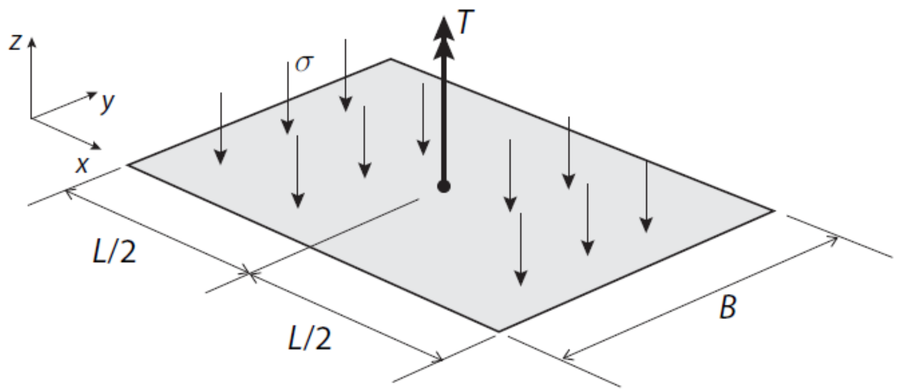
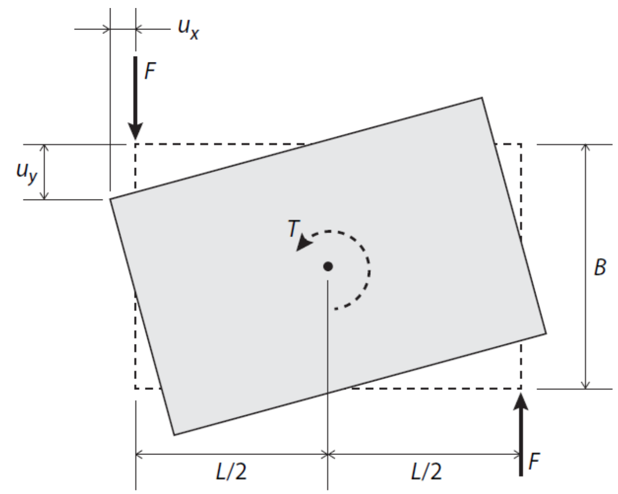


Torsional capacity for $V \rightarrow 0$:

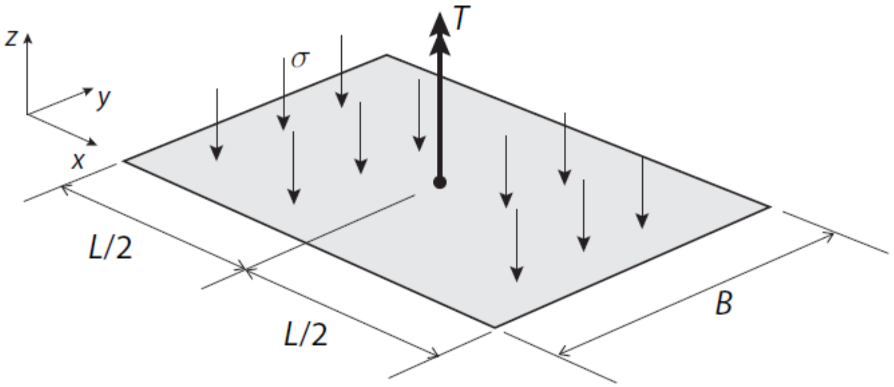


Failure mechanism:



TORSION

Torsional capacity for V $\rightarrow$ 0:



Solution:

$$T/B = \tfrac{1}{2}\sqrt{1+\left(\frac{L}{B}\right)^2}\,V\tan\phi$$

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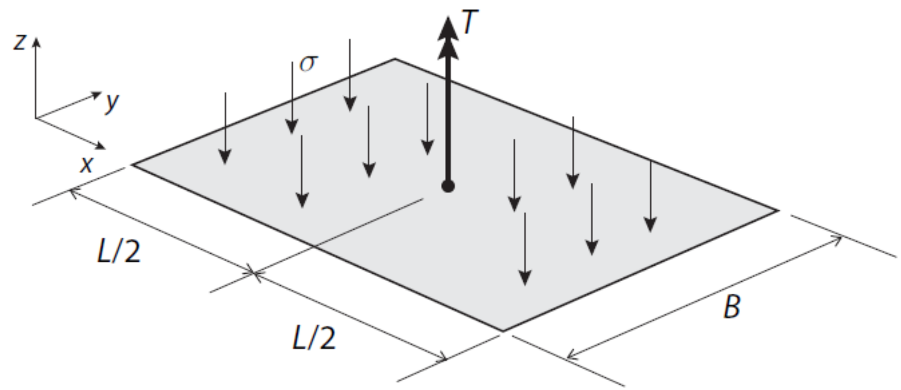


Solution:

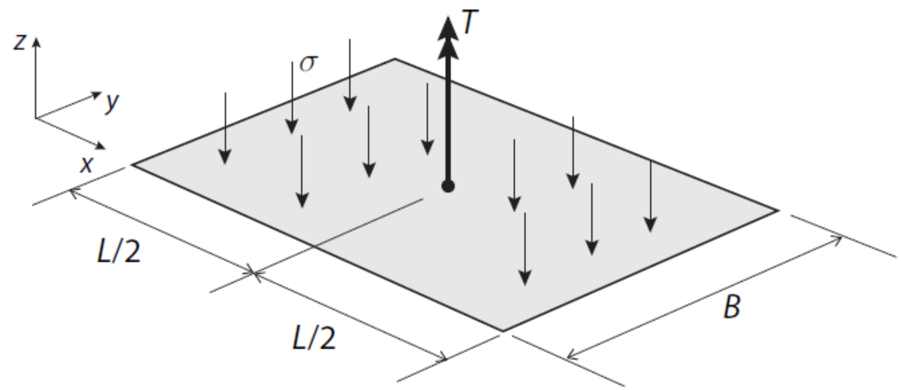
$$T/B = \mu_T V$$

where:

$$\mu_T = \tfrac{1}{2}\sqrt{1+\left(\frac{L}{B}\right)^2}\tan\phi$$

("torsional friction coefficient")

Torsional capacity for V → 0:



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(“torsional friction coefficient”)

Analogous to sliding limit:

$$H = V\tan\phi$$

TORSION

Torsional capacity for V $\rightarrow$ 0:

Solution:

$$\frac{T}{LV\tan\phi} = \tfrac{1}{2}\sqrt{1+\left(\frac{B}{L}\right)^2}$$

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Murff et al (undrained):

$$\frac{T}{ALs_u} = \frac{L}{6B}\left(\frac{\sin\theta}{2\cos^2 t} + \tfrac{1}{2}\ln\left[\tan(\tfrac{\pi}{4}+\tfrac{\theta}{2})\right]\right) + \frac{L^2}{6B^2}\left(\frac{\cos\theta}{2\sin^2 t} - \tfrac{1}{2}\ln(\tan\tfrac{\theta}{2})\right)$$

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Adapt to drained case:

$$\tau_{\max}A = s_u A \sim V\tan\phi$$

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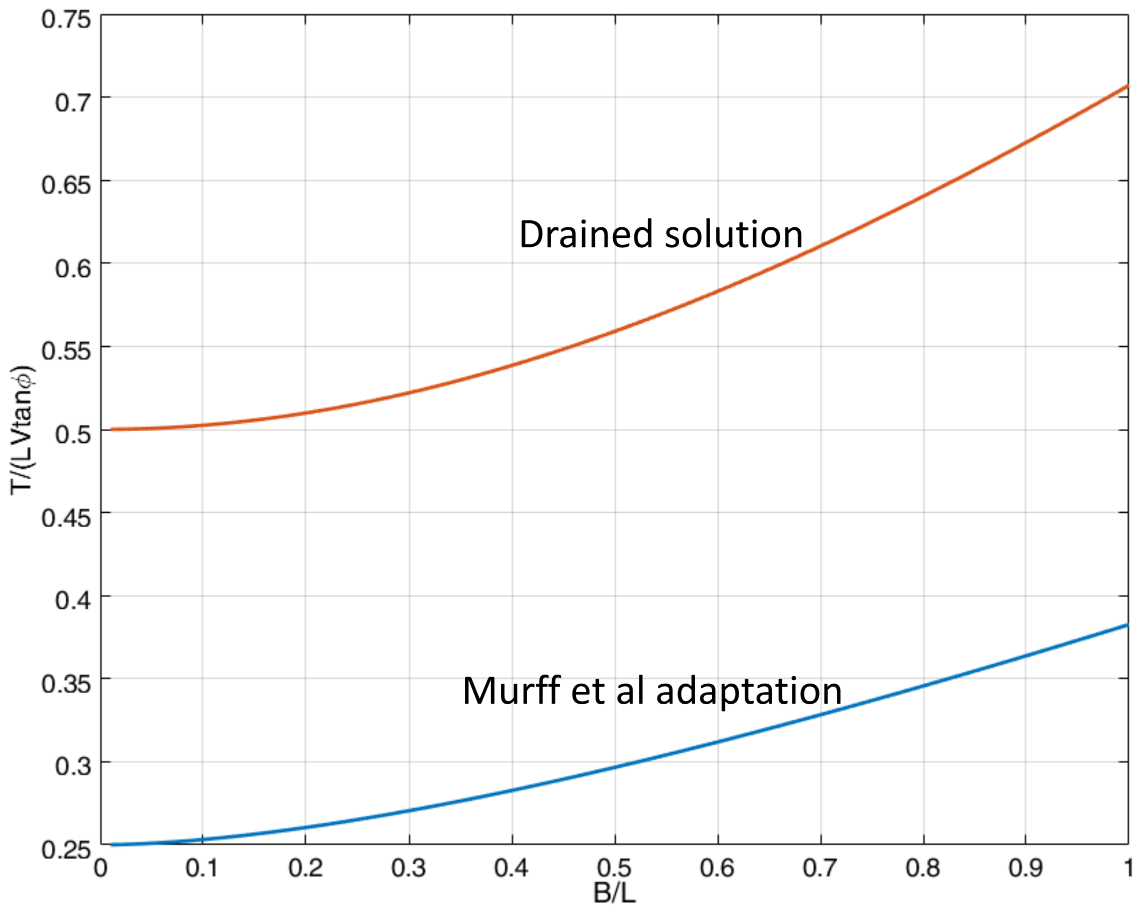
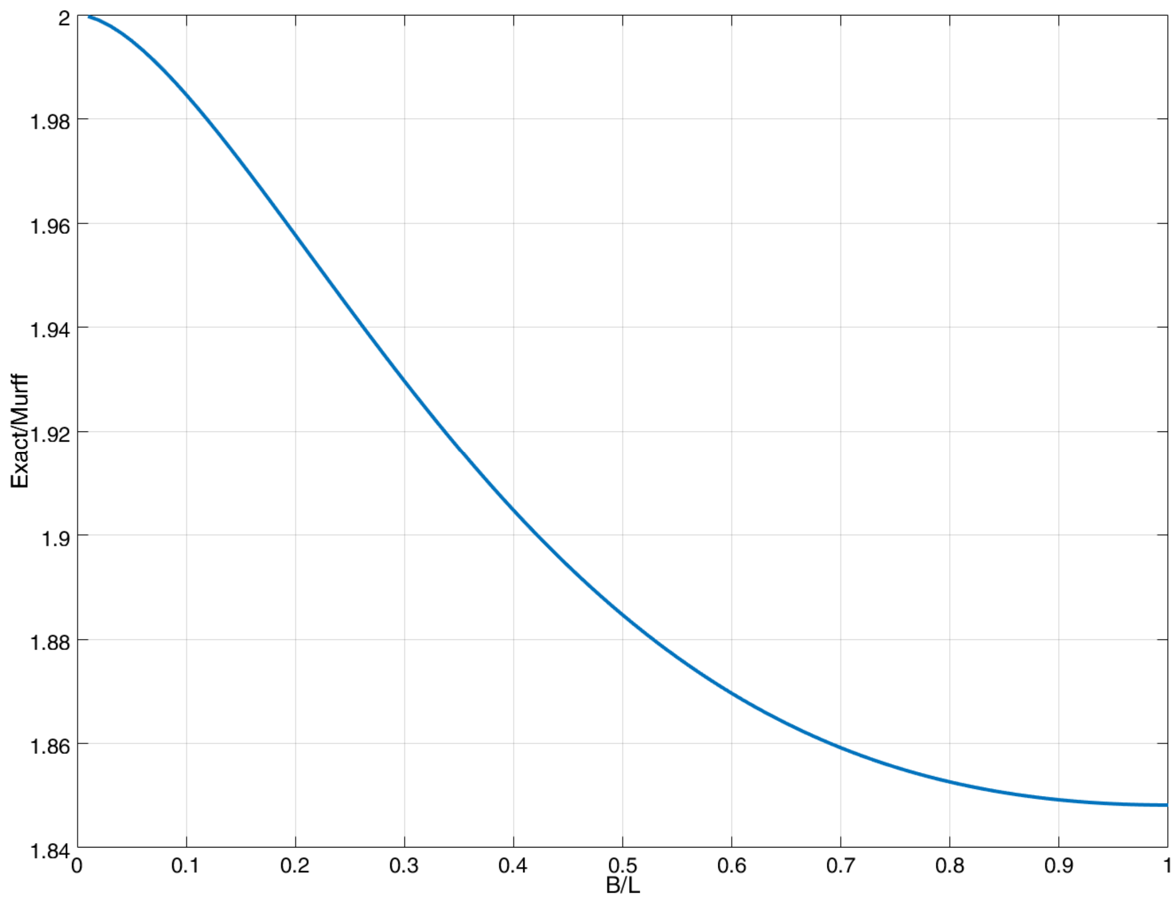
Torsional capacity for $V \to 0$:



TORSION

Torsional capacity for V → 0:

(Drained solution)/(Murff et al adaptation)



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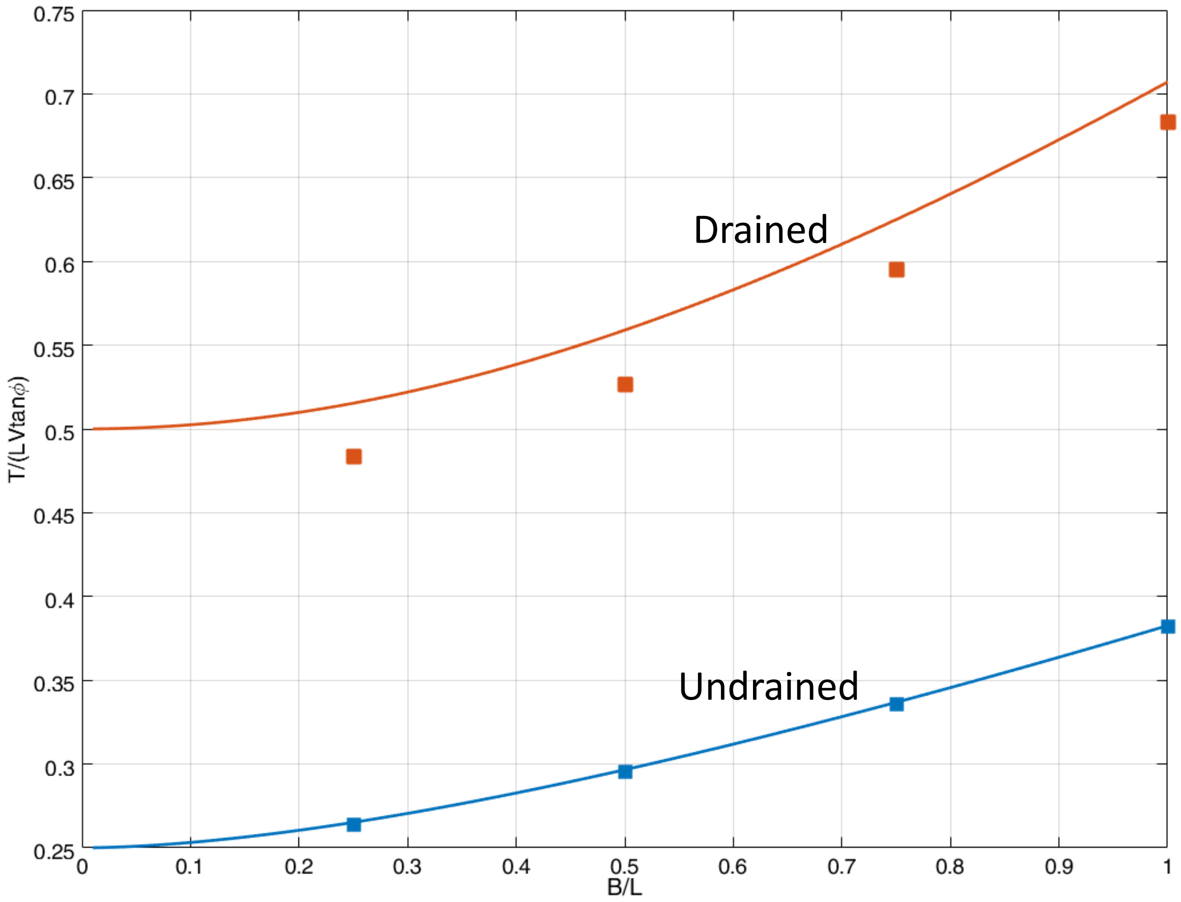

TORSION

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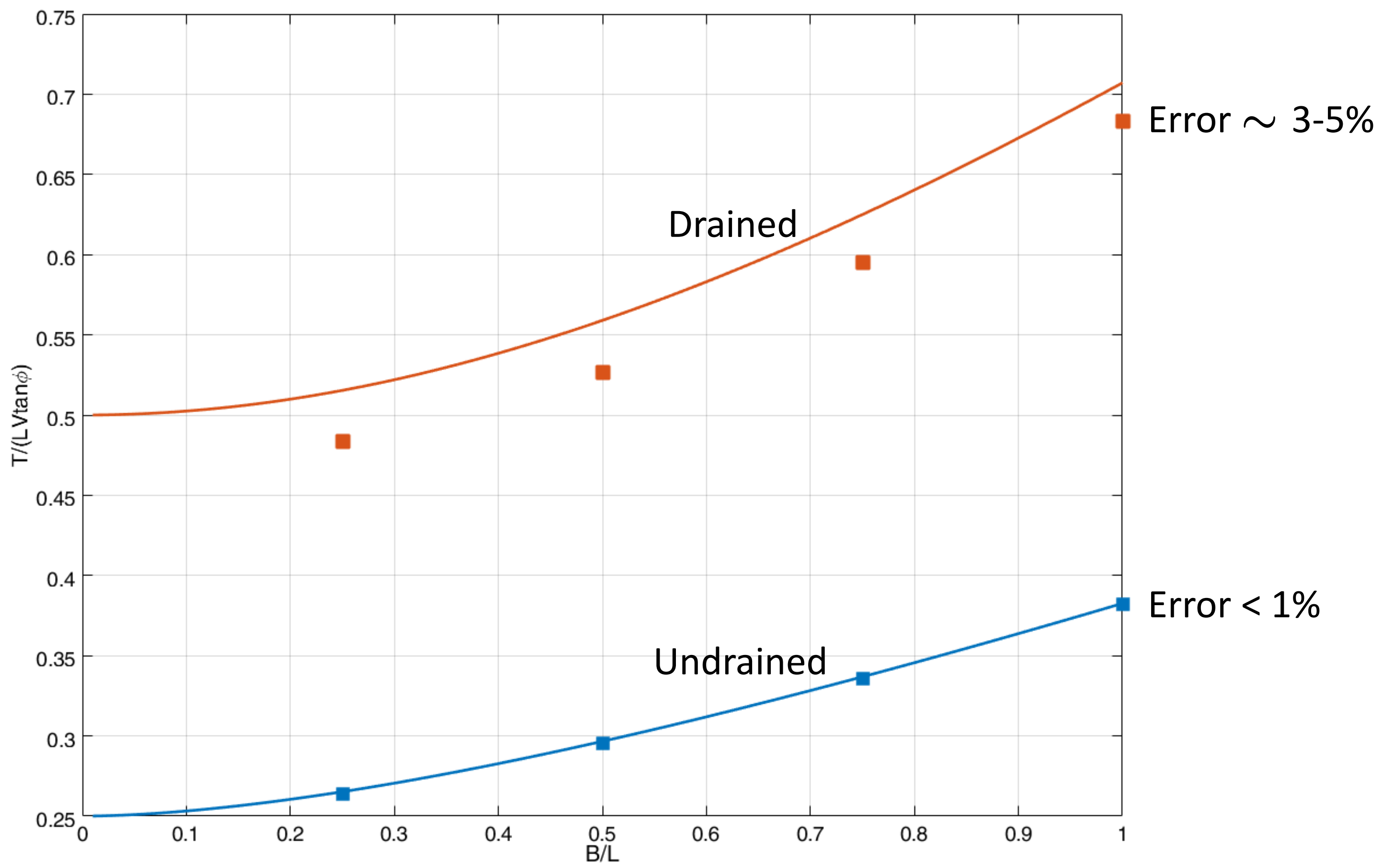

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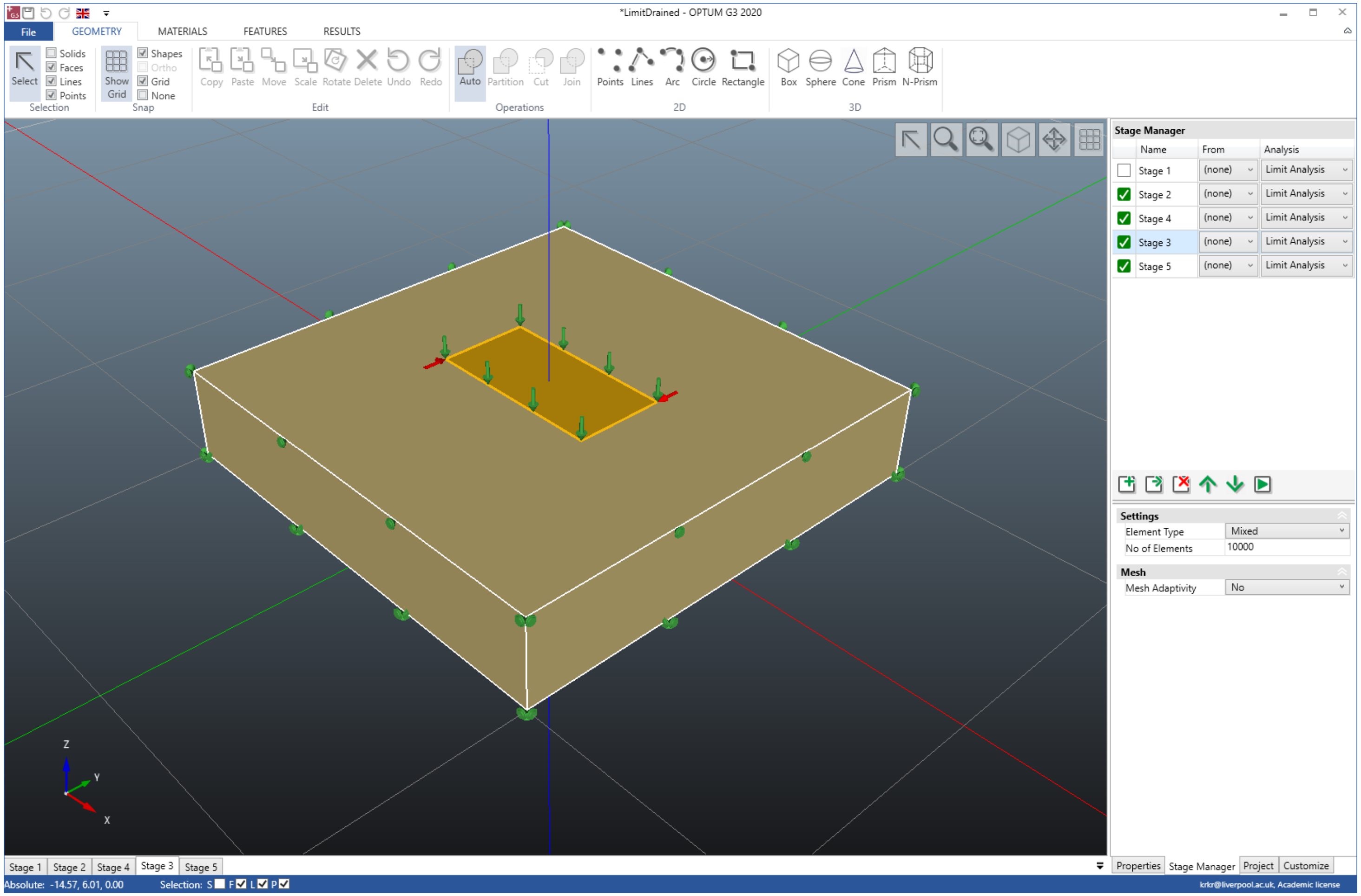

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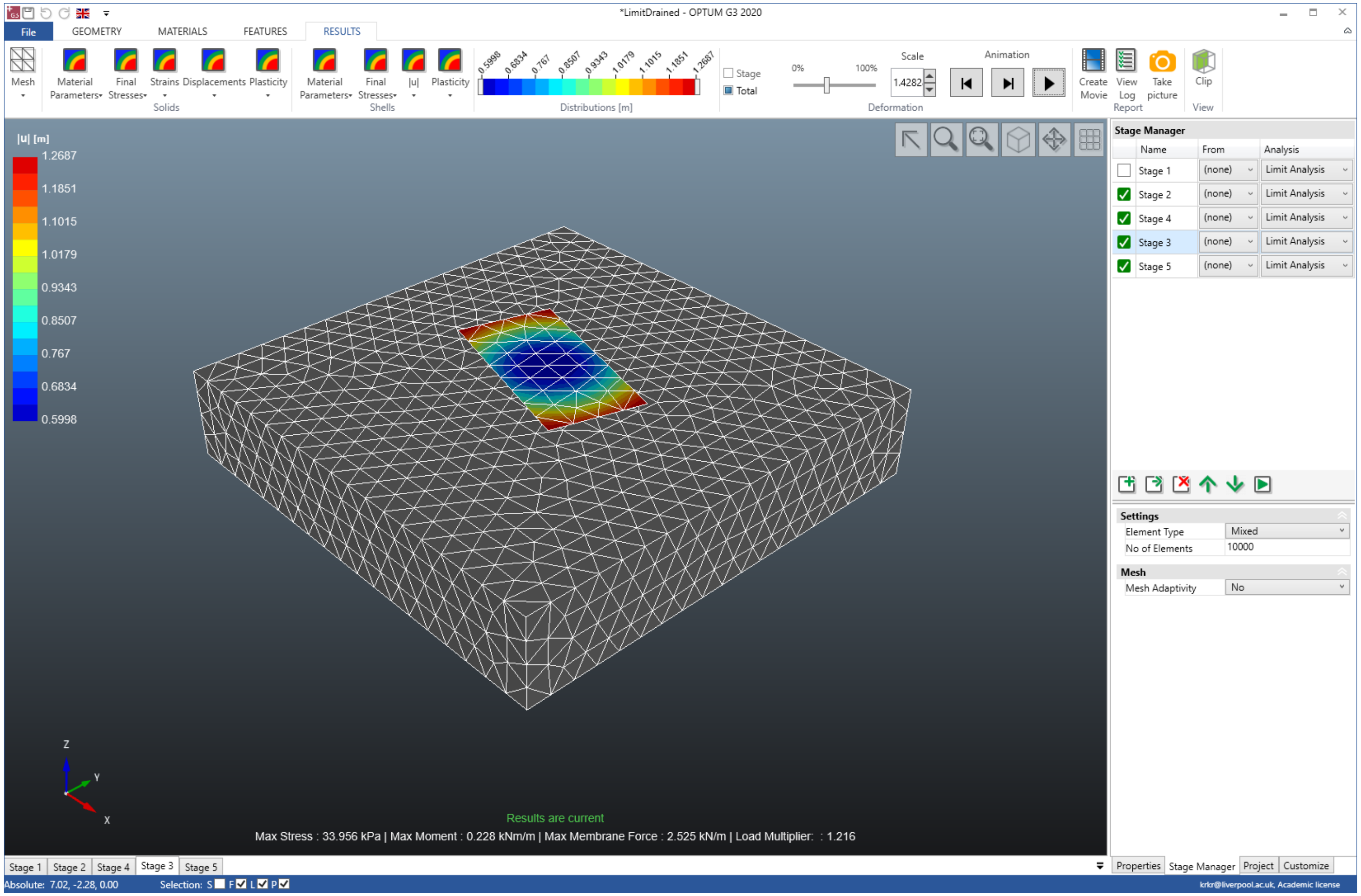
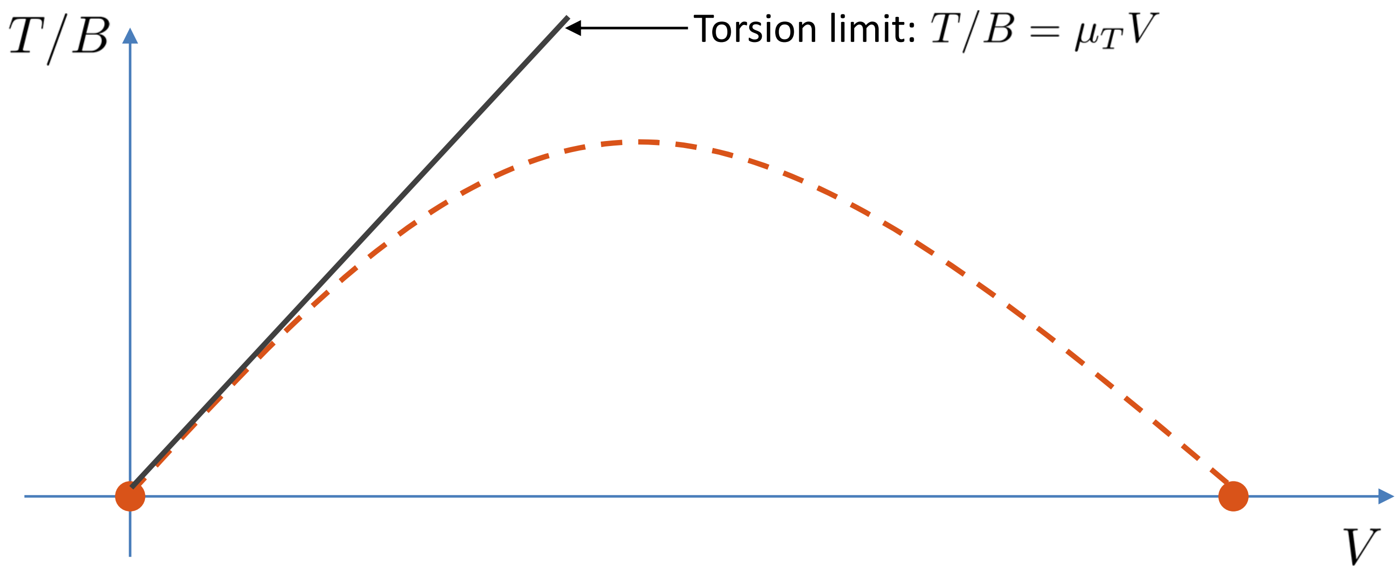


Optum CE

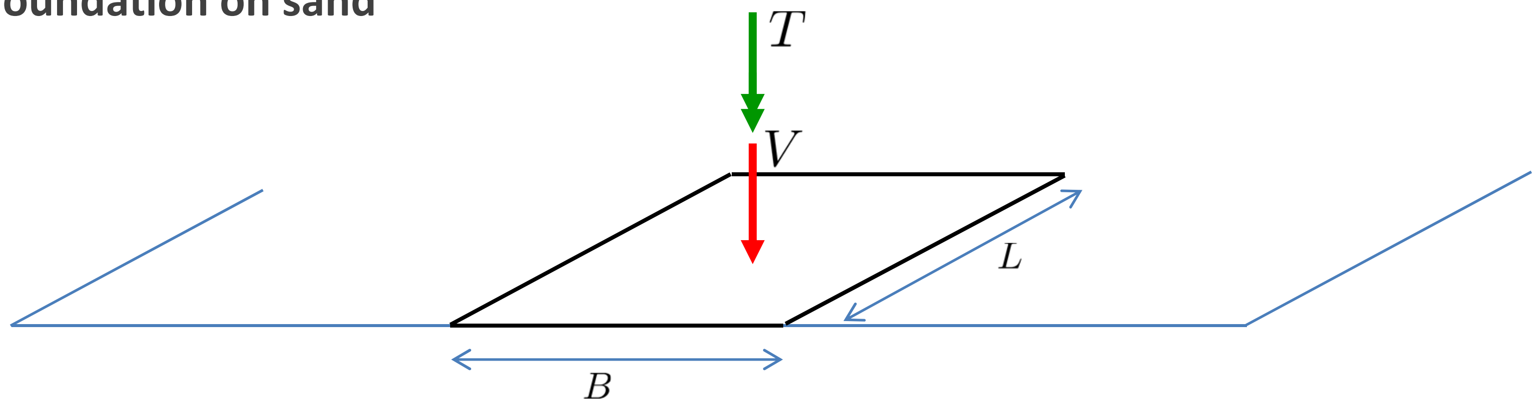
V-T diagram



TORSION

Optum CE

Surface foundation on sand



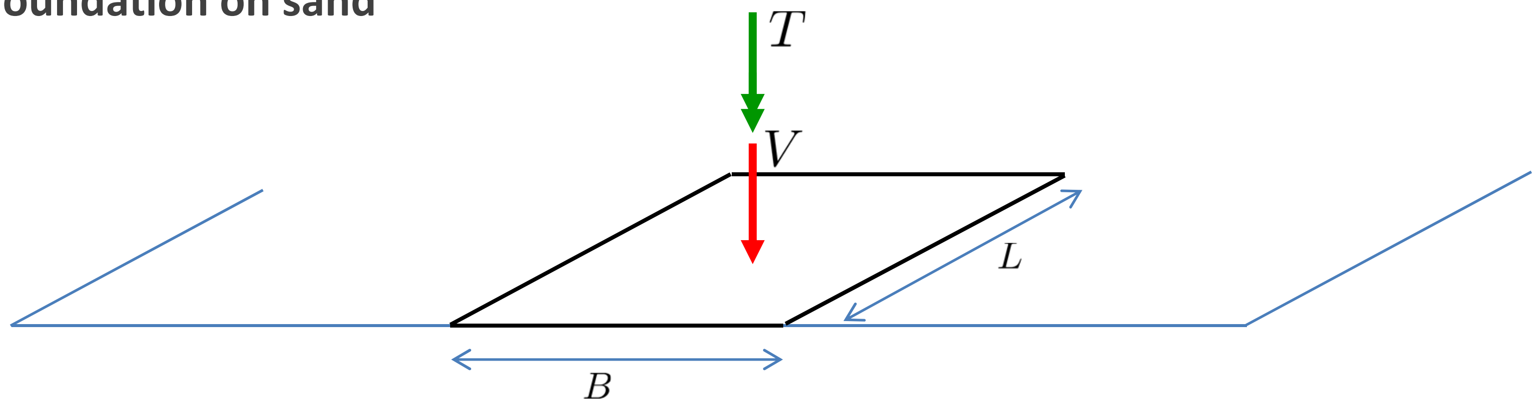
Modified bearing capacity equation:

$$\frac{V_u}{A} = \tfrac{1}{2}\gamma B N_\gamma^* t_\gamma$$

Requirement:

$$t_\gamma = 0 \quad \text{for} \quad T/B = \mu_T V$$

Surface foundation on sand



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Possibility:

$$t_\gamma = 1 - \left(\frac{T}{B\mu_T V}\right)^m$$

Optum CE

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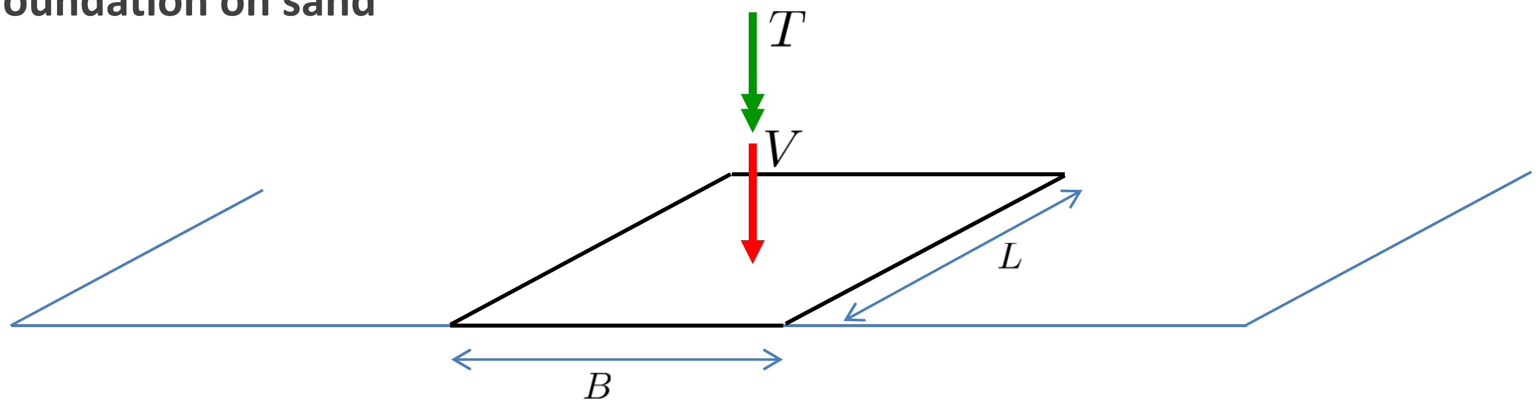

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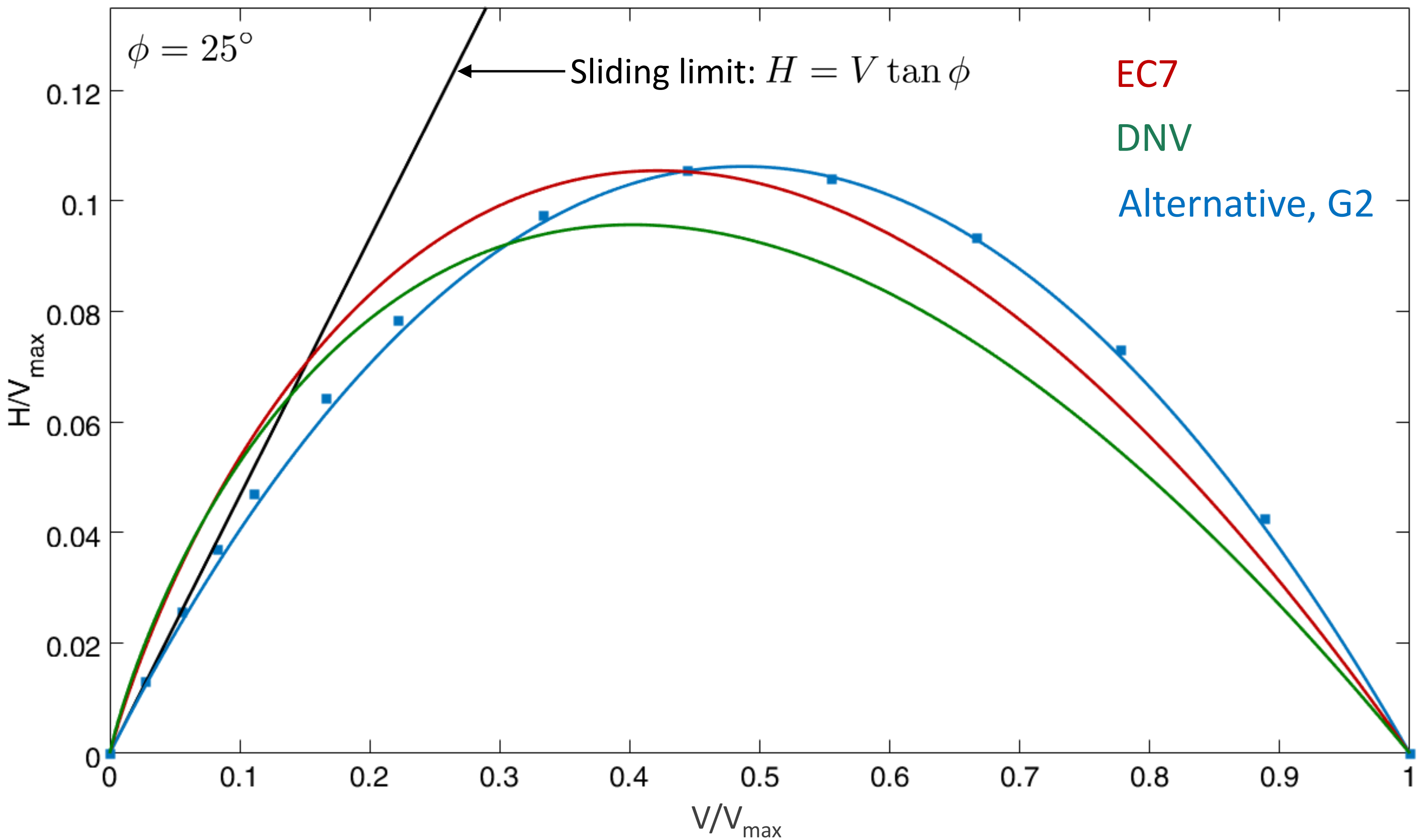
Analogous to inclination factor (strip):

$$i_\gamma = 1 - \left(\frac{H}{V\tan\phi}\right)^m, \quad m = \frac{40.6\tan\phi}{20.7 - 8.8\tan\phi}$$

Optum CE

Inclined load



Optum CE

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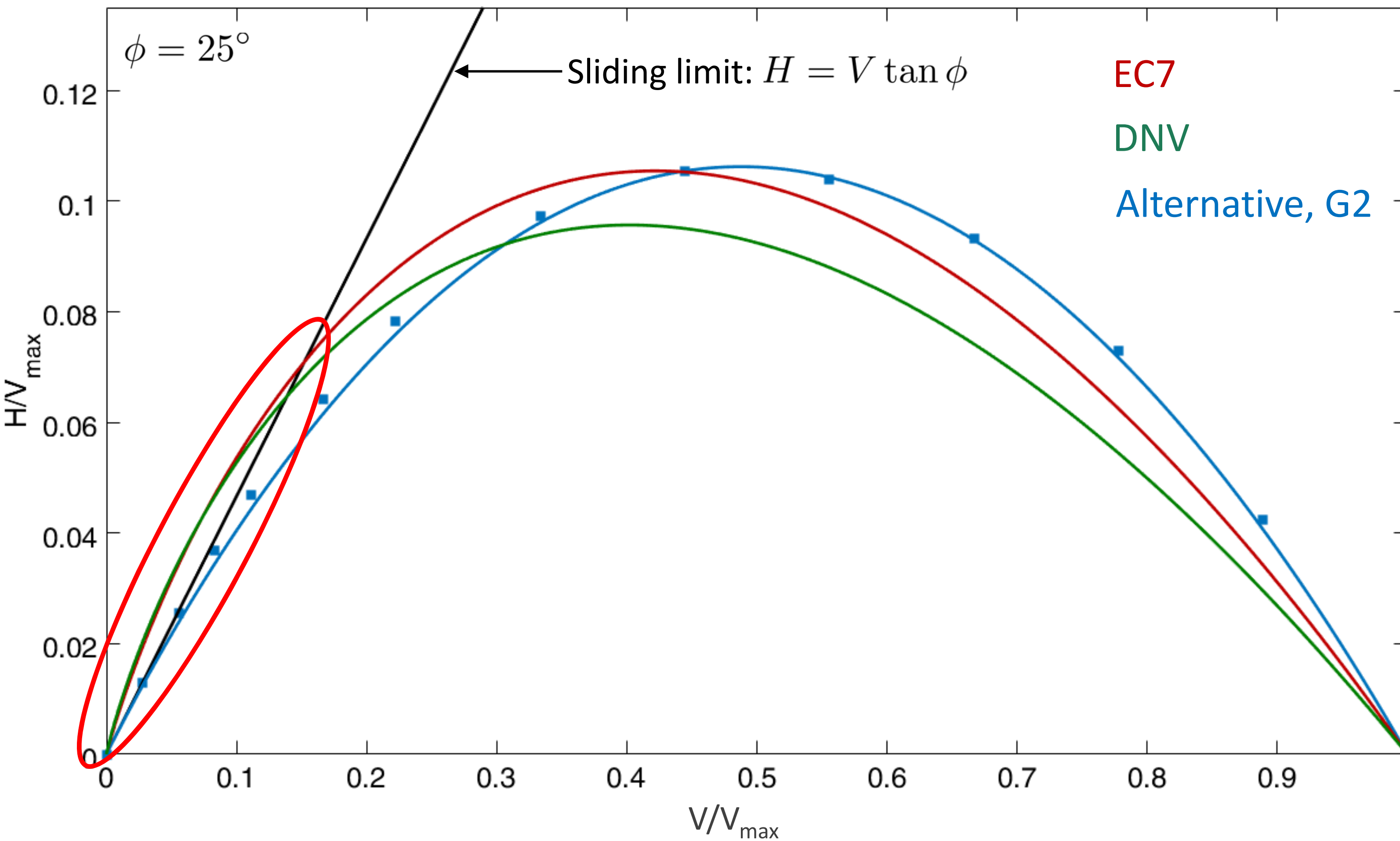

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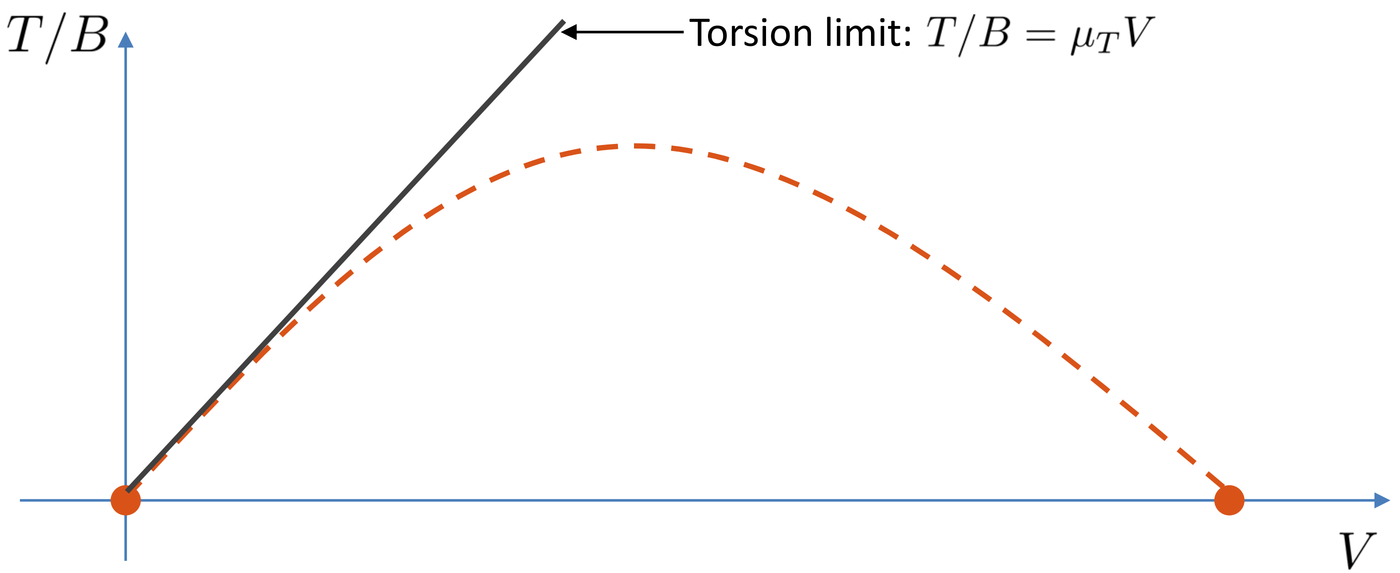

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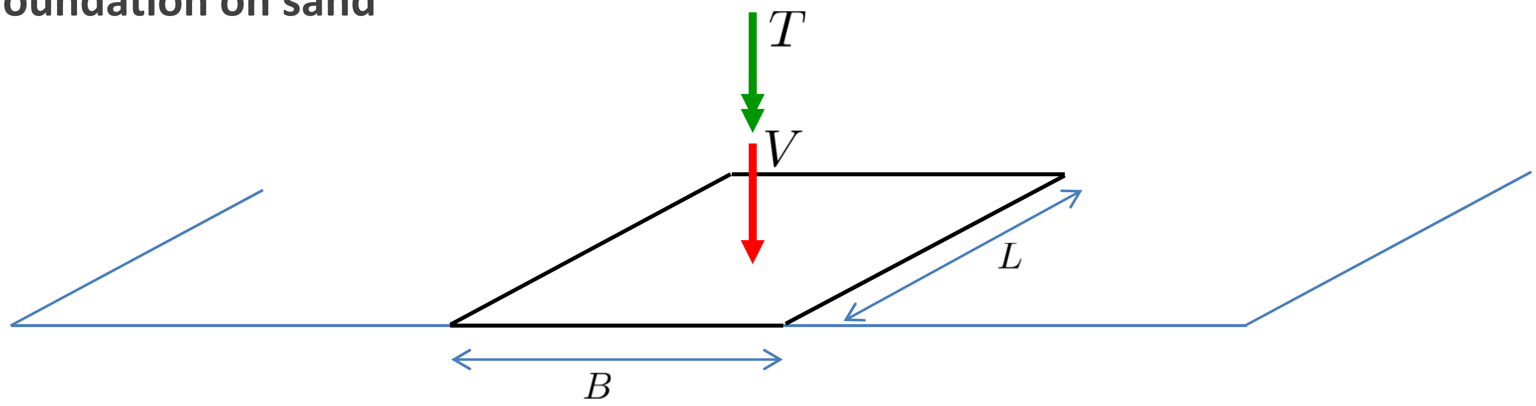

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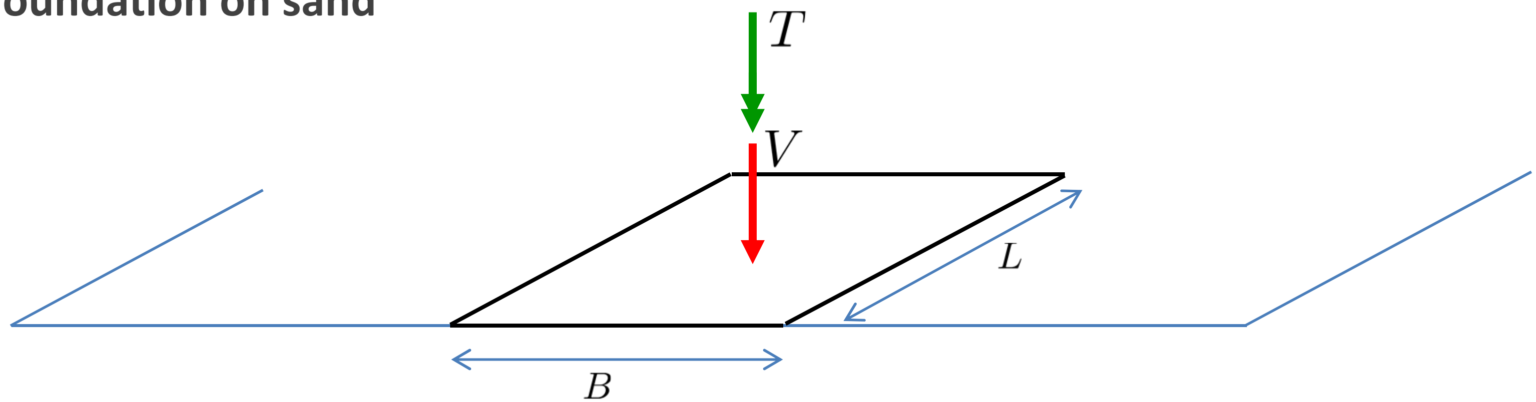

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TORSION

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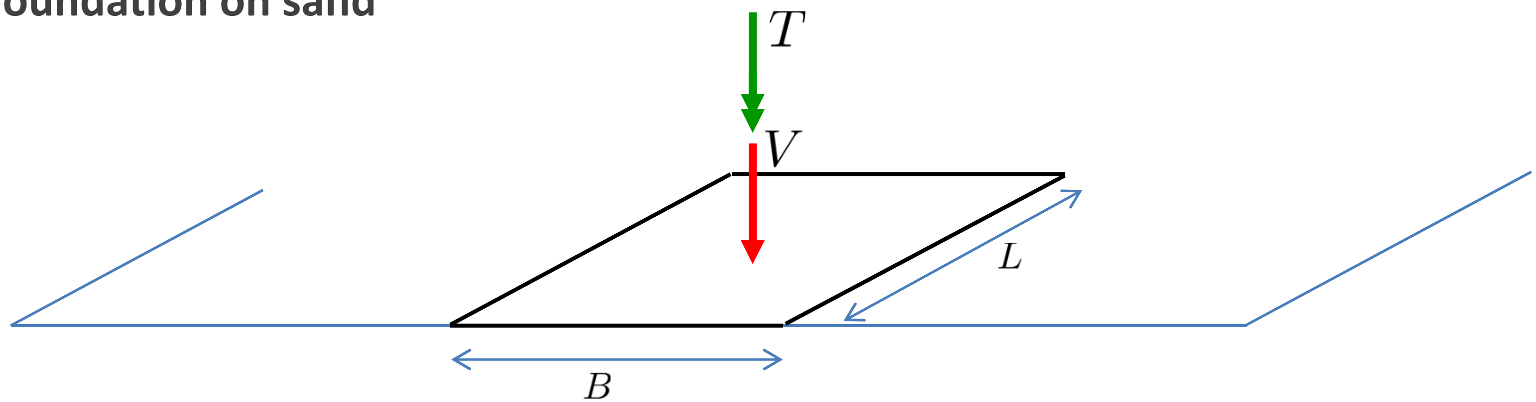

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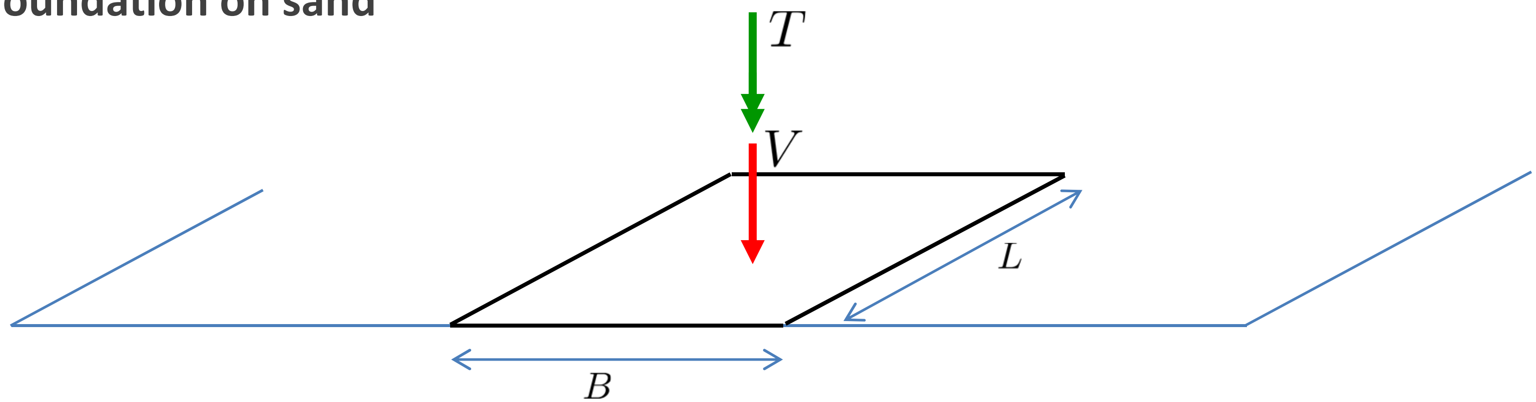

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where

$$m = m(\phi), \quad n = n(B/L)$$

(hopefully)

TORSION

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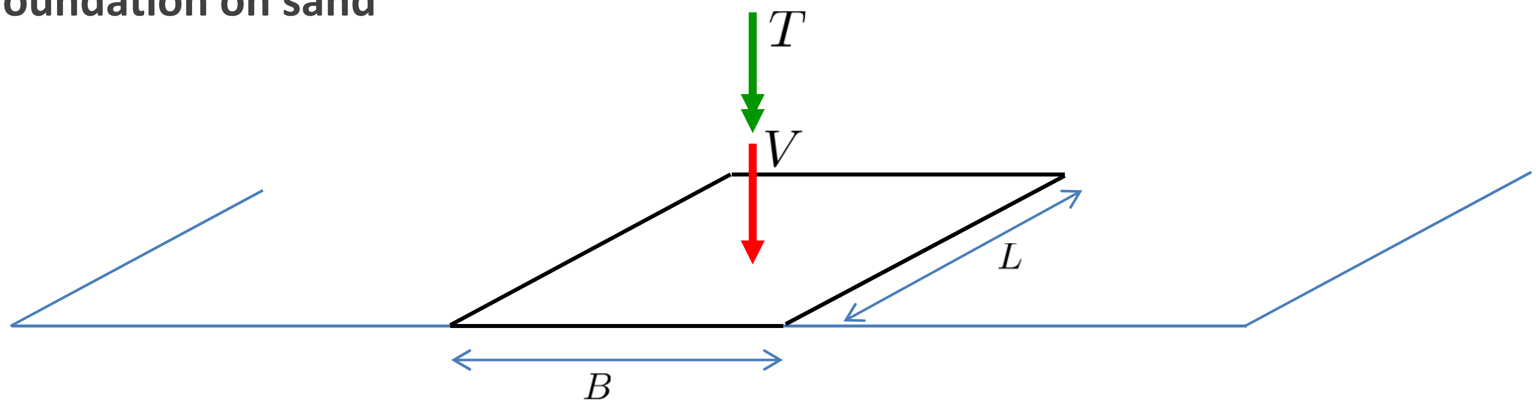

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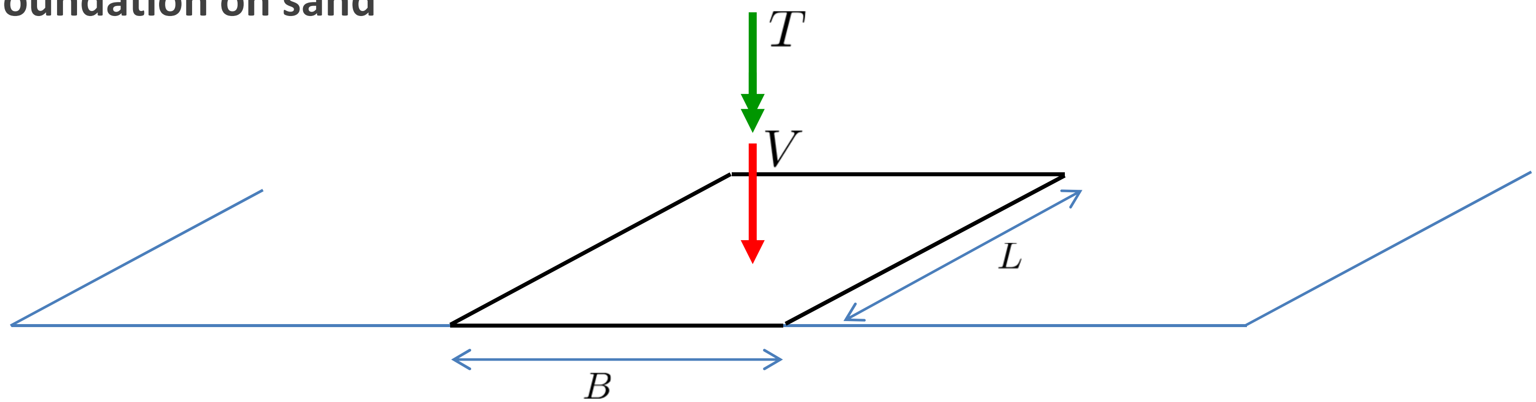
Optum^CE

Surface foundation on sand



V-T diagram:

$$\frac{T}{B\mu_T V_{\max}} = \left[1 - \left(\frac{V}{V_{\max}}\right)^{\frac{1}{m}}\right]^{\frac{1}{n}} \frac{V}{V_{\max}}$$

where

$$V_{\max} = \tfrac{1}{2}\gamma B N_\gamma^*$$

$$\mu_T = \tfrac{1}{2}\sqrt{1 + \left(\frac{L}{B}\right)^2}\,\tan\phi$$

V-T diagram

Optum[CE]

V-T diagram

Optum^CE

V-T diagram

Optum CE

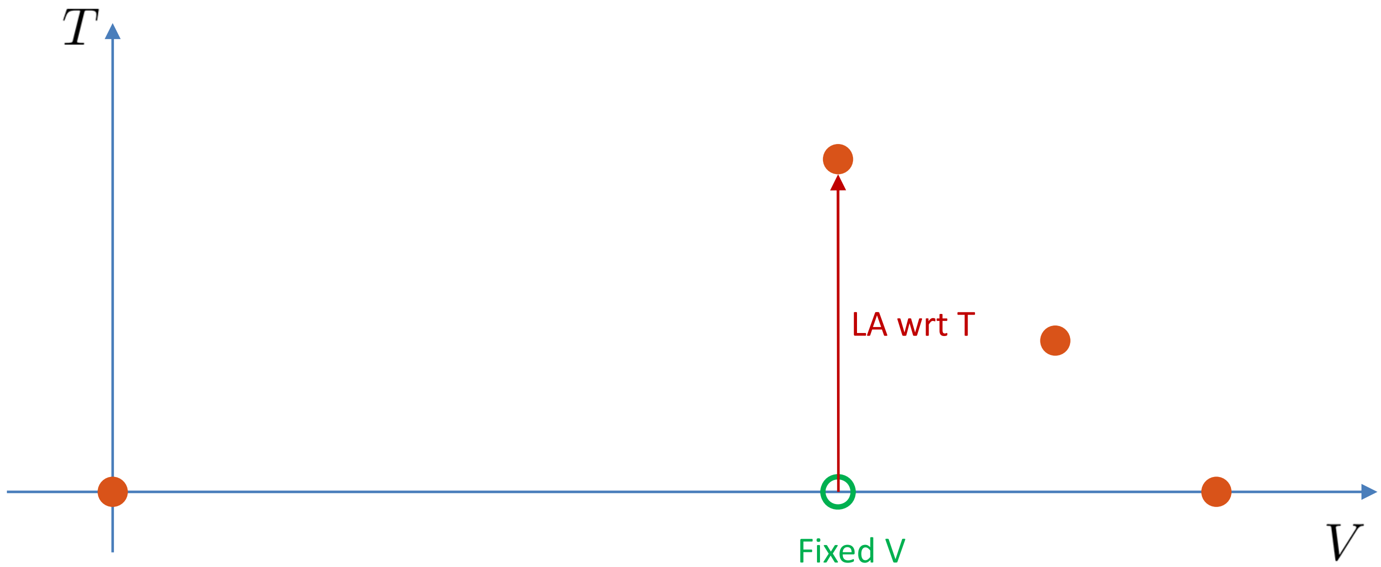
V-T diagram

TORSION

V-T diagram

V-T diagram

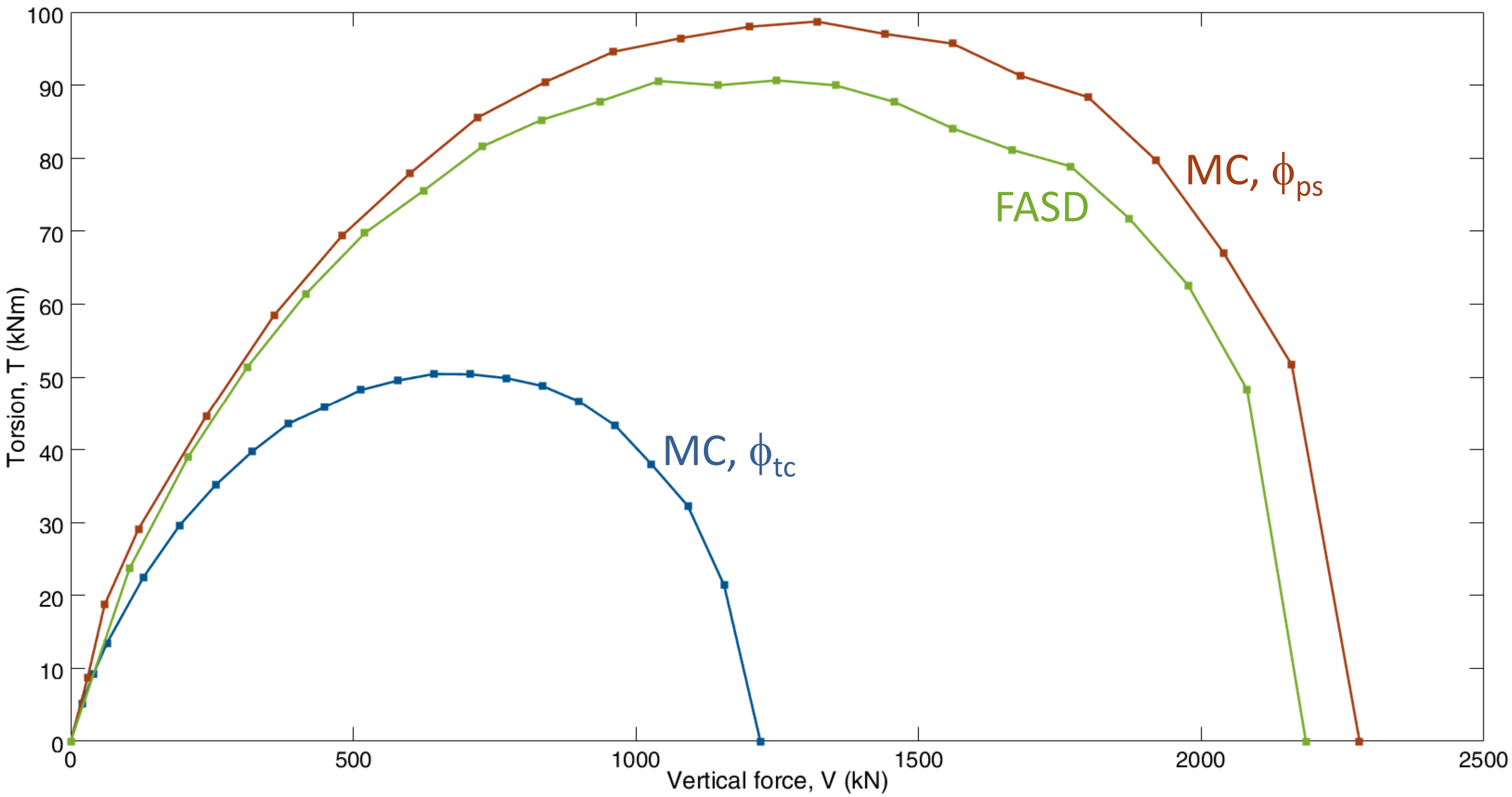
V-T diagram



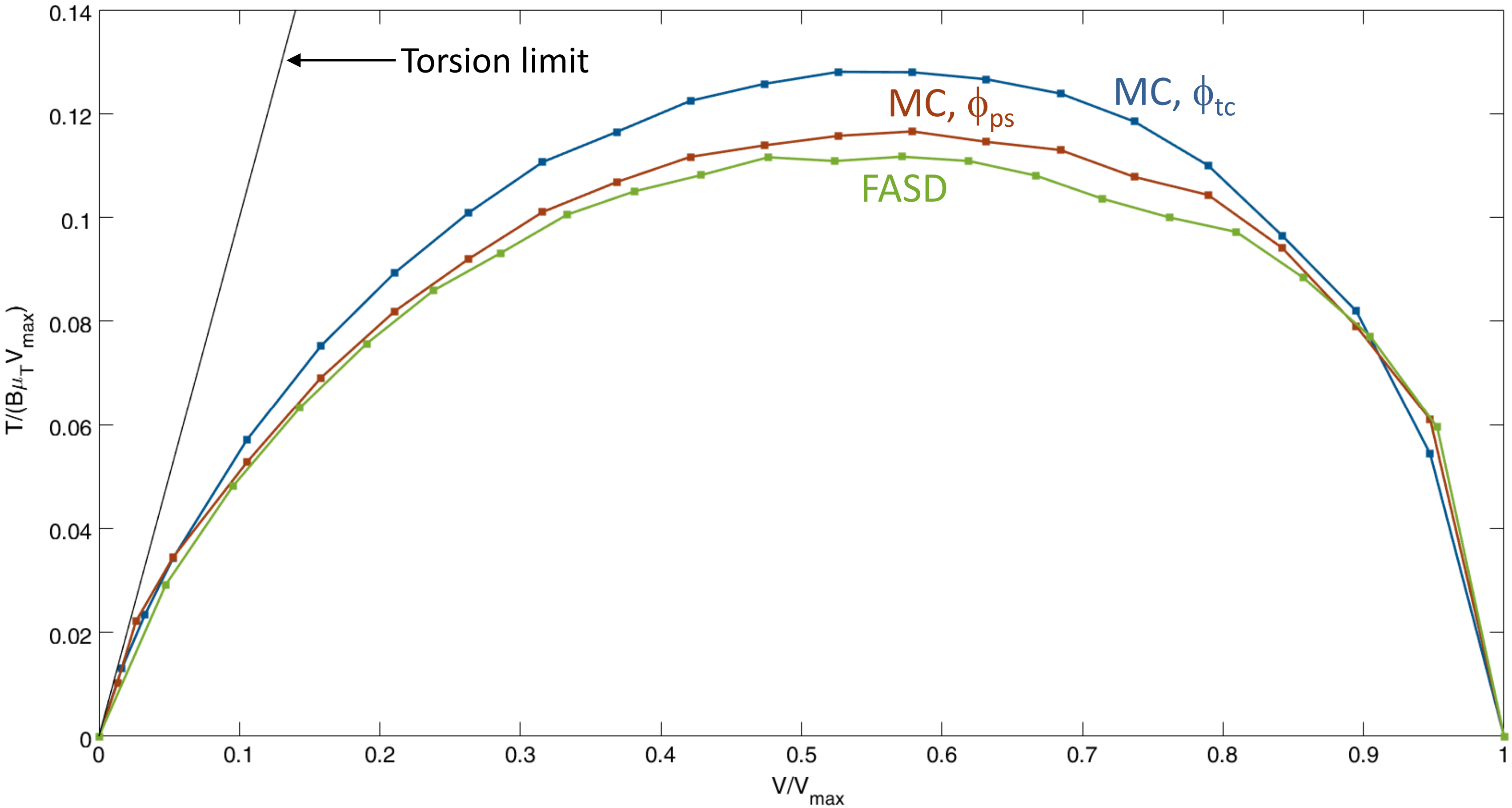
V-T diagram

TORSION

V-T diagrams

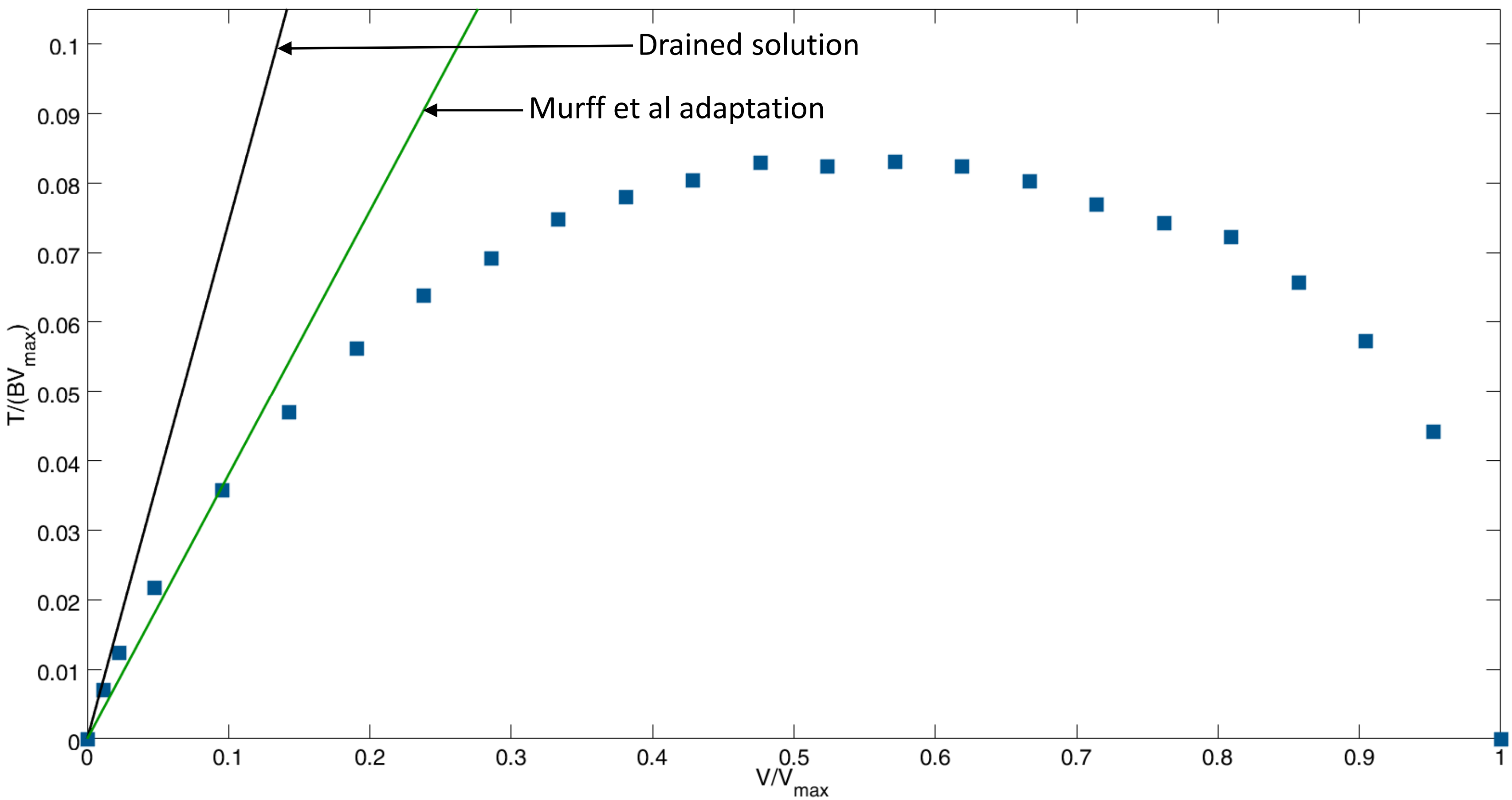


V-T diagrams


Torsion limit
MC, ϕtc
MC, ϕps
FASD
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
T/(BμTVmax)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
V/Vmax

TORSION

V-T diagram - FASD



V-T diagram - FASD