THE BEARING CAPACITY EQUATION
- why don't I get the same result with OPTUM?

KRISTIAN KRABBENHOFT

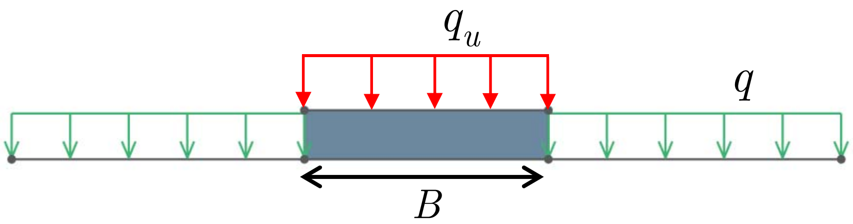
Optum Computational Engineering

Optum[CE]

Outline

+ Bearing capacity factor N_γ
+ Superposition principle
+ Inclined loading
+ Eccentricity
+ Shape – 2D to 3D

BEARING CAPACITY EQUATION

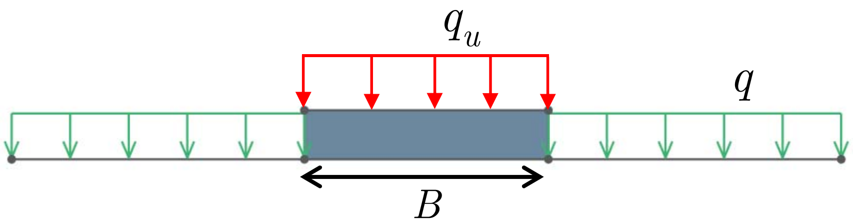


The bearing capacity equation:

$$q_u = cN_c + qN_q + \frac{1}{2}\gamma B N_\gamma$$

where N_c, N_q, N_γ = bearing capacity factors – functions of ϕ

BEARING CAPACITY EQUATION



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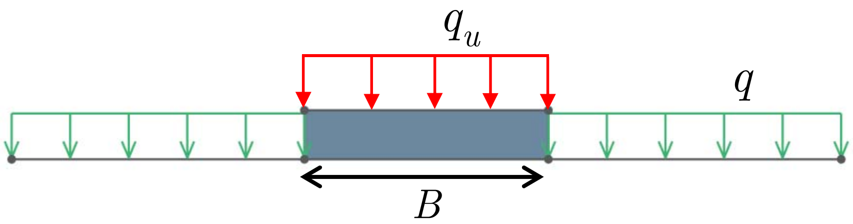
Undrained:

$$q_u = (2+\pi)s_u + q$$

Drained:

$$q_u = qN_q + \tfrac{1}{2}\gamma B N_\gamma$$

BEARING CAPACITY EQUATION



The bearing capacity equation:

$$q_u = cN_c + qN_q + \tfrac{1}{2}\gamma B N_\gamma$$

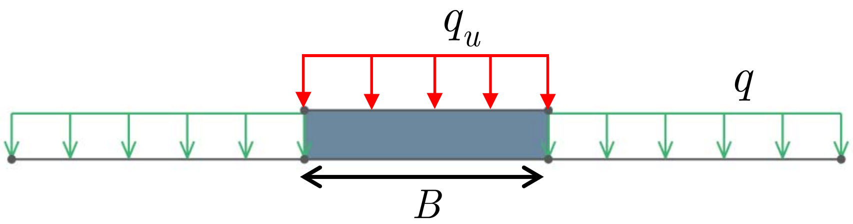
where

$$N_q = \exp(\pi \tan\phi)\tan^2(45^\circ + \tfrac{1}{2}\phi)$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = 2(N_q - 1)\tan\phi$$

BEARING CAPACITY EQUATION



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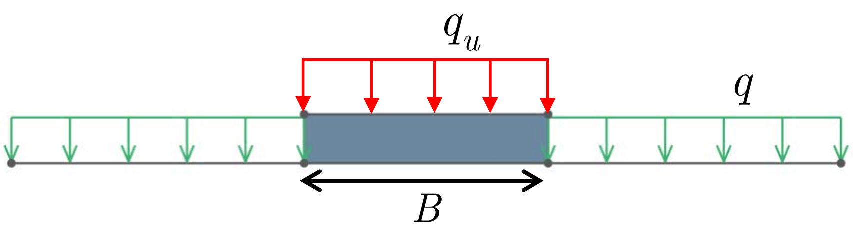

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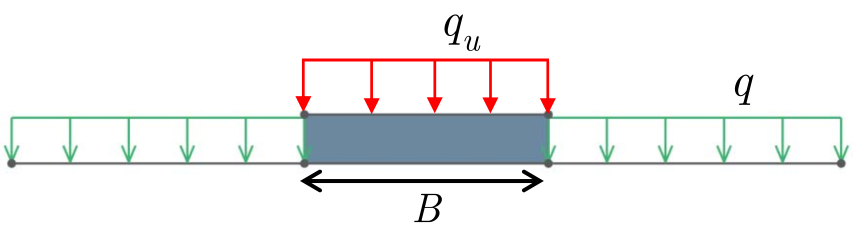
BEARING CAPACITY EQUATION



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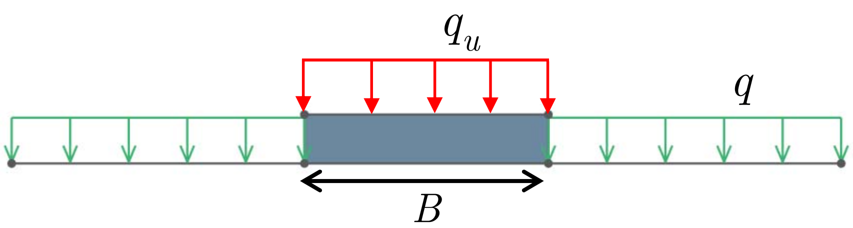
$$N_\gamma = 2(N_q - 1)\tan\phi \qquad \text{(EC7)}$$

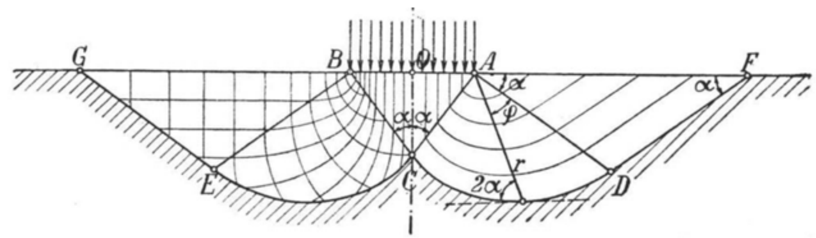
$$= 1.5(N_q - 1)\tan\phi \qquad \text{(Brinch Hansen)}$$

$$= (N_q - 1)\tan(1.4\phi) \qquad \text{(Meyerhof)}$$

$$\vdots$$

BEARING CAPACITY EQUATION





Figur 1.

Weightless soil:

$$q_u = cN_c + qN_q$$

where

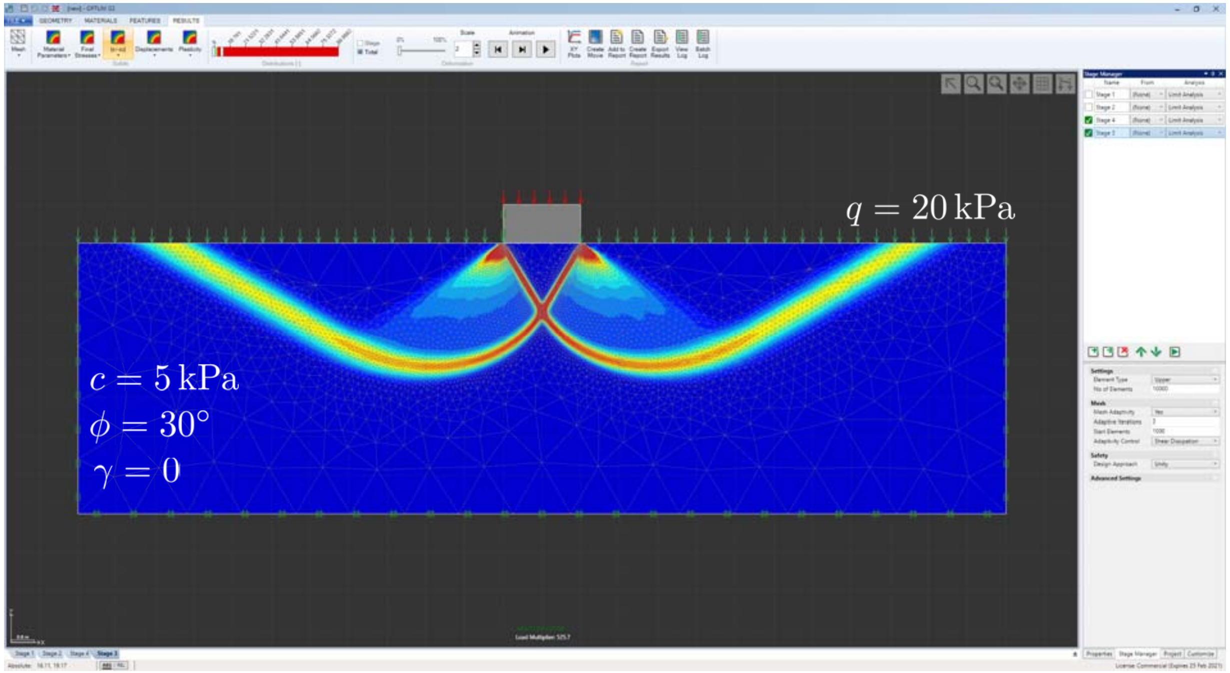
$$N_q = \exp(\pi \tan \phi) \tan^2(45^\circ + \tfrac{1}{2}\phi)$$

$$N_c = (N_q - 1) \cot \phi$$

This solution is exact **(Prandtl 1921, Reissner 1924)**

BEARING CAPACITY EQUATION

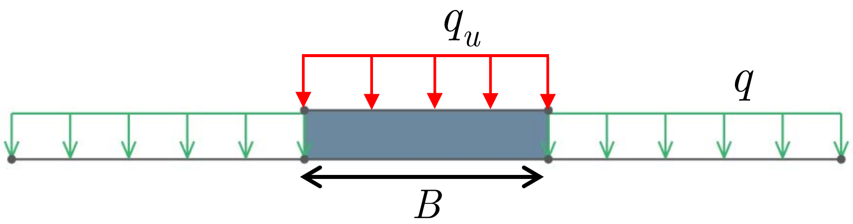
OPTUM G2



10,000 elements (LB/UB) + 3 adaptive iterations (sol time ≈ 20 sec):

q_u (kN/m²)				
LB	**UB**	**(LB+UB)/2**	**Exact**	**Error (%)**
509.5	525.7	517.6	518.7	-0.2%

BEARING CAPACITY EQUATION



Weightless soil:

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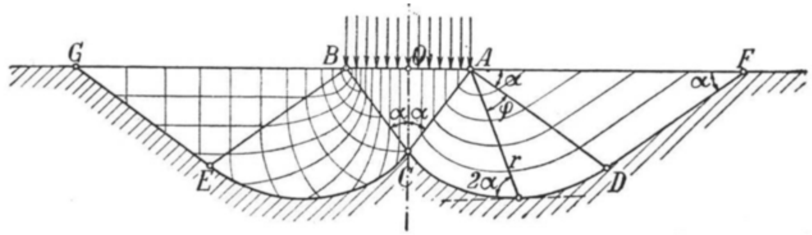

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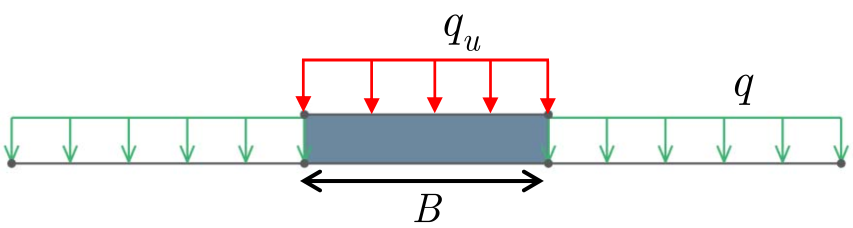


Ponderable soil – superposition:

$$q_u = cN_c + qN_q + \tfrac{1}{2}\gamma B N_\gamma$$

where

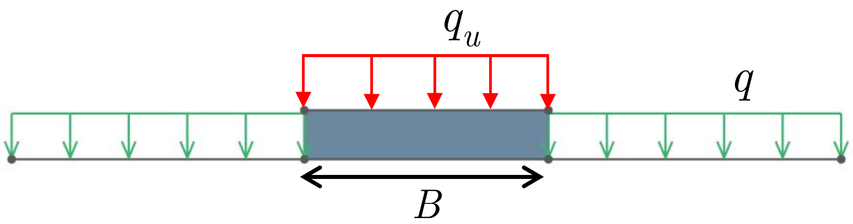
$$N_q = \exp(\pi \tan\phi)\tan^2(45^\circ + \tfrac{1}{2}\phi)$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = \ ?$$

This solution is conservative – provided that N_γ is exact

BEARING CAPACITY EQUATION



Exact N_γ – zero cohesion + zero surcharge:

$$q_u = \frac{1}{2}\gamma B N_\gamma$$

or

$$N_\gamma = \frac{2q_u}{\gamma B}$$

100 year search for exact N_γ

60 N_γ expressions (Diaz-Segura 2013)

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NRC Research Press

NOTE

Assessment of the range of variation of N_γ from 60 estimation methods for footings on sand

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Can. Geotech. J. **50**: 793–800 (2013) dx.doi.org/10.1139/cgj-2012-0426

Published at www.nrcresearchpress.com/cgj on 9 May 2013.

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60 N_γ expressions (Diaz-Segura 2013)

Table 1. Expressions for the estimation of the N_γ factor.

Author	Expression
Terzaghi (1943); fitted expression; *limit equilibrium*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) + 3.0\right]\tan(1.34\phi)$
Taylor (1948); *limit equilibrium*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) - 1\right]\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)$
Caquot and Kérisel (1953); fitted from Ukritchon et al. (2003); *method of characteristics*	$N_\gamma = \left[1.413\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) + 1.794\right]\tan(1.27\phi)$
Biarez et al. (1961); *equilibrium limit*	$N_\gamma = 1.8\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) - 1\right]\tan\phi$
Feda (1961); *empirical*	$N_\gamma = 0.01\exp(\phi/4)$ (for $\phi < 35°$; ϕ in degrees)
Meyerhof (1963); *semi-empirical based on limit equilibrium*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) - 1\right]\tan(1.4\phi)$
Hu (1964); fitted expression; *equilibrium limit*	$N_\gamma = \left[1.901\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) + 0.27\right]\tan(1.285\phi)$
Krizek (1965); *empirical*	$N_\gamma = \frac{6\phi}{40-\phi}$ (for $\phi < 35°$; ϕ in degrees)
Booker (1969); *method of characteristics*	$N_\gamma = 0.1045\exp(9.6\phi)$
Hansen and Christensen (1969); fitted expression; *method of characteristics*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) - 1\right]\tan(1.33\phi)$
Muhs and Weiss (1969); (Eurocode 7); *semi-empirical expression*	$N_\gamma = 2\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) - 1\right]\tan\phi$
Abdul-Baki and Beik (1970); fitted expression; *limit equilibrium*	$N_\gamma = \left[1.752\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) + 0.186\right]\tan(1.32\phi)$
Brinch-Hansen (1970); *semi-empirical based on Lundgren-Mortensen (1953) failure mechanics*	$N_\gamma = 1.5\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) - 1\right]\tan\phi$
Davis and Booker (1971); fitted expression; *limit equilibrium*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) + 2.33\right]\tan(1.316\phi)$
Chummar (1972); fitted expression; *semi-empirical*	$N_\gamma = \left[7.12\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) + 65.5\right]\tan(0.27\phi)$
Vesic (1973); *approximation based on Caquot and Kérisel (1953) analysis using the method of characteristics*	$N_\gamma = 2\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) + 1\right]\tan\phi$
Chen (1975); *upper bound limit analysis*	$N_\gamma = 2\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) + 1\right]\tan\phi\tan\left(\frac{\pi}{4}+\frac{\phi}{5}\right)$
Chen (1975); fitted from mechanics two values; *upper bound limit analysis*	$N_\gamma = \left[1.45\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) + 0.754\right]\tan(1.41\phi)$
Salençon et al. (1976); fitted expression; *limit equilibrium*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) - 1.0\right]\tan(1.405\phi)$
Steenfelt (1977); *empirical fitting from N_γ values obtained from Lundgren and Mortensen (1953)*	$N_\gamma = [0.08705 + 0.3231\sin(2\phi) - 0.04836\sin^2(2\phi)]\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(1.5\pi\tan\phi) - 1\right]$
Craig and Pariti (1978); fitted expression; *limit equilibrium*	$N_\gamma = \left[2.22\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) + 0.222\right]\tan\phi$
Spangler and Handy (1982); *approximation from Terzaghi's Mechanism*	$N_\gamma = 1.1\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi) - 1\right]\tan(1.3\phi)$
Ingra and Baecher (1983); *statistical analysis of footing load test data*	$N_\gamma = \exp(0.173\phi - 1.646)$ (ϕ in degrees)

60 N_γ expressions (Diaz-Segura 2013)

Table 1 (*continued*).

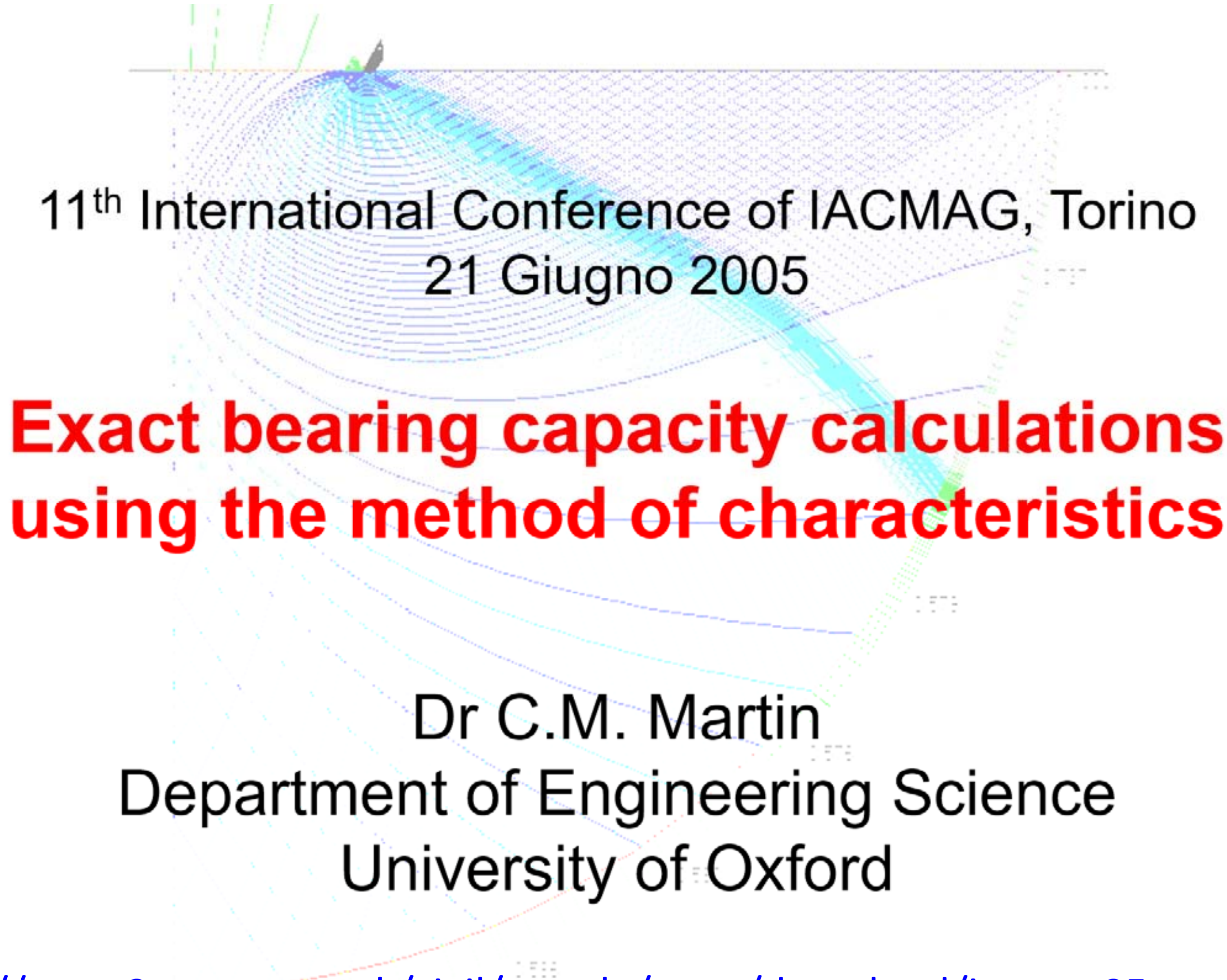
Author	Expression
Simone and Restaino (1984); fitted expression; *method of characteristics*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) - 1\right]\tan(1.341\phi)$
Hettler and Gudehus (1988); *empirical*	$N_\gamma = \exp[5.71(\tan\phi)^{1.15}] - 1.0$
Saran and Agarwal (1991); fitted expression; *limit equilibrium*	$N_\gamma = \exp\left(\frac{0.757}{\ln\phi} + 15.286\phi - 3.452\right)$
Bolton and Lau (1993); *method of characteristics*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) - 1\right]\tan(1.5\phi)$
Bolton and Lau (1993); fitted expression from original values; *method of characteristics*	$N_\gamma = \left[1.274\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) + 3.736\right]\tan(1.367\phi)$
Kumbhojkar (1993); fitted expression; *numerical solution by graphical method*	$N_\gamma = \left[1.2\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) + 1.324\right]\tan(1.417\phi)$
Zadroga (1994); *empirical expression*	$N_\gamma = 0.657\exp(0.141\phi)$ (ϕ in degrees)
Manoharan and Dasgupta (1995); fitted expression; *finite element nonassociated flow rule*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) + 3.464\right]\tan(1.279\phi)$
Bowles (1996); fitted expression from $K_{p\gamma}$ values; *limit equilibrium*	$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$ $K_{p\gamma} = \exp\left(1.708 + 3.287\phi - \frac{0.34}{\ln\phi}\right)$
Frydman and Burd (1997); fitted expression; *finite difference analysis*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) + 1.0\right]\tan(1.4\phi)$
Michalowski (1997); *upper bound limit analysis*	$N_\gamma = \exp(0.66 + 5.11\tan\phi)\tan\phi$
Paolucci and Pecker (1997); fitted expression; *upper bound limit analysis*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) + 1.0\right]\tan(1.71\phi)$
Danish standard DS415 (Danish Standards Association 1998); *empirical fitting from* N_γ *values obtained from Lundgren and Mortensen (1953)*	$N_\gamma = \frac{1}{4}\left\{\left[\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) - 1.0\right]\cos\phi\right\}^{1.5}$
Soubra (1999); fitted expression; *upper bound limit analysis*	$N_\gamma = \left[1.374\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) - 0.162\right]\tan(1.343\phi)$
Coduto (2001); *approximation from Terzaghi's Mechanism*	$N_\gamma = \frac{2\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1\right]\tan\phi}{1+0.4\sin(4.0\phi)}$
Perkins and Madson (2000); *upper-bound analysis based on Chen (1975)*	$N_\gamma = \frac{1}{2}\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\left[\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(1.5\pi\tan\phi) - 1\right] + \frac{\sin\phi\cos\phi}{(1+8\sin^2\phi)(1-\sin\phi)}\left[\left(\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) - \frac{\cot\phi}{3}\right)\exp(1.5\pi\tan\phi) + \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\frac{\cot\phi}{3} + 1\right]$
Poulos et al. (2001); *solution based on Davis and Booker (1971)*	$N_\gamma \approx 0.1054\exp(9.6\phi)$
Ueno et al. (2001); fitted expression; *method of characteristics*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) - 1.0\right]\tan(1.461\phi)$
Wang et al. (2001); fitted expression for mechanics one; *upper bound limit analysis*	$N_\gamma = 1.2\left[\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) + 4.6\right]\tan(1.436\phi)$
Wang et al. (2001); fitted expression for mechanics two; *upper bound limit analysis*	$N_\gamma = \left[1.234\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) + 4.151\right]\tan(1.394\phi)$
Zhu et al. (2001); case 1; *limit equilibrium*	$N_\gamma = \left[2\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\exp(\pi\tan\phi) + 1\right](\tan\phi)^{1.35}$

BEARING CAPACITY EQUATION

60 N_γ expressions (Diaz-Segura 2013)

Table 1 (*concluded*).

Author	Expression
Zhu et al. (2001); case 2; *limit equilibrium*	$N_\gamma = \left[2\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1\right]\tan(1.07\phi)$
Cassidy and Houlsby (2002); fitted expression; *method of characteristics*	$N_\gamma = \left[0.85\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-3.884\right]\tan(1.716\phi)$
Dewaikar and Mohapatra (2003); fitted expression; *limit equilibrium — Terzaghi's mechanism*	$N_\gamma = \left[1.626\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+2.019\right]\tan(1.373\phi)$
Kumar (2003); fitted expression; *method of characteristics*	$N_\gamma = \left[0.96\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+0.508\right]\tan(1.352\phi)$
Kumar (2003); fitted expression; *upper bound analysis — both sides failure mechanism*	$N_\gamma = \left[1.379\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-0.461\right]\tan(1.337\phi)$
Ukritchon et al. (2003); fitted expression from mean values; *lower and upper bound analysis*	$N_\gamma = \left[1.279\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-3.057\right]\tan(1.219\phi)$
Hjiaj et al. (2005); *lower and upper bound analysis*	$N_\gamma = \exp\left[\frac{\pi}{6}(1+3\pi\tan\phi)\right](\tan\phi)^{2\pi/5}$
Martin (2005); fitted expression *method of characteristics*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.338\phi)$
Smith (2005); *method of characteristics*	$N_\gamma = 1.75\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp[(0.75\pi+\phi)\tan\phi]-1.0\right]\tan\phi$
Kumar and Kouzer (2007); fitted expression; *upper bound limit analysis*	$N_\gamma = \left[1.012\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-0.226\right]\tan(1.426\phi)$
Lyamin et al. (2007); *lower and upper bound analysis*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-0.6\right]\tan(1.33\phi)$
Kumar and Khatri (2008); fitted expression; *lower bound finite elements and linear programming*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.264\phi)$
Salgado (2008); *approximation expression from N_γ values of Martin (2005) and Lyamin et al. (2007)*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.32\phi)$
Yang and Yang (2008); fitted expression; *upper bound limit analysis*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.0\right]\tan(1.396\phi)$
Jahanandish et al. (2010); fitted expression; *zero extension lines method*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.0\right]\tan(1.5\phi)$
Kumar and Khatri (2011); fitted expression; *lower bound with finite element and linear programming*	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-5.115\right]\tan(1.577\phi)$



http://www2.eng.ox.ac.uk/civil/people/cmm/download/iacmag05_cmm.ppt

Optum^CE

Exact bearing capacity calculations using the method of characteristics

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Keywords: bearing capacity, shallow foundation, cohesive-frictional, limit analysis

ABSTRACT: This paper discusses the use of the method of characteristics (commonly referred to as the slip-line method) to solve the classic geotechnical bearing capacity problem of a vertically loaded, rigid strip footing resting on a cohesive-frictional halfspace. It would appear that, contrary to popular belief, the method of characteristics can be used to establish the exact plastic collapse load for any combination of the parameters c, ϕ, γ, B and q – including the infamous 'N_γ problem'. This applies to footings of arbitrary roughness, though only the extreme cases (smooth and fully rough) are considered in detail here.

http://www2.eng.ox.ac.uk/civil/people/cmm/download/iacmag05_cmm.pdf

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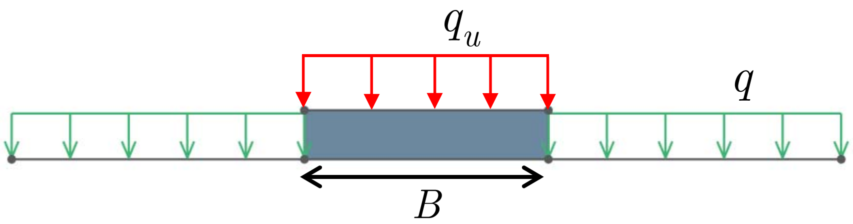
N_γ ($\delta = \phi$) by common formulae: error [%]

ϕ [°]	Meyerhof (1963)	Hansen (1970)	Vesić (1975)	Eurocode (1996)	Poulos et al. (2001)
5	-38.5	-34.3	296.3	-12.4	114.9
10	-15.3	-10.2	182.6	19.8	30.0
15	-4.4	0.1	124.1	33.4	10.1
20	1.1	3.8	89.7	38.4	5.9
25	4.2	4.1	67.6	38.8	7.1
30	6.2	2.1	51.8	36.2	8.9
35	7.8	-1.6	39.3	31.2	7.7
40	9.5	-7.0	27.9	23.9	0.3
45	12.2	-14.3	16.0	14.3	-15.3

http://www2.eng.ox.ac.uk/civil/people/cmm/download/iacmag05_cmm.ppt

BEARING CAPACITY EQUATION



Exact N_γ – zero cohesion + zero surcharge:

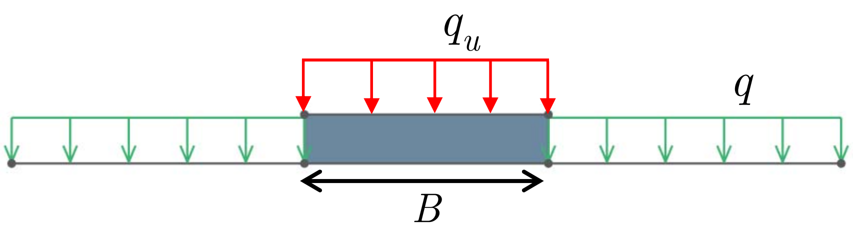
$$q_u = \frac{1}{2}\gamma B N_\gamma$$

or

$$N_\gamma = \frac{2q_u}{\gamma B}$$

Exact N_γ determined by CM Martin in 2005 using method of characteristics

BEARING CAPACITY EQUATION



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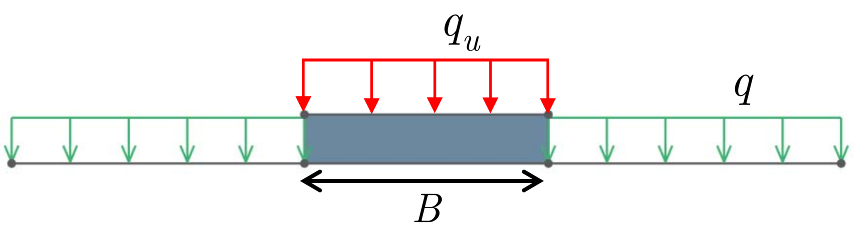
BEARING CAPACITY EQUATION



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Good approximation:

$$N_\gamma = (N_q - 1)\tan(1.34\phi)$$

OptumCE

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$$N_\gamma = (N_q - 1)\tan(1.34\phi)$$

ϕ (°)	Exact	Approximate	N_γ error (%)
15	1.1814	1.0763	-8.9
20	2.8389	2.7274	-3.9
25	6.4913	6.3952	-1.5
30	14.754	14.705	-0.33
35	34.476	34.512	+0.11
40	85.566	85.716	+0.18
45	234.21	234.71	+0.21

OptumCE

Good approximation:

$$N_\gamma = (N_q - 1)\tan(1.34\phi)$$

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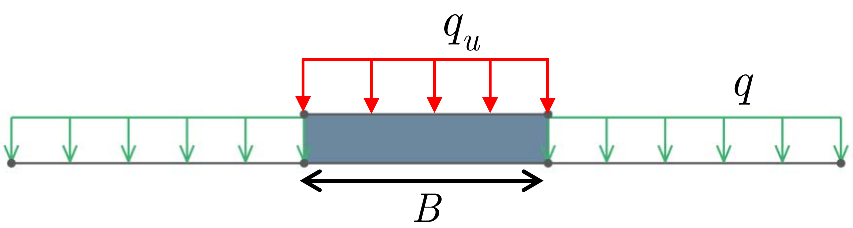
BEARING CAPACITY EQUATION



Final equation:

$$q_u = cN_c + qN_q + \tfrac{1}{2}\gamma B N_\gamma$$

where

$$N_q = \exp(\pi \tan\phi)\tan^2(45^\circ + \tfrac{1}{2}\phi)$$

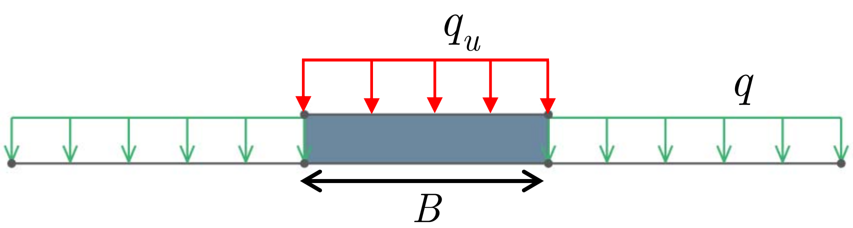
$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.34\phi)$$

***This solution is conservative*[1]**

[1]*To within the approximation of* N_γ

BEARING CAPACITY EQUATION



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General solution: numerical analysis

[1] *To within the approximation of* N_γ

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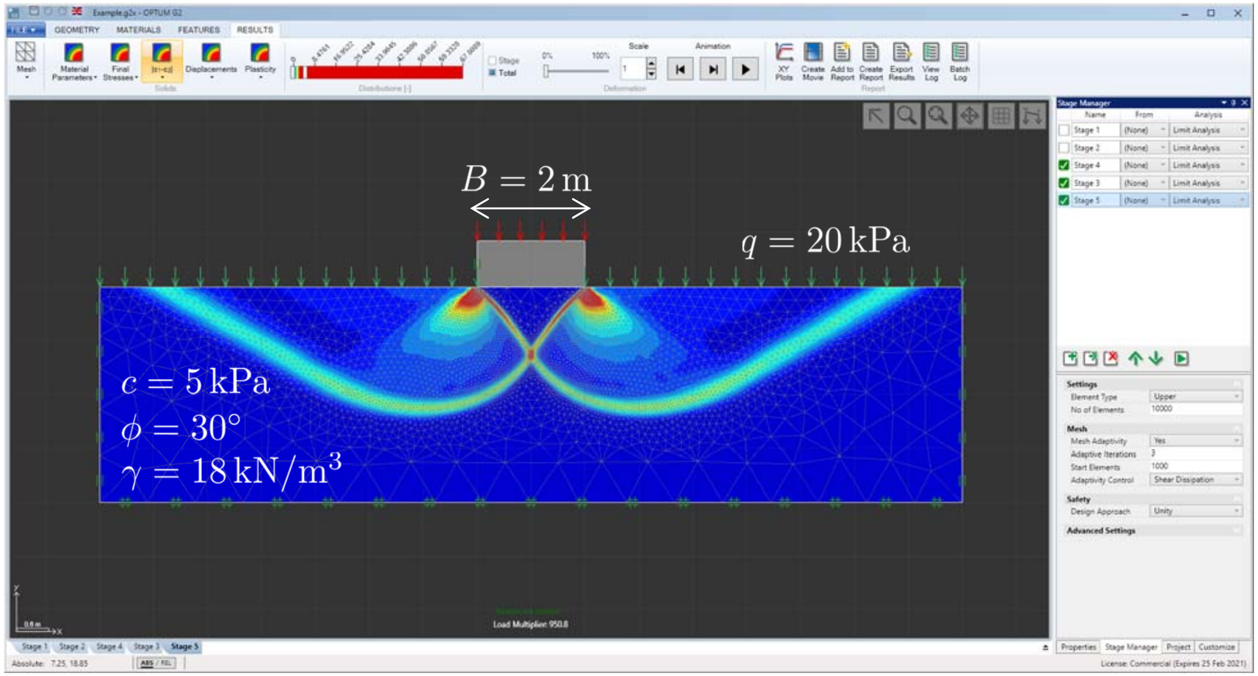
OPTUM G2

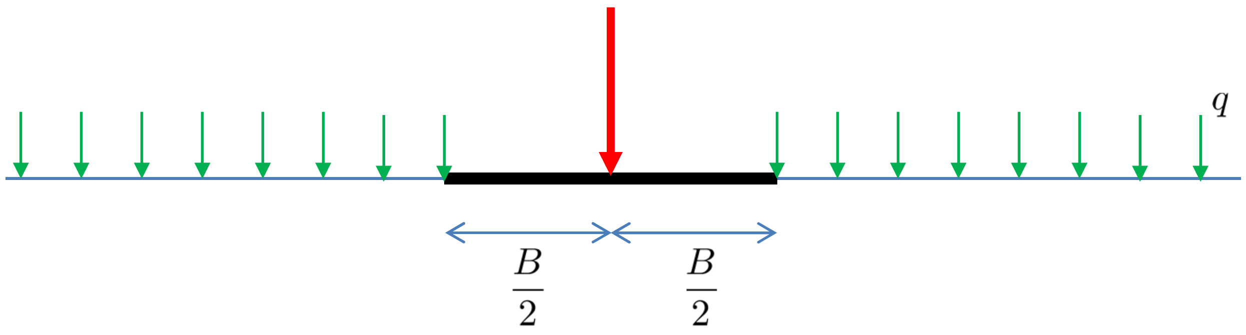


10,000 elements (LB/UB) + 3 adaptive iterations (sol time ≈ 20 sec):

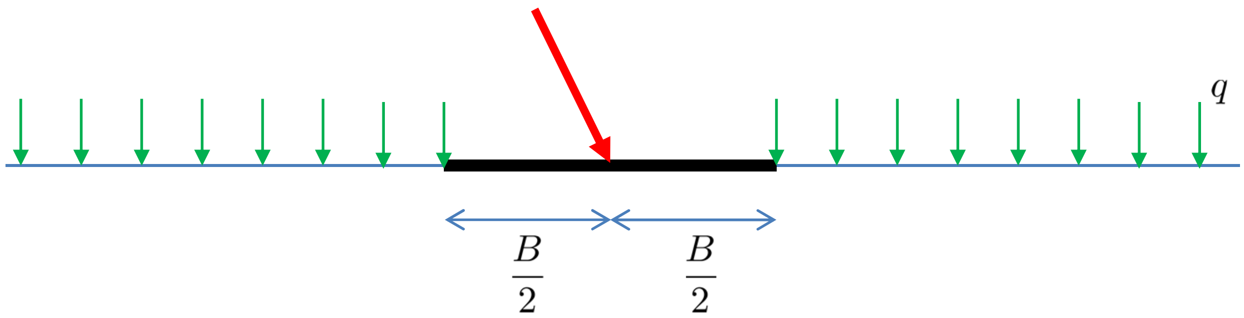
q_u (kN/m²)				
LB	**UB**	**(LB+UB)/2**	**BCE**	**Dev. (%)**
915.4	950.8	933.1	783.4	+16%

Optum CE

Inclined load



Inclined load



BEARING CAPACITY EQUATION

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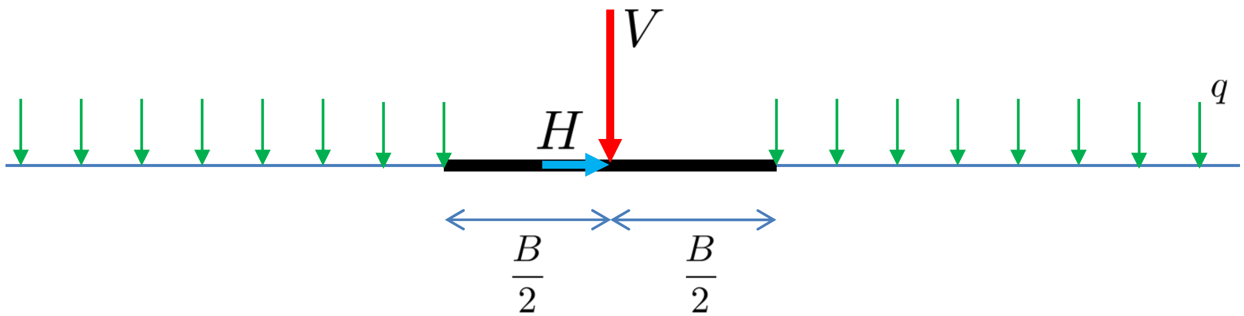


Modified equation:

$$\frac{V_u}{B} = cN_c i_c + qN_q i_q + \tfrac{1}{2}\gamma B N_\gamma i_\gamma$$

where (EC7, strip)

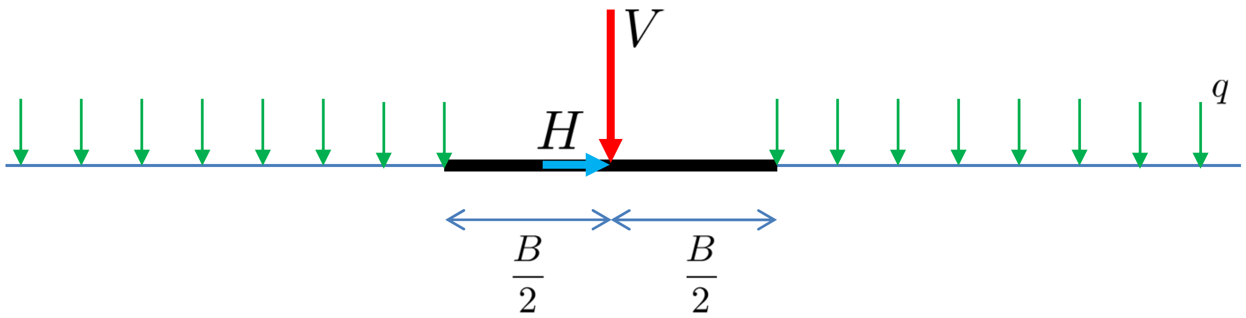
$$i_q = \left(1 - \frac{H}{V + Bc/\tan\phi}\right)^2$$

$$i_c = i_q - \frac{1 - i_q}{N_c \tan\phi}$$

$$i_\gamma = \left(1 - \frac{H}{V + Bc/\tan\phi}\right)^3$$

Optum CE

Inclined load



Surface foundation, c = 0:

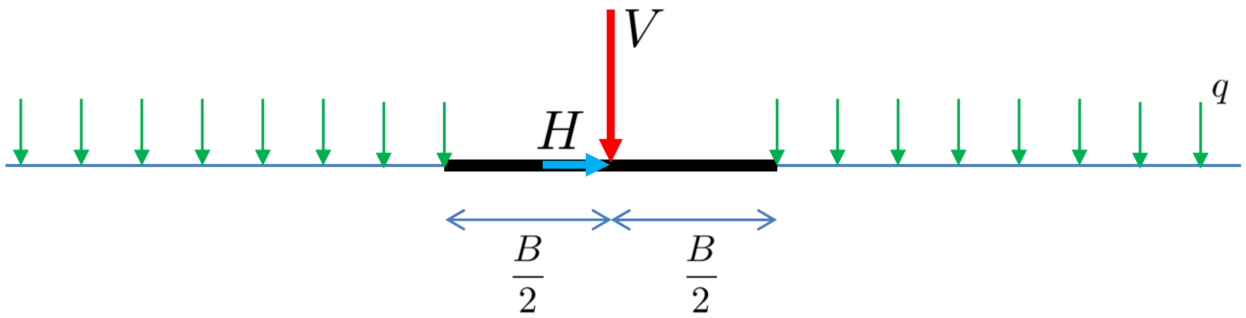
$$\frac{V_u}{B} = \tfrac{1}{2}\gamma B N_\gamma i_\gamma$$

where

$$i_\gamma = \left(1 - \frac{H}{V}\right)^3 \qquad \text{(EC7)}$$

OptumCE

Inclined load



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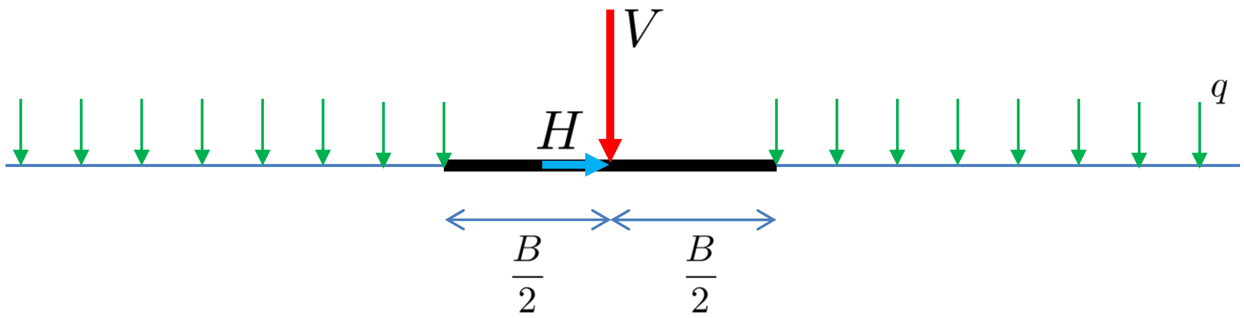
$$i_\gamma = \left(1 - \frac{0.7H}{V}\right)^5 \qquad \text{(DNV)}$$

Optum^CE

Inclined load



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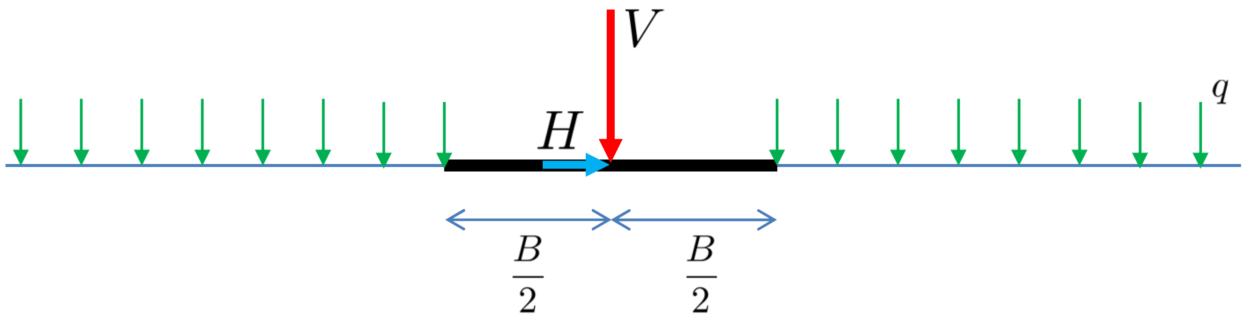
However:

$$i_\gamma > 0 \quad \text{for} \quad H = V \tan\phi \qquad \text{(sliding)}$$

Optum CE

Inclined load



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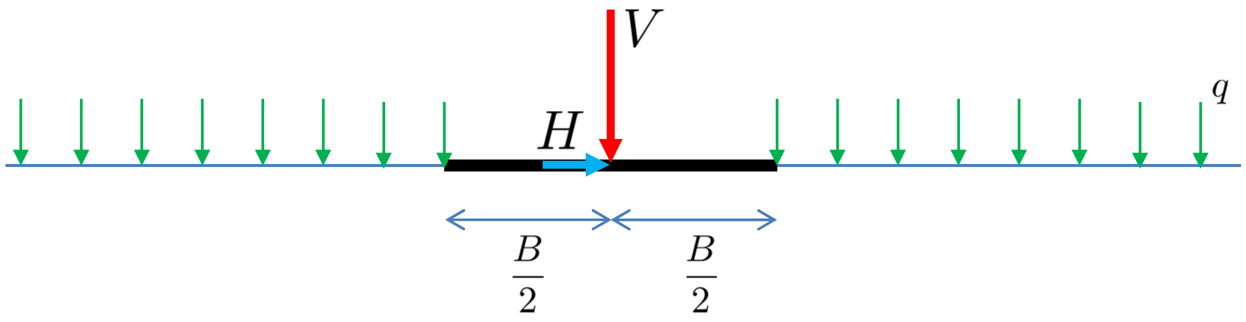
$$i_\gamma = \left(1 - \frac{0.7H}{V}\right)^5 \qquad \text{(DNV)}$$

Alternative:

$$i_\gamma = 1 - \left(\frac{H}{V\tan\phi}\right)^m \quad , \qquad m = \frac{40.6\tan\phi}{20.7 - 8.8\tan\phi}$$

Inclined load

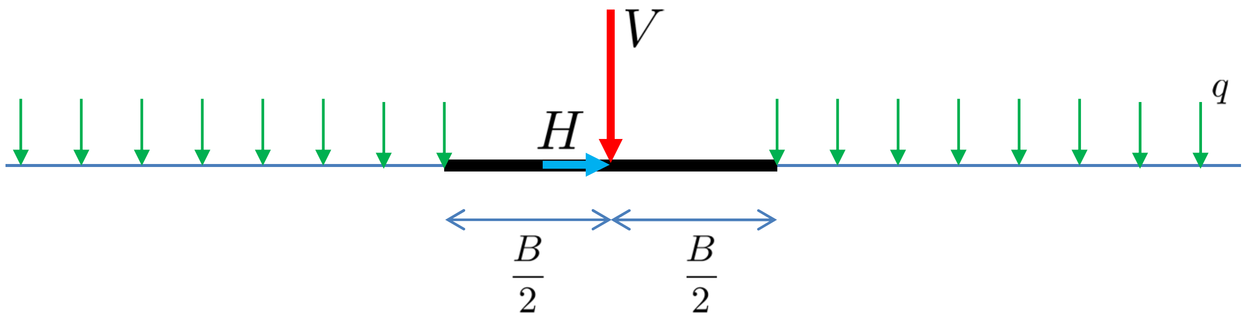


VH diagram:

$$\frac{V_u}{B} = \tfrac{1}{2}\gamma B N_\gamma \left[1 - \left(\frac{H}{V\tan\phi}\right)^m\right]$$

Optum CE

Inclined load



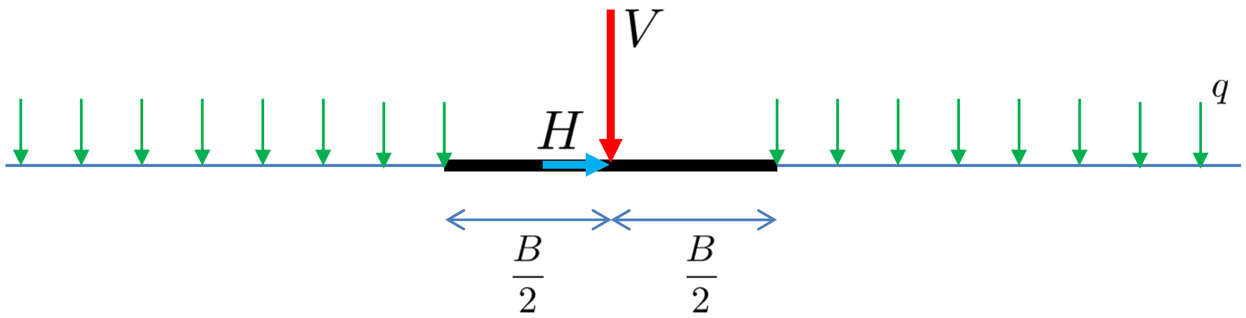
VH diagram:

$$\frac{V_u}{B} = \tfrac{1}{2}\gamma B N_\gamma \left[1 - \left(\frac{H}{V\tan\phi}\right)^m\right]$$

or:

$$H = \left[1 - \left(\frac{V}{\frac{1}{2}\gamma B^2 N_\gamma}\right)^{\frac{1}{m}}\right] V\tan\phi$$

Inclined load



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or:

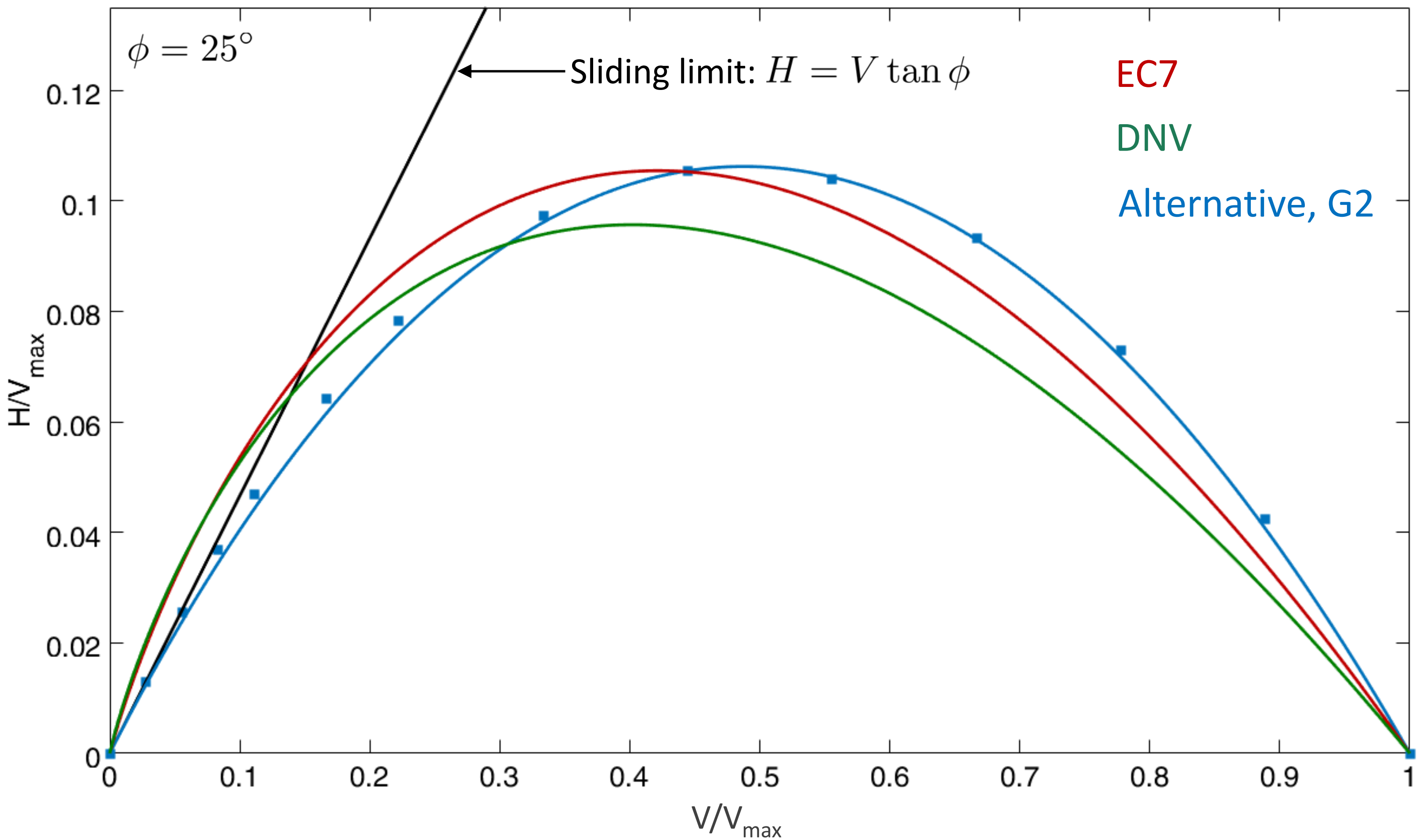
$$\frac{H}{V_{\max}} = \left[1 - \left(\frac{V}{V_{\max}}\right)^{\frac{1}{m}}\right]\frac{V}{V_{\max}}\tan\phi$$

where

$$V_{\max} = \tfrac{1}{2}\gamma B N_\gamma$$

Optum CE

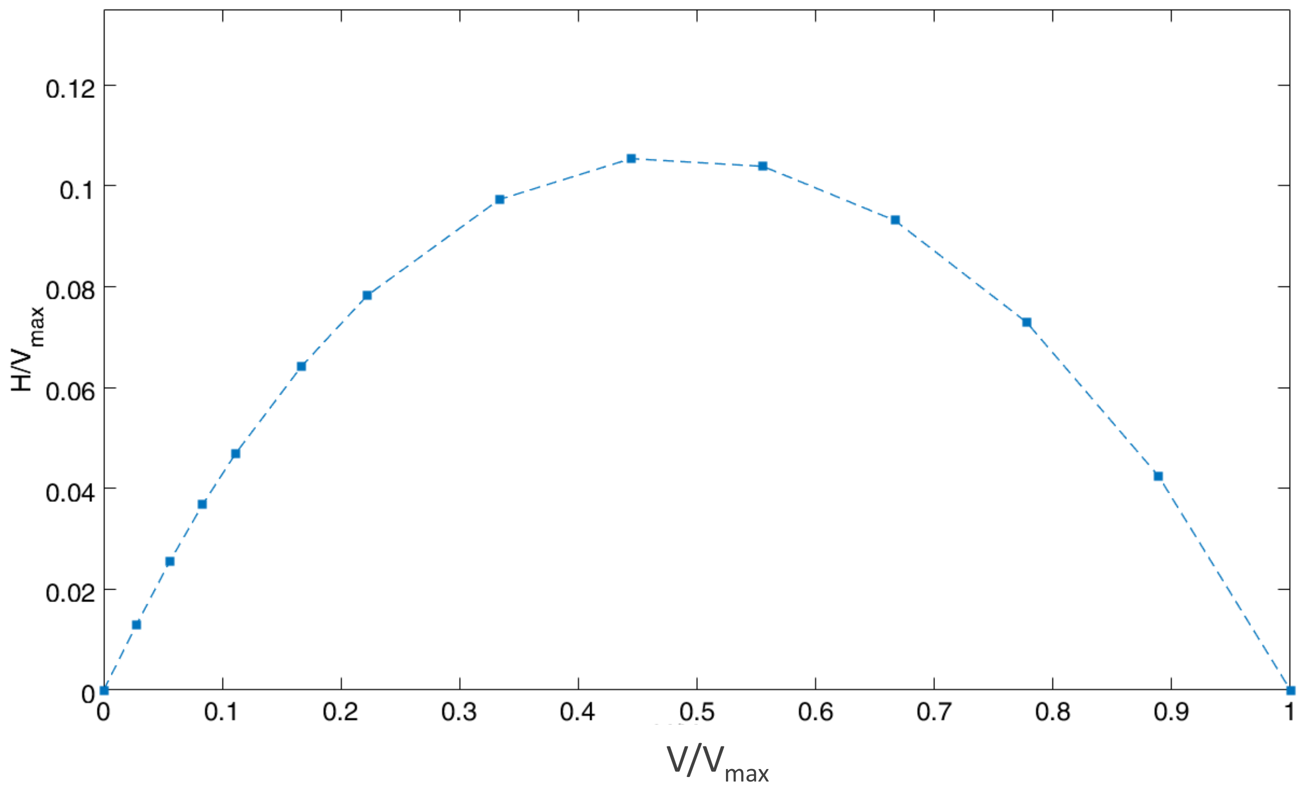
Inclined load



Optum^CE

VH diagrams using Limit Analysis

1. Determine V_{min} and V_{max}



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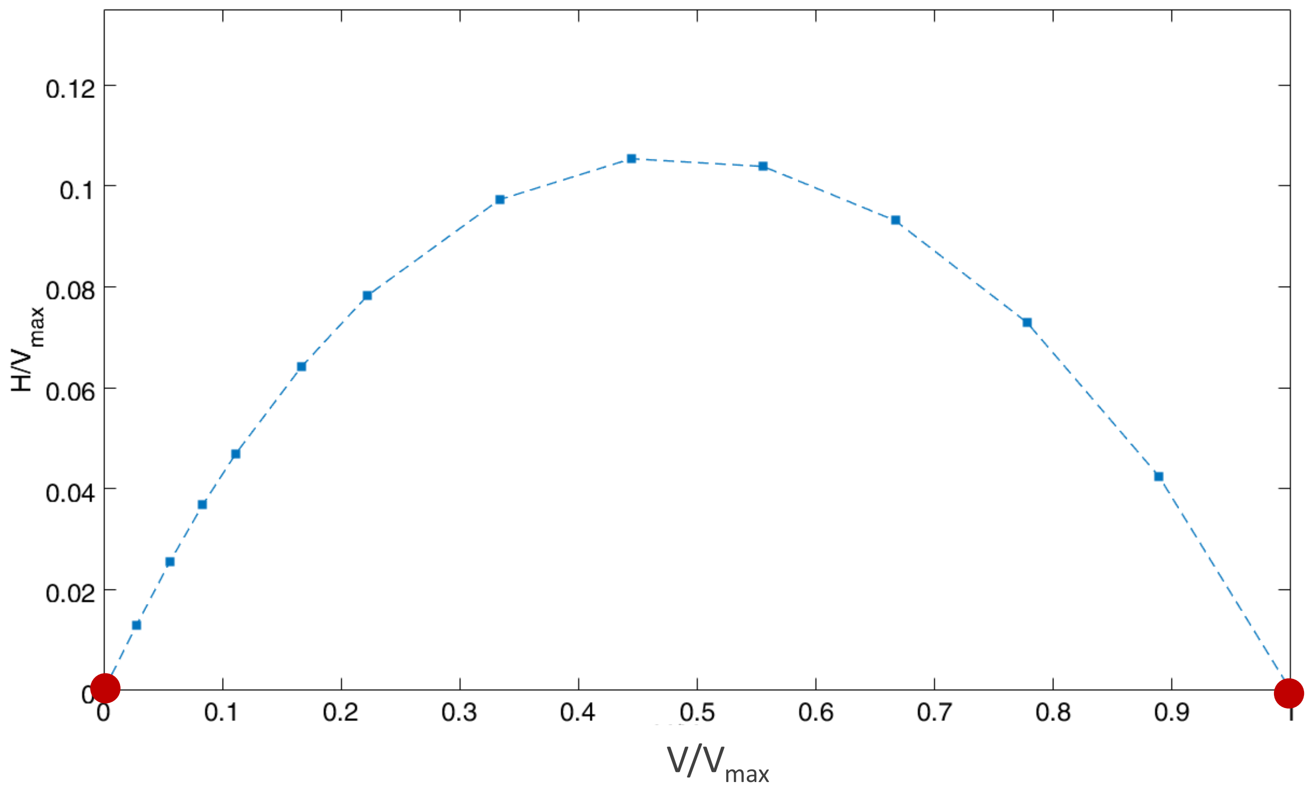
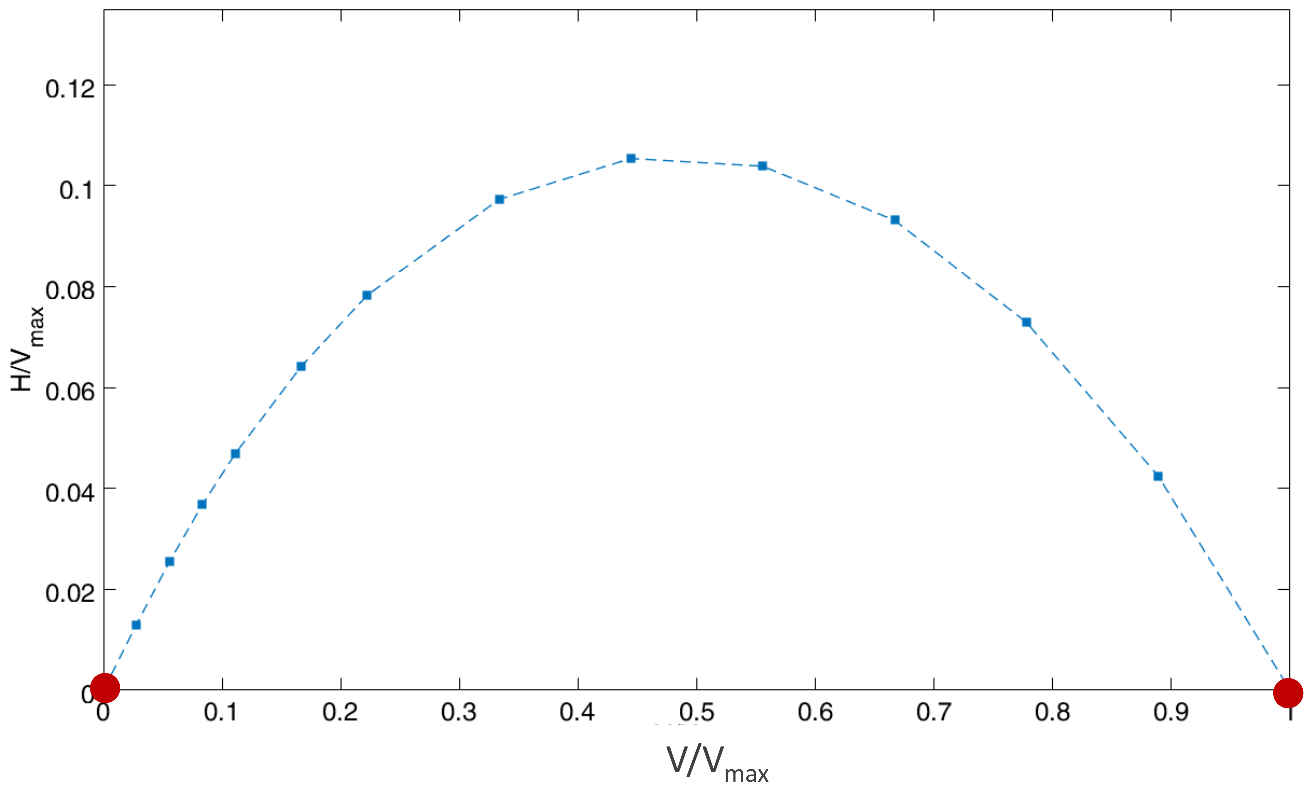


BEARING CAPACITY EQUATION

VH diagrams using Limit Analysis

1. Determine V_{min} and V_{max}
2. Determine H for fixed V in between V_{min} and V_{max}



OptumCE

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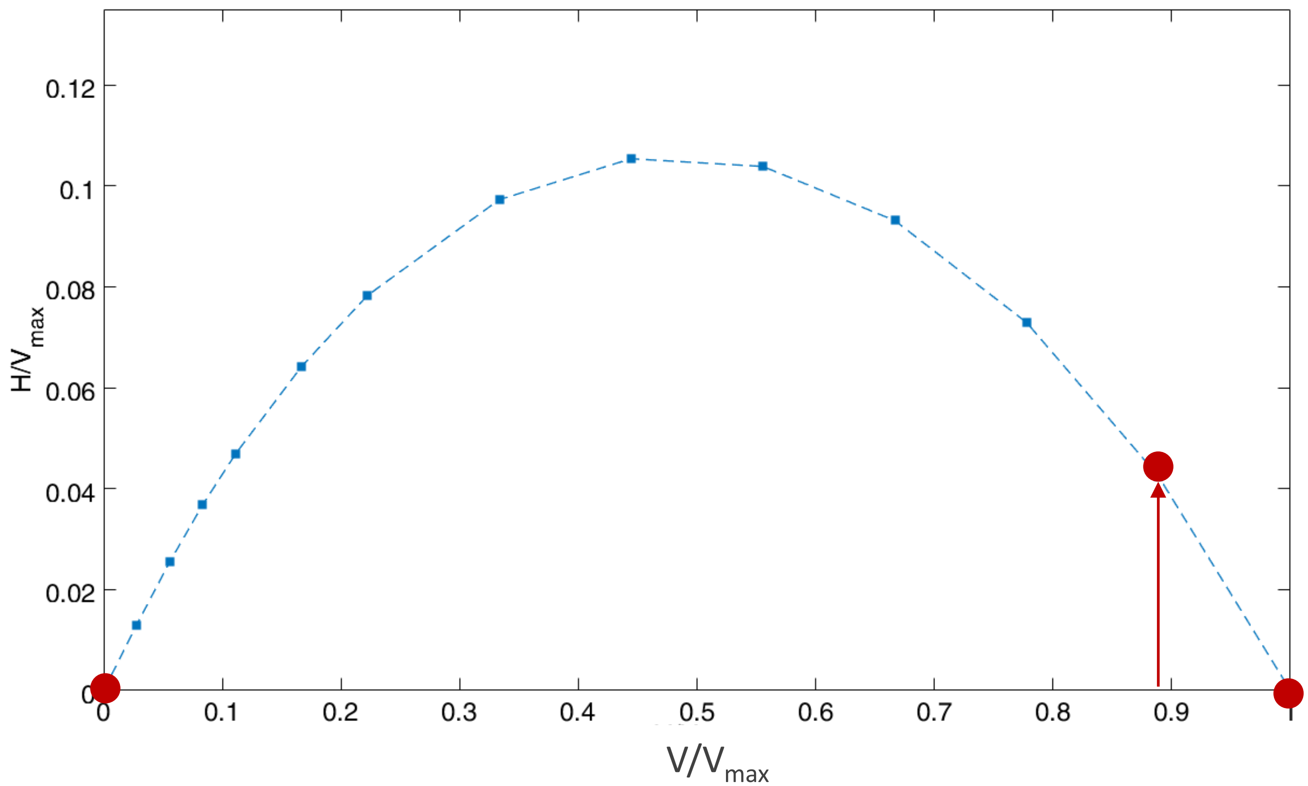

OptumCE

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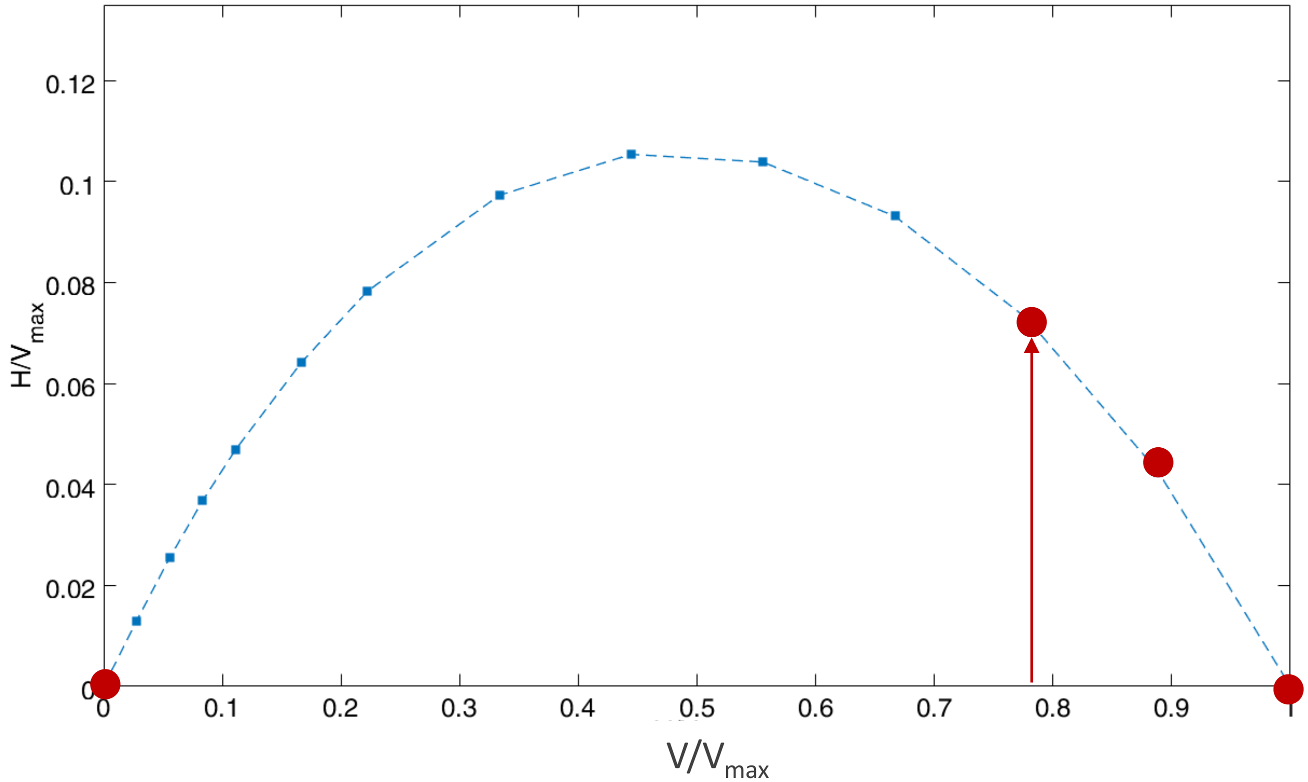

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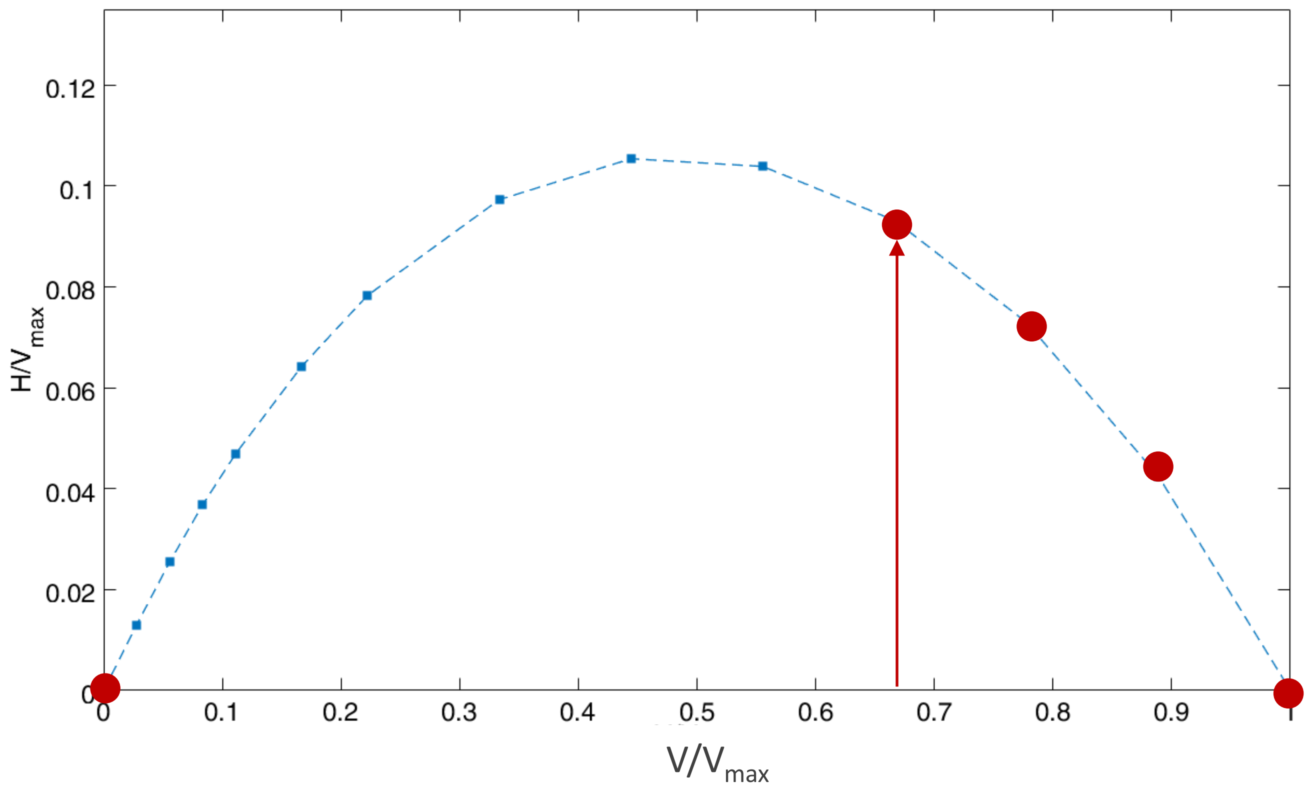

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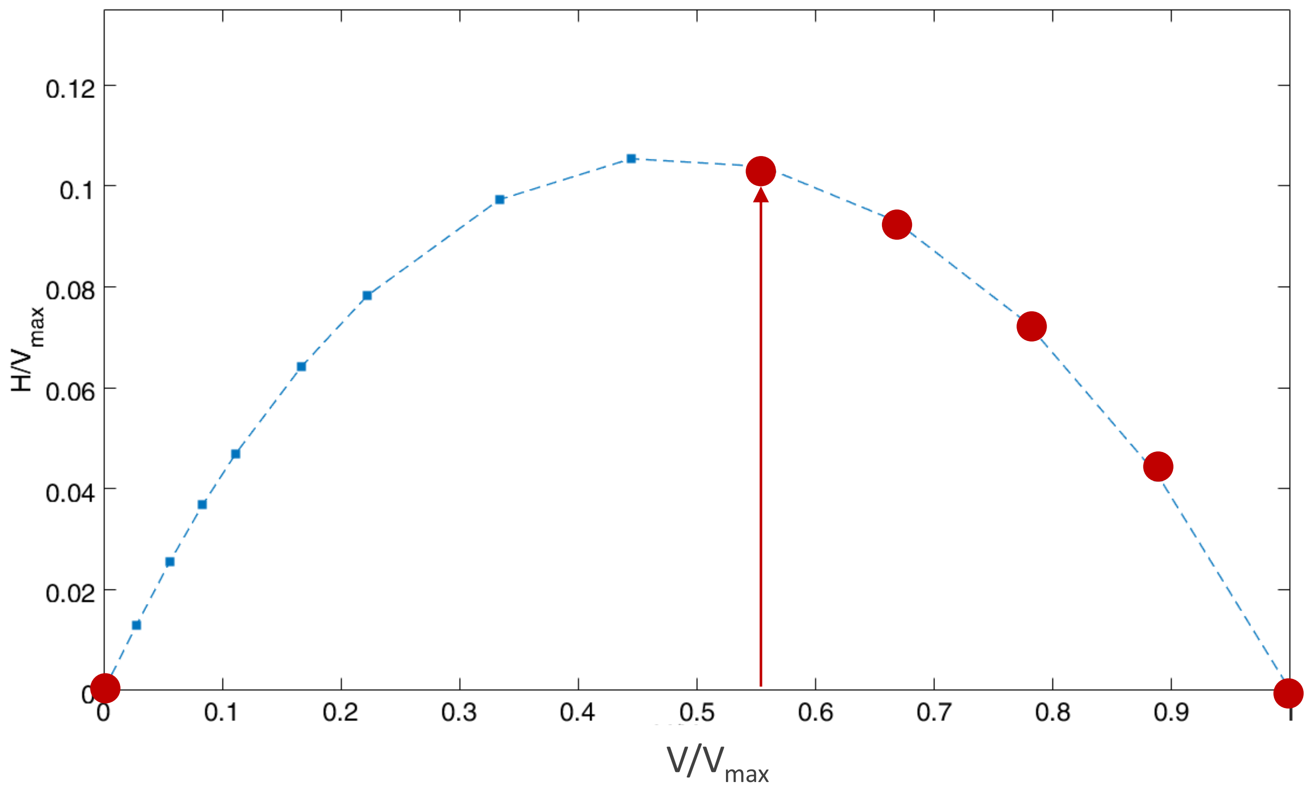

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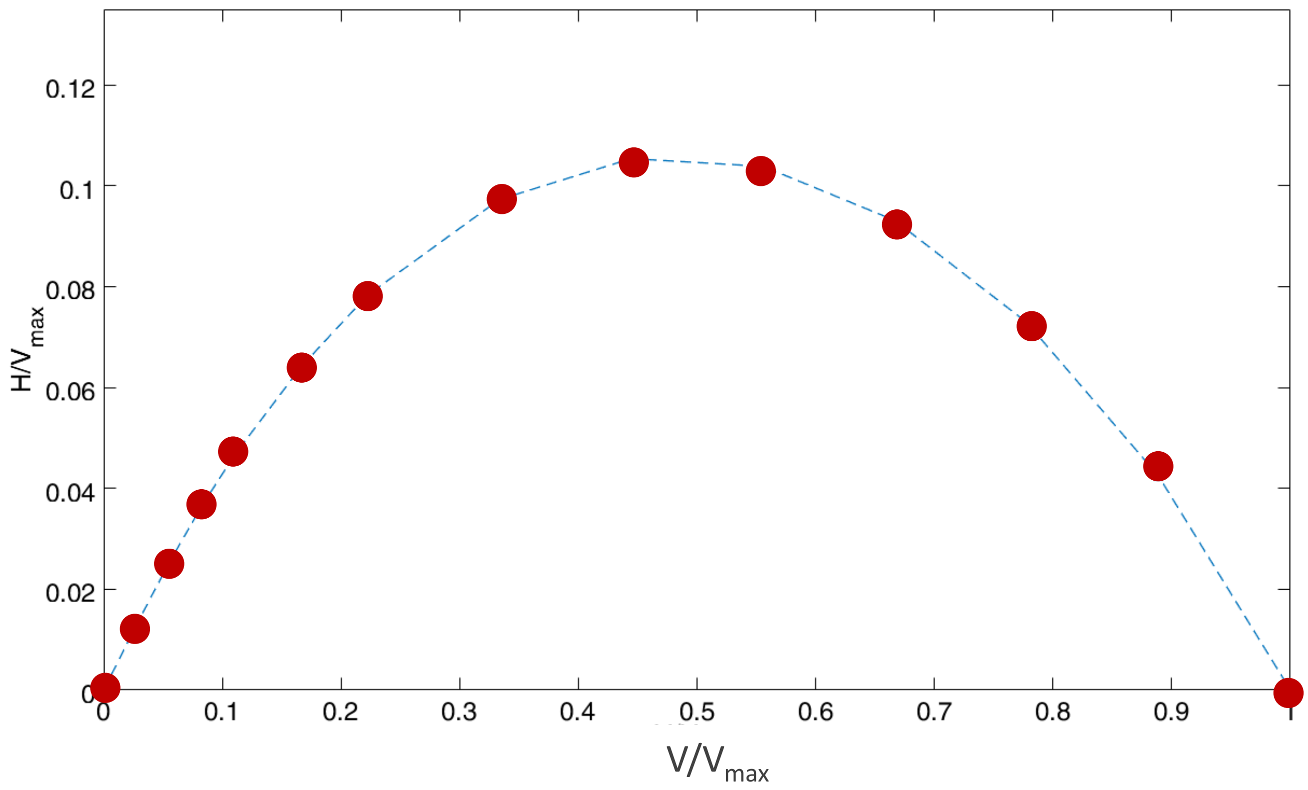

BEARING CAPACITY EQUATION

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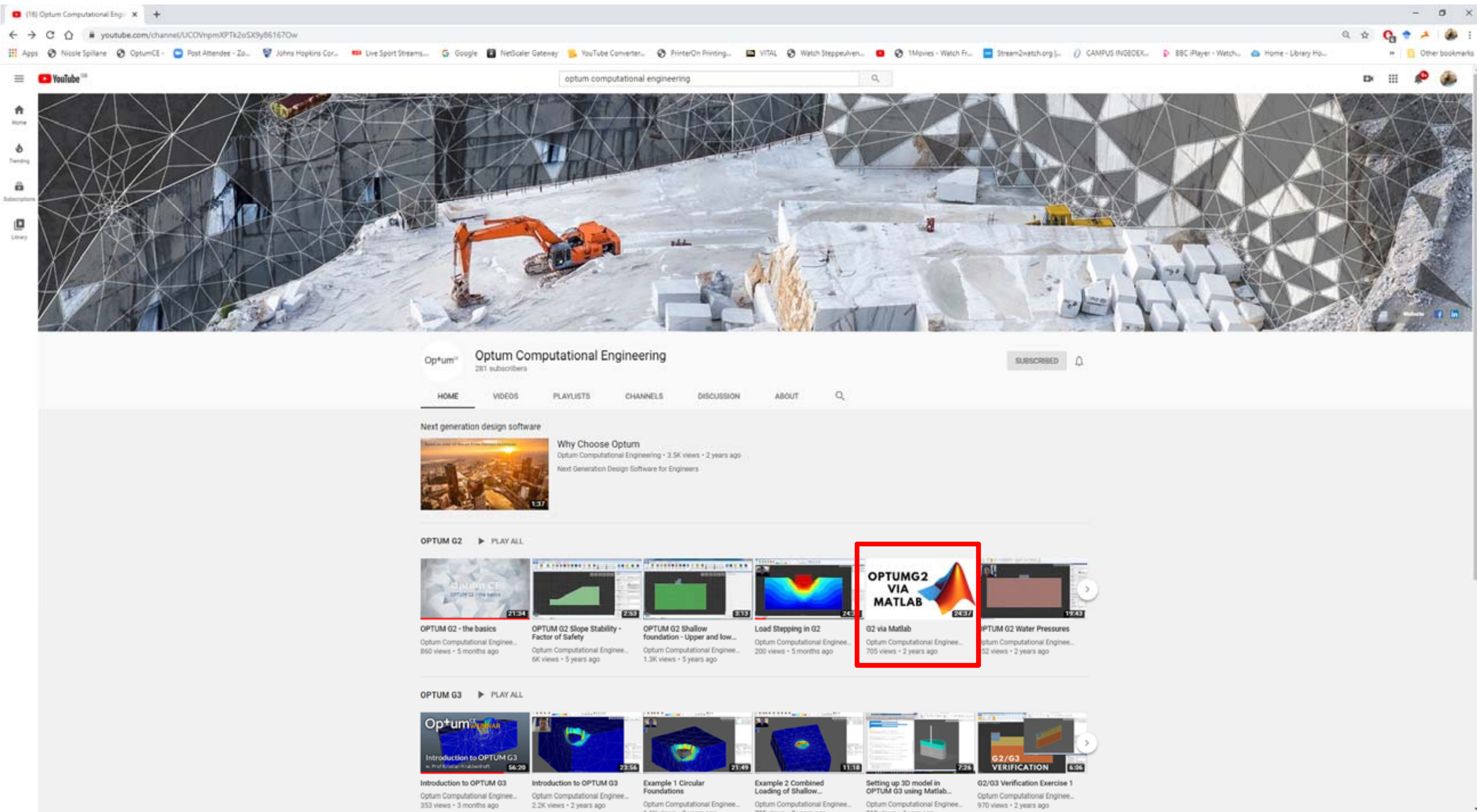
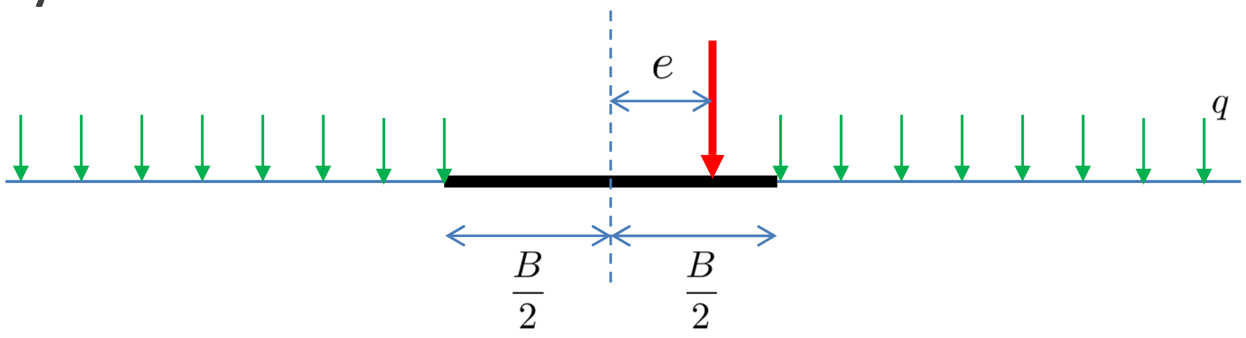


Eccentricity



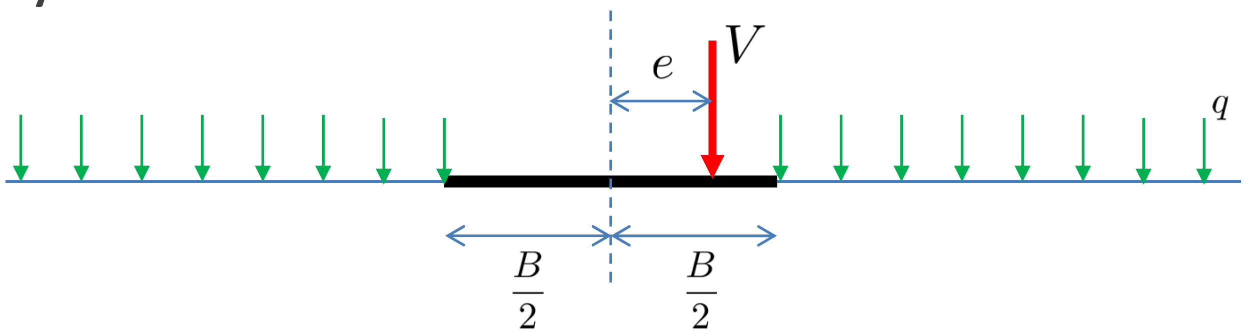
Eccentricity

Optum CE

Eccentricity

BEARING CAPACITY EQUATION

Eccentricity



Modified equation:

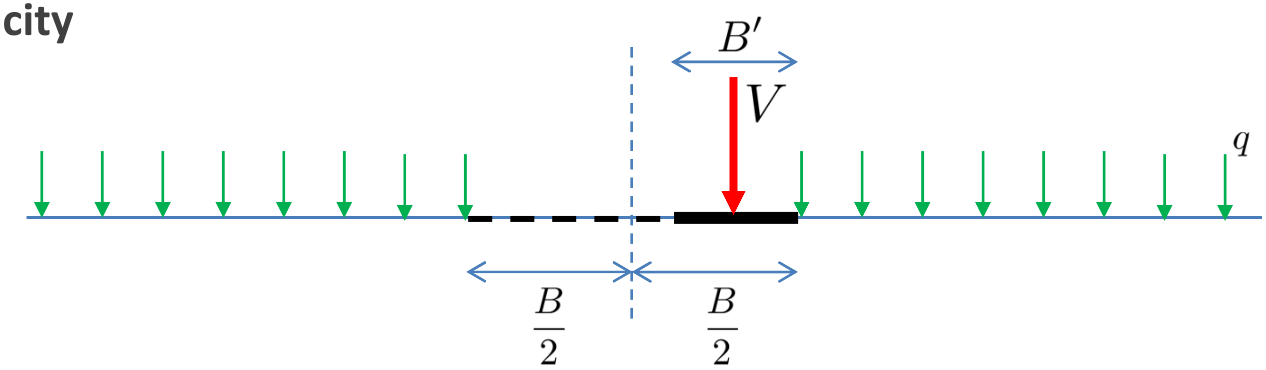
$$\frac{V_u}{B'} = cN_c i_c + qN_q i_q + \tfrac{1}{2}\gamma B' N_\gamma i_\gamma$$

where

$$B' = B - 2e$$

Optum^{CE}

Eccentricity



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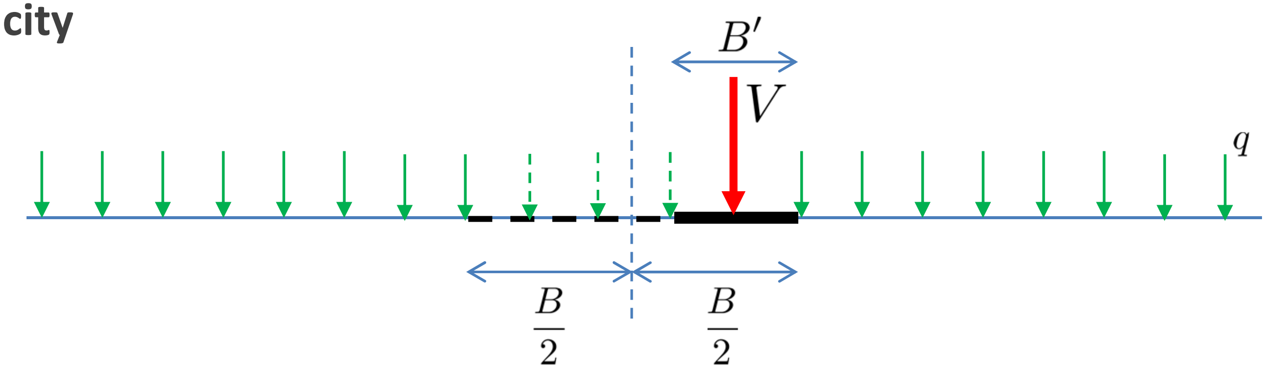

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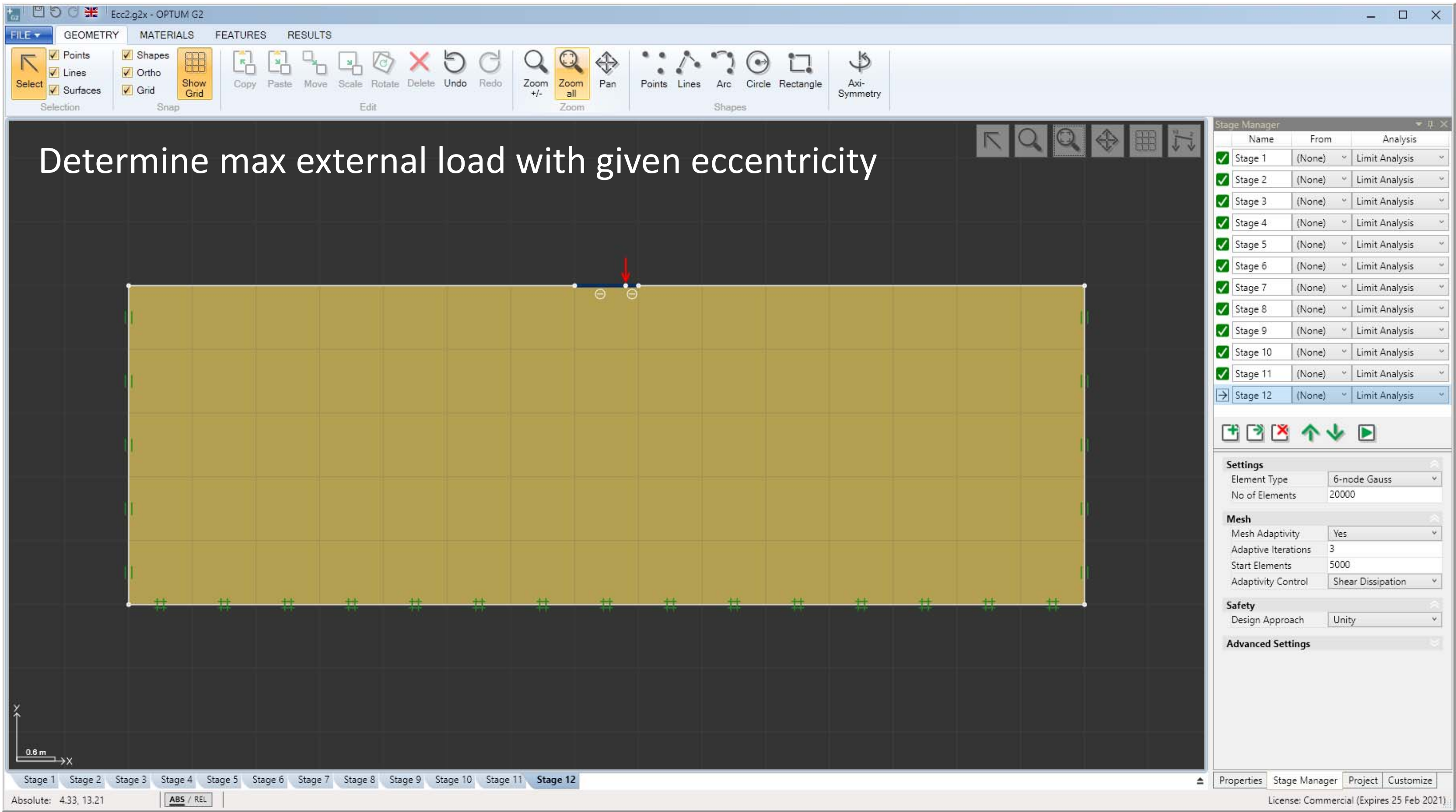

OptumCE

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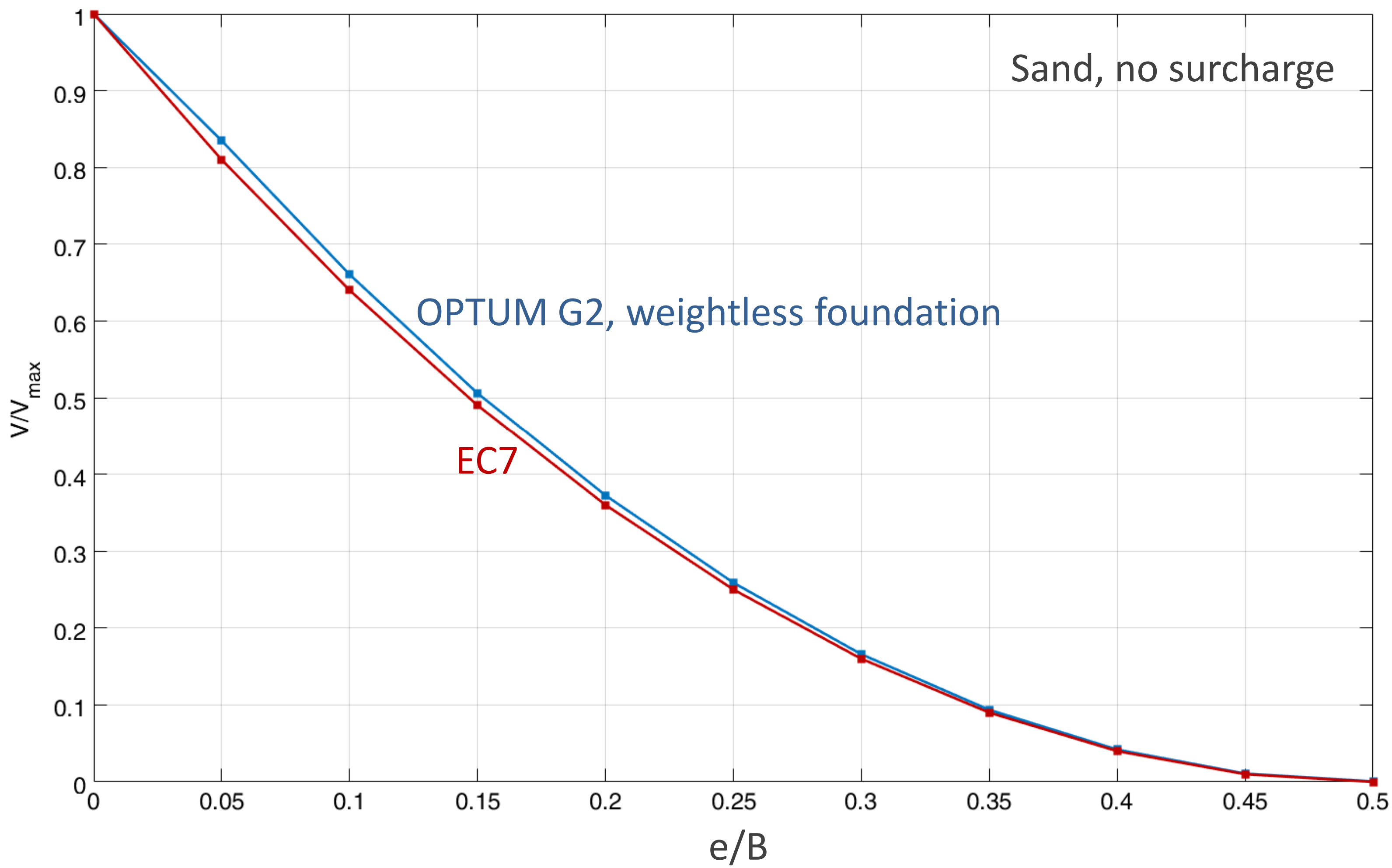
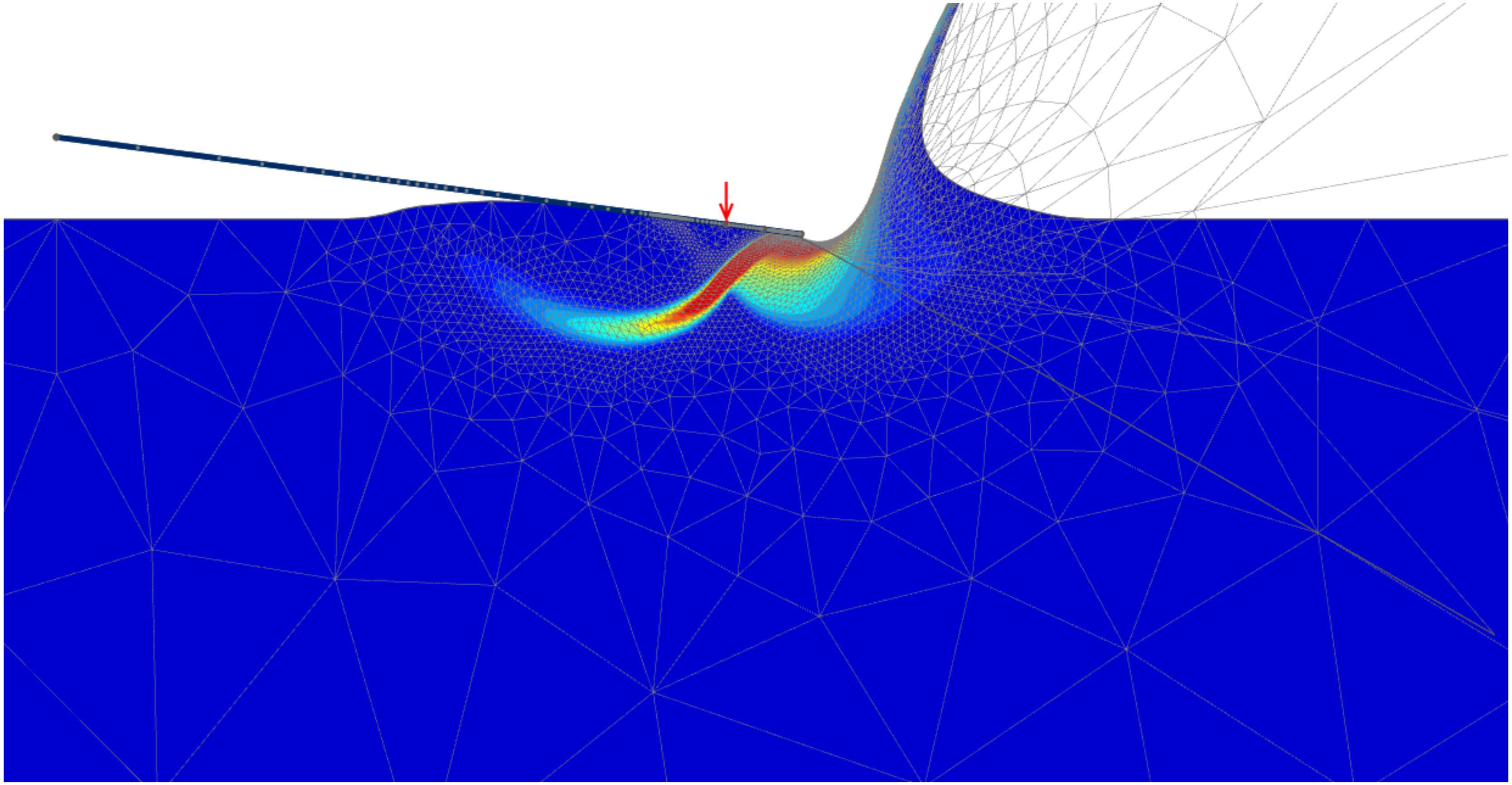


OptumCE

Eccentricity

e/B = 0.4, weightless foundation



Eccentricity

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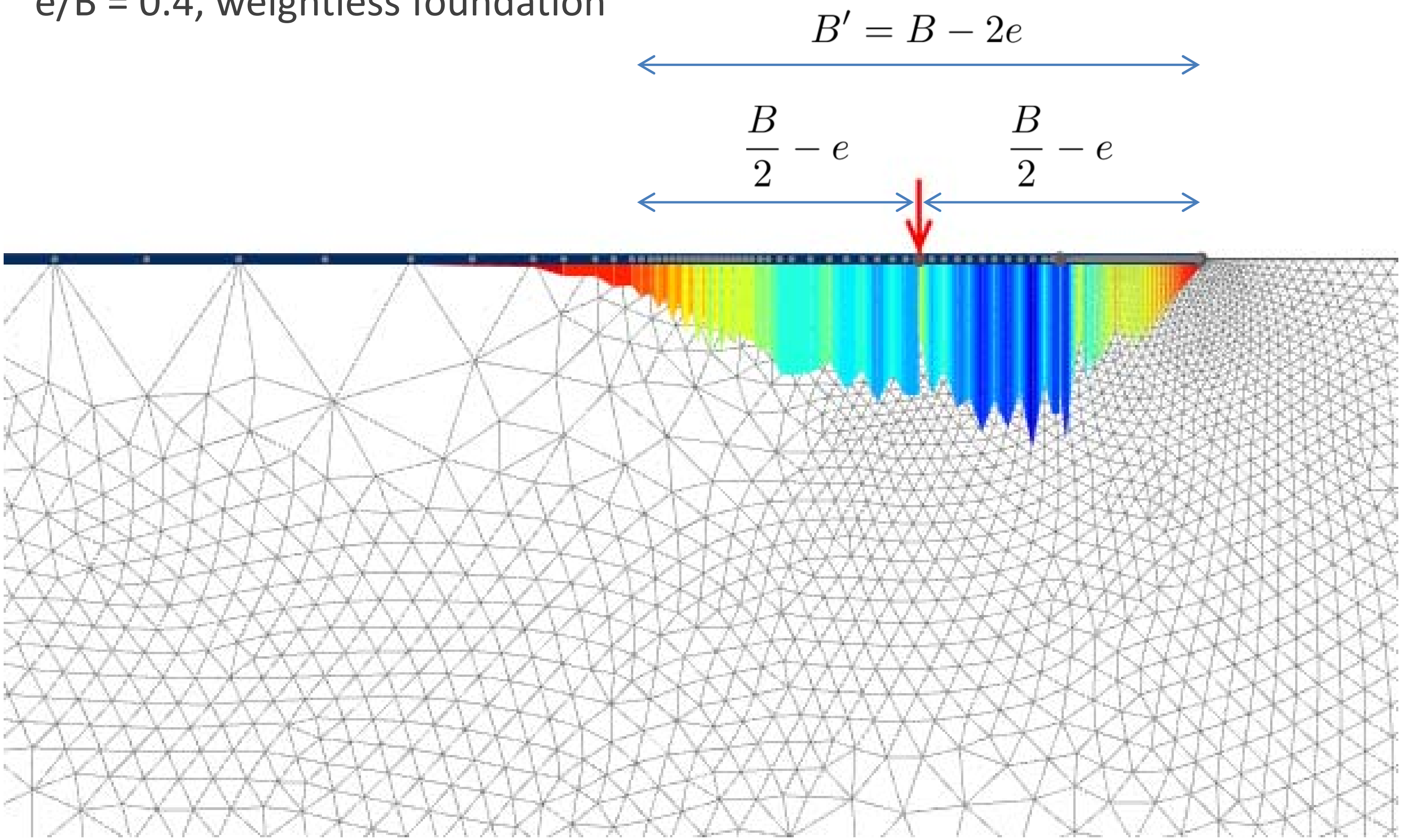
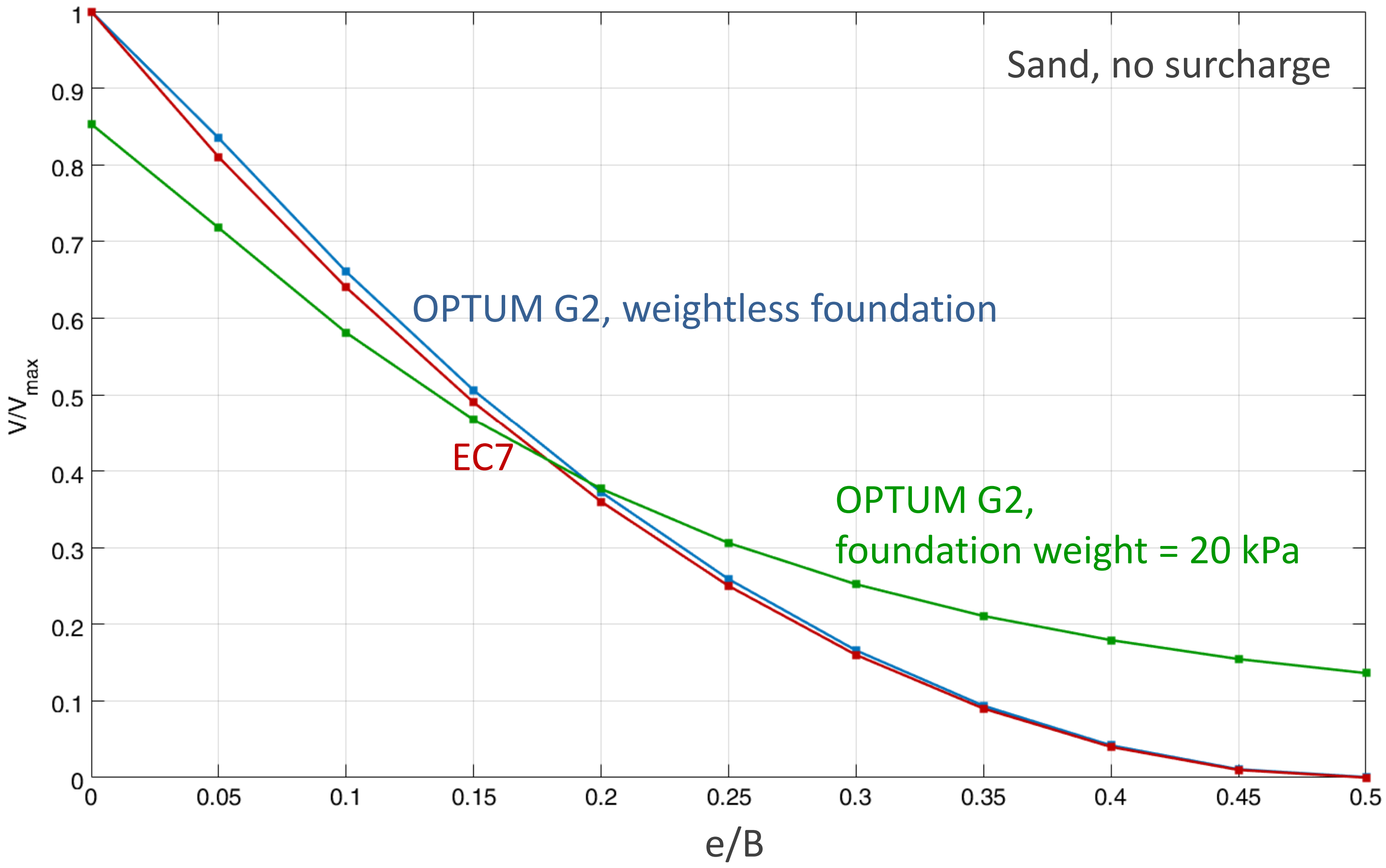


BEARING CAPACITY EQUATION

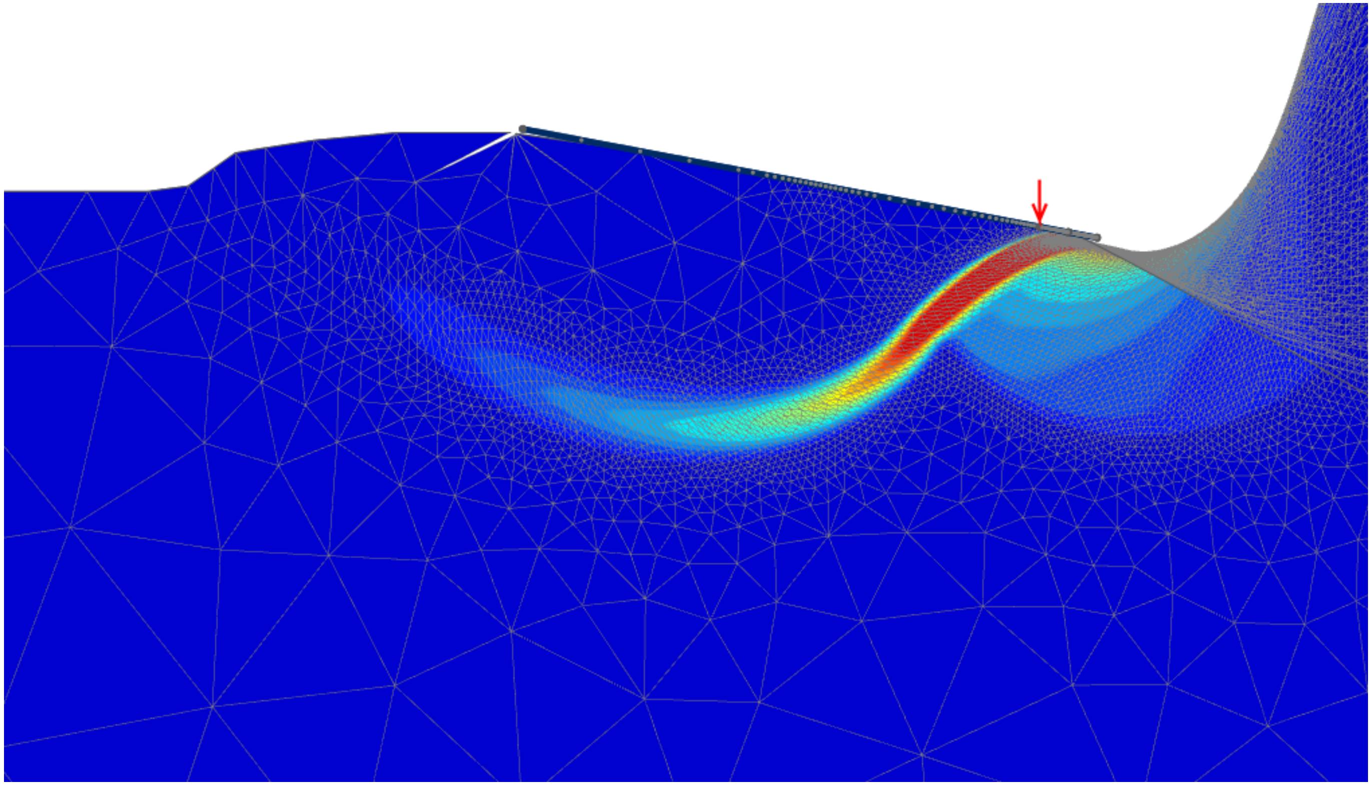
Eccentricity



Optum^{CE}

Eccentricity

e/B = 0.4, foundation weight = 20 kPa



Eccentricity

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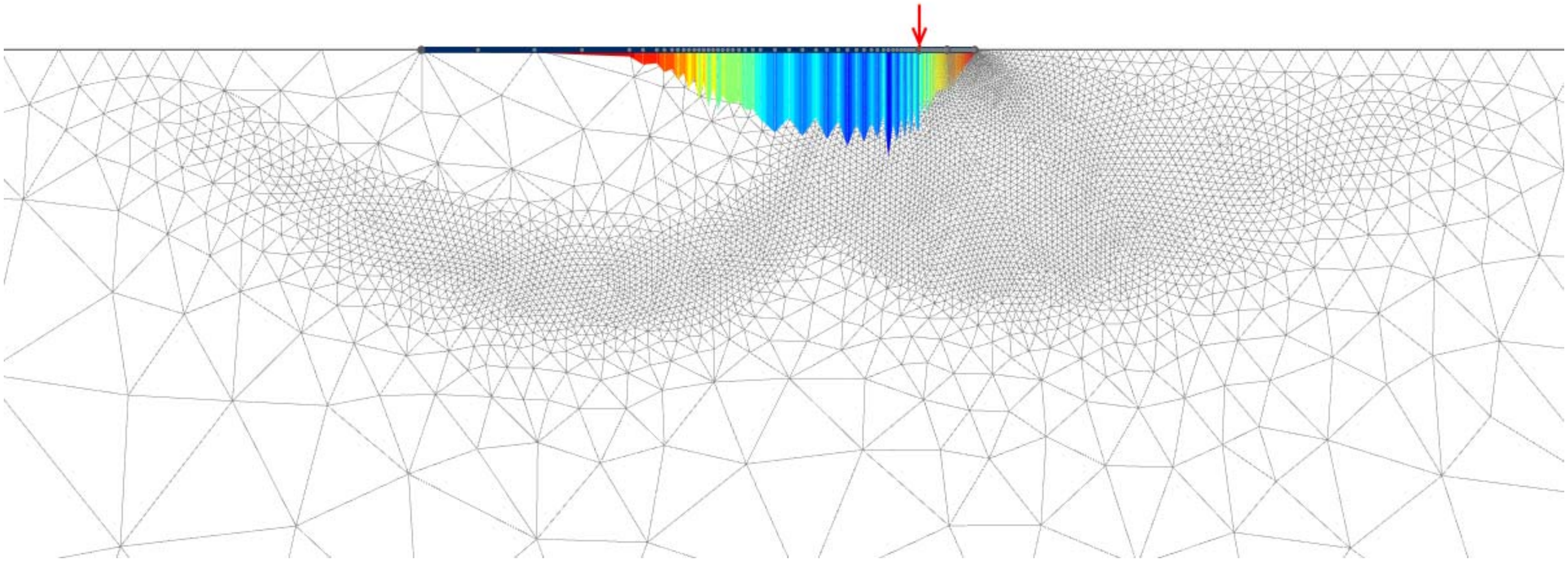
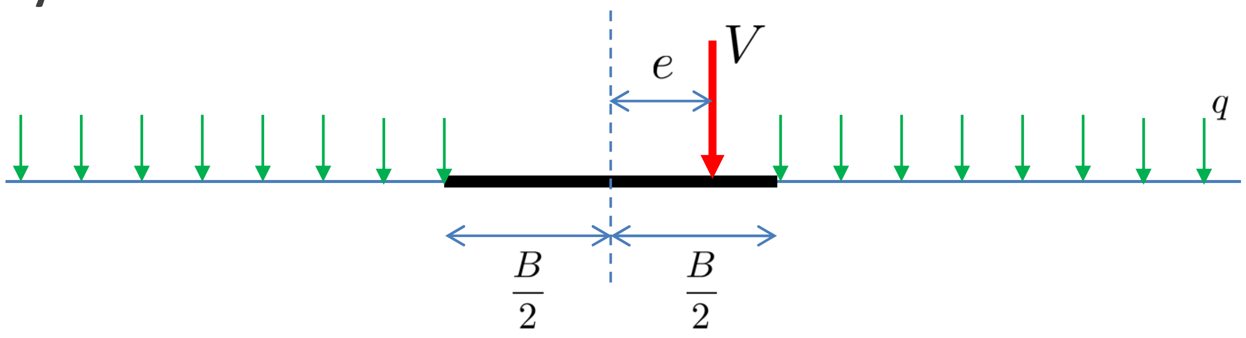


Eccentricity



Undrained, no surcharge:

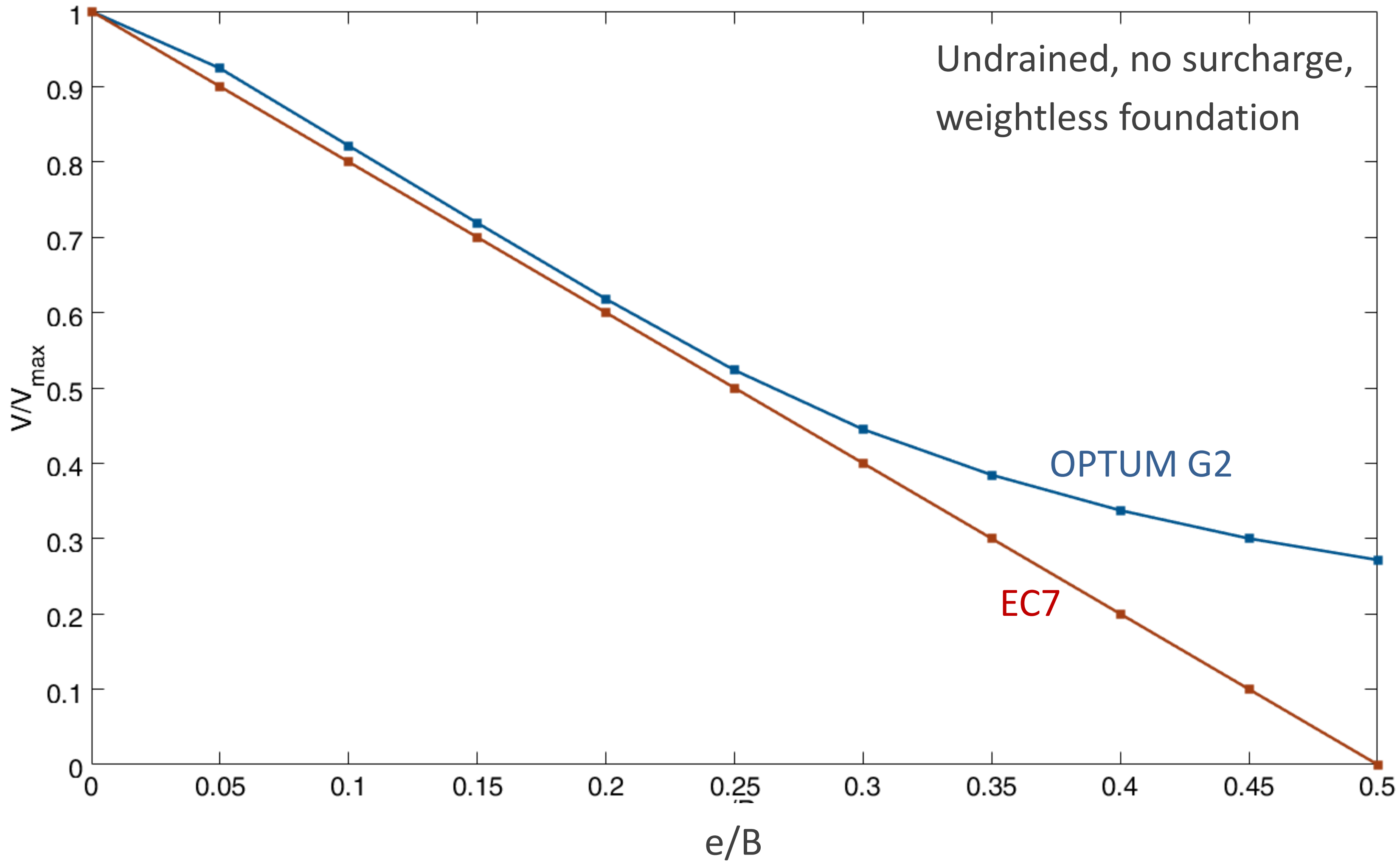
$$\frac{V_u}{B'} = (2+\pi)s_u$$

where

$$B' = B - 2e$$

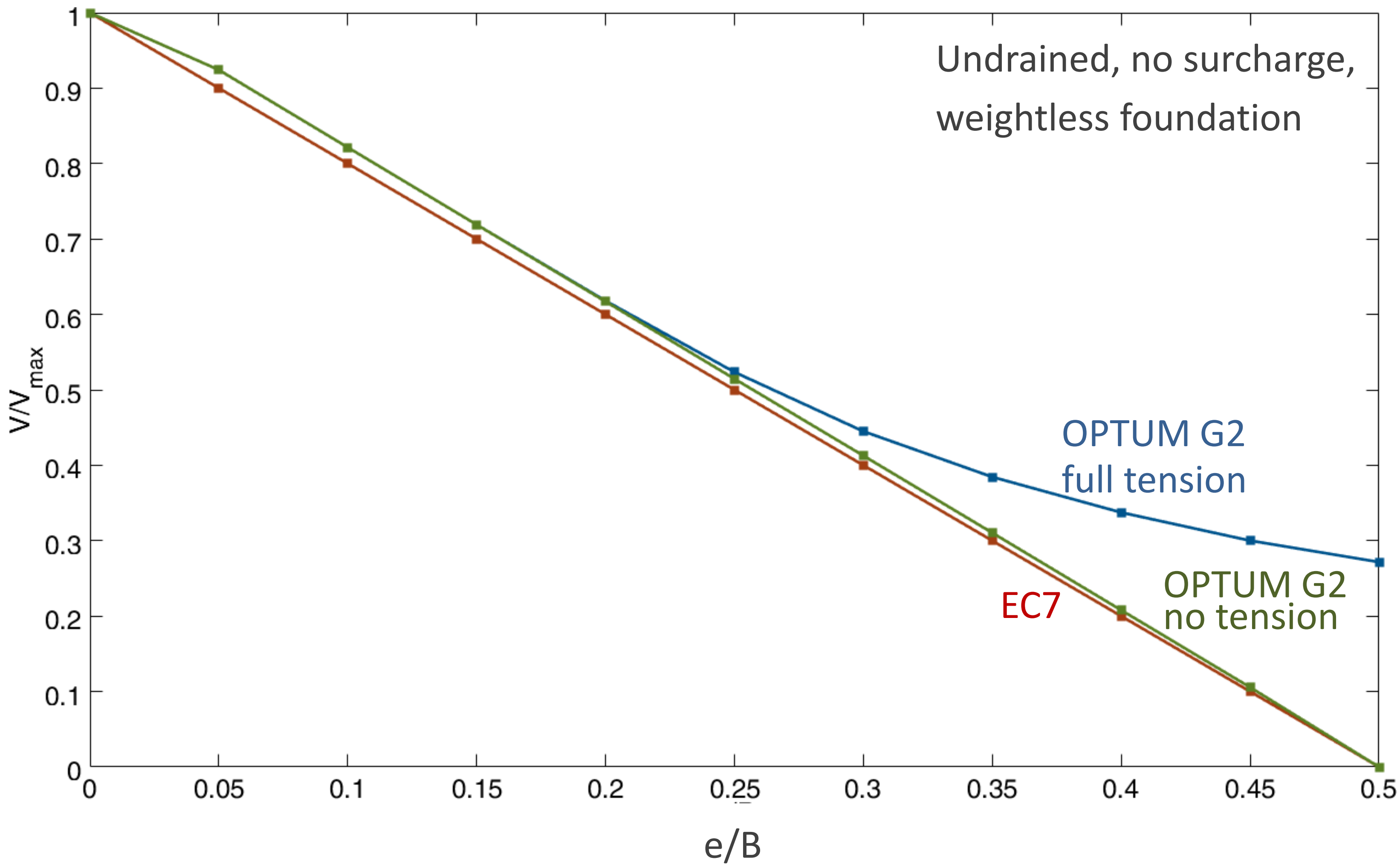
Optum CE

Eccentricity



BEARING CAPACITY EQUATION

Eccentricity



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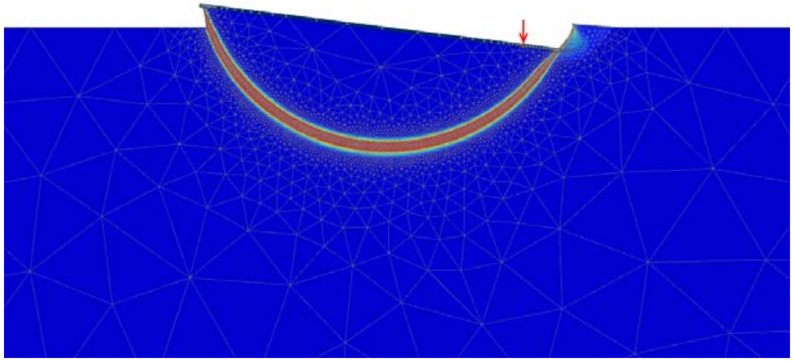
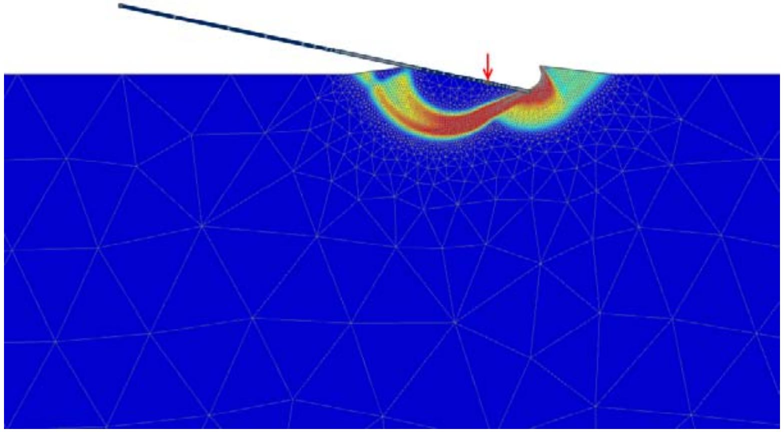


Full tension at interface

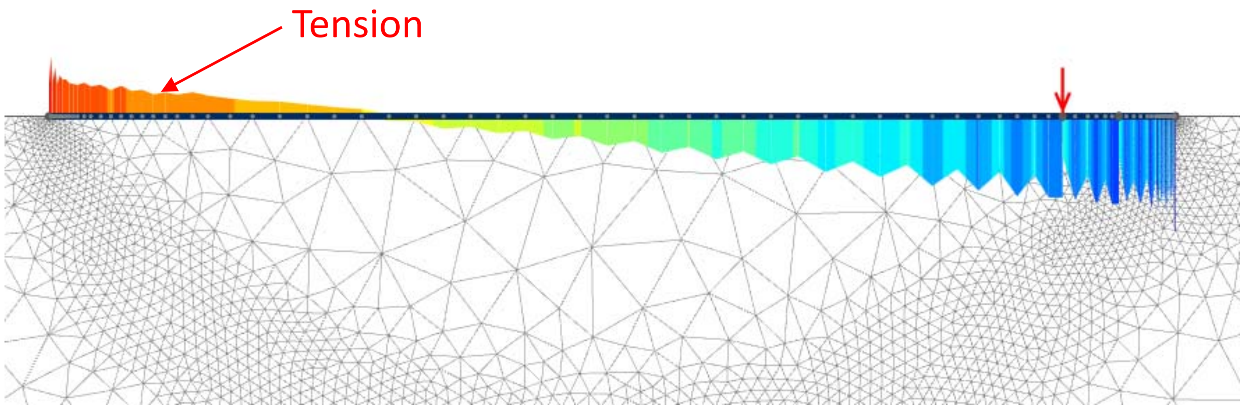


No tension at interface (tension cut-off)

BEARING CAPACITY EQUATION

Eccentricity

e/B = 0.4



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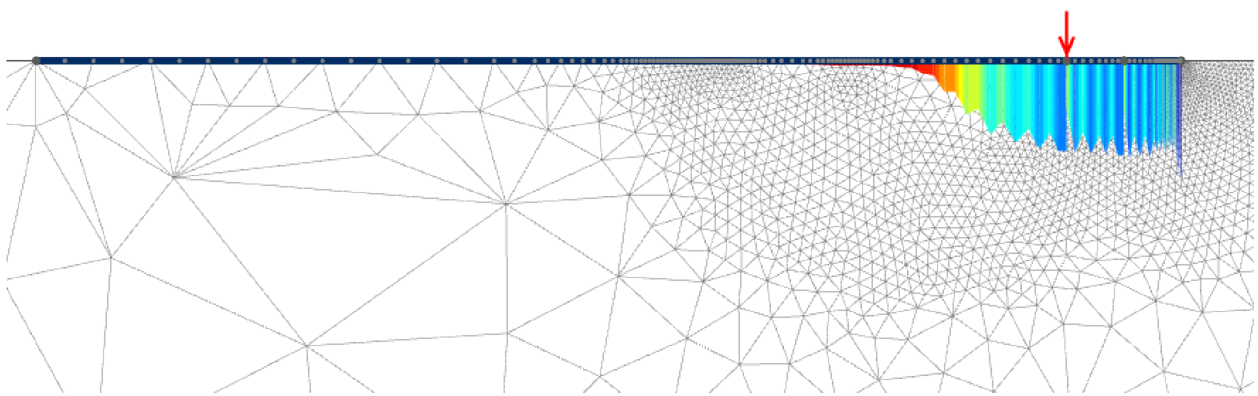
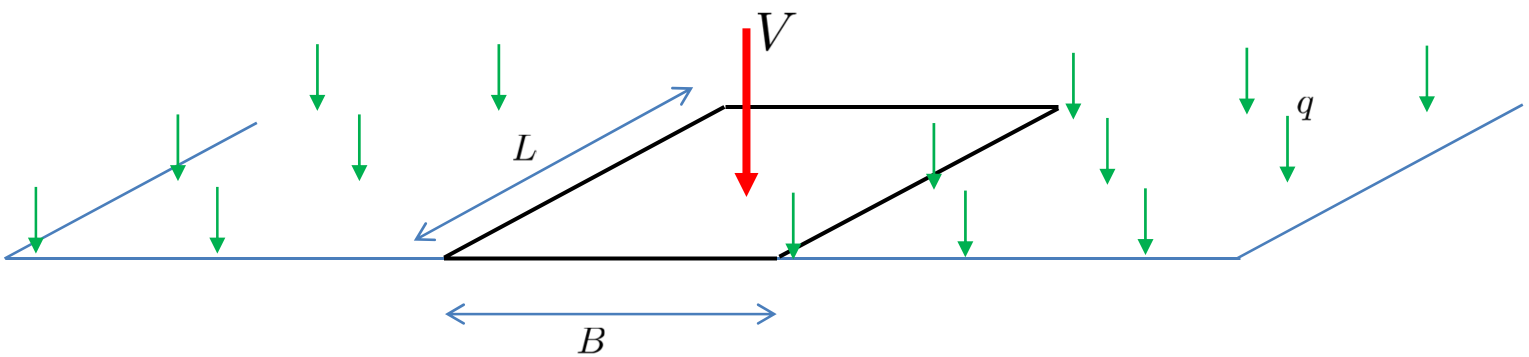


No tension at interface (tension cut-off)

Optum^CE

Shape



Modified equation:

$$\frac{V_u}{A} = cN_c i_c s_c + qN_q i_q s_q + \tfrac{1}{2}\gamma B N_\gamma i_\gamma s_\gamma$$

where (EC7)

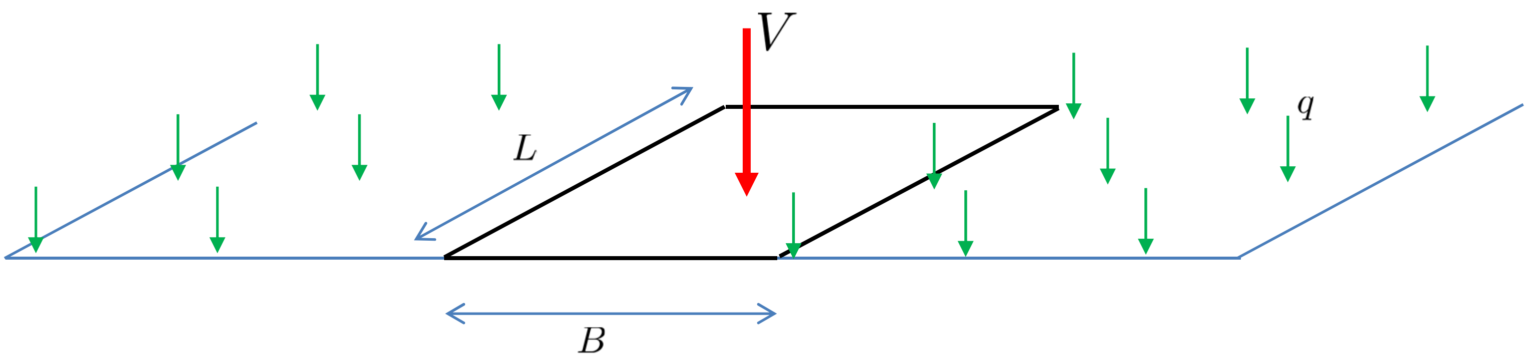
$$s_q = 1 + \frac{B'}{L'}\sin\phi$$

$$s_c = \frac{s_q N_q - 1}{N_q - 1}$$

$$s_\gamma = 1 - 0.3\frac{B'}{L'}$$

Optum^CE

Shape



Modified equation:

$$\frac{V_u}{A} = cN_c i_c s_c + qN_q i_q s_q + \frac{1}{2}\gamma B N_\gamma i_\gamma s_\gamma$$

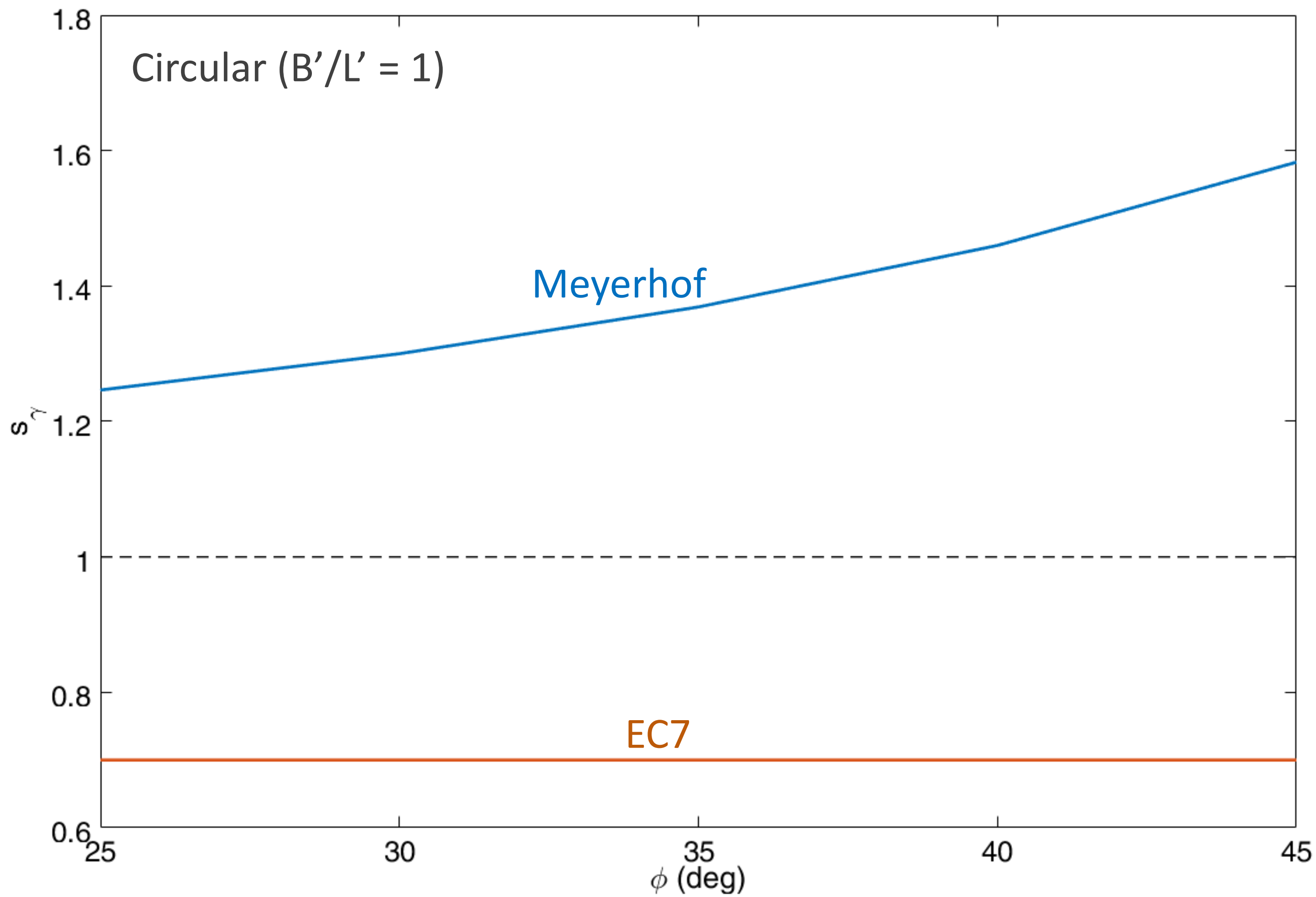
Two families of shape factors:

EC7: $s_\gamma = 1 - 0.3\frac{B'}{L'}$: independent of ϕ and always ≤ 1

Meyerhof: $s_\gamma = 1 + 0.1\tan^2(45 + \frac{1}{2}\phi)$: dependent on ϕ and always ≥ 1

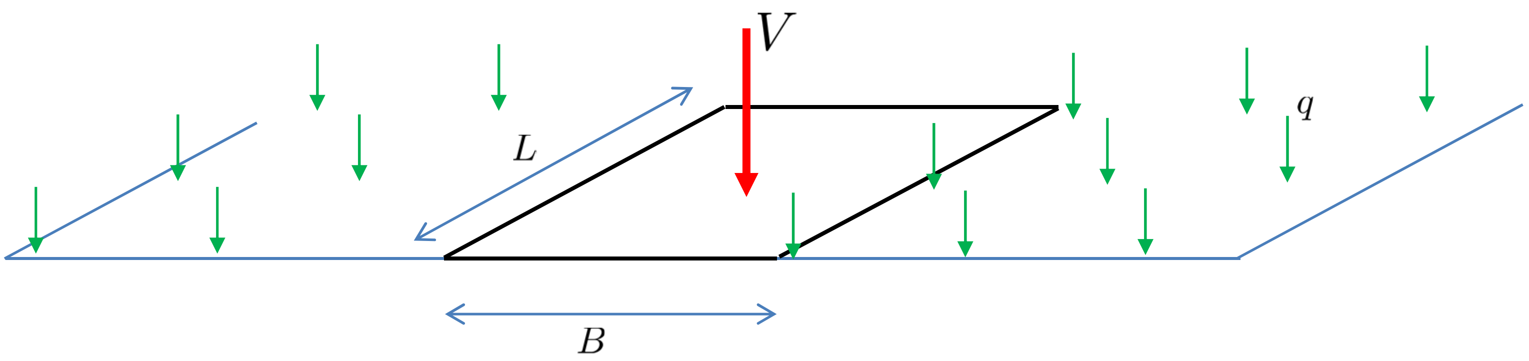
BEARING CAPACITY EQUATION

Shape



Optum CE

Shape



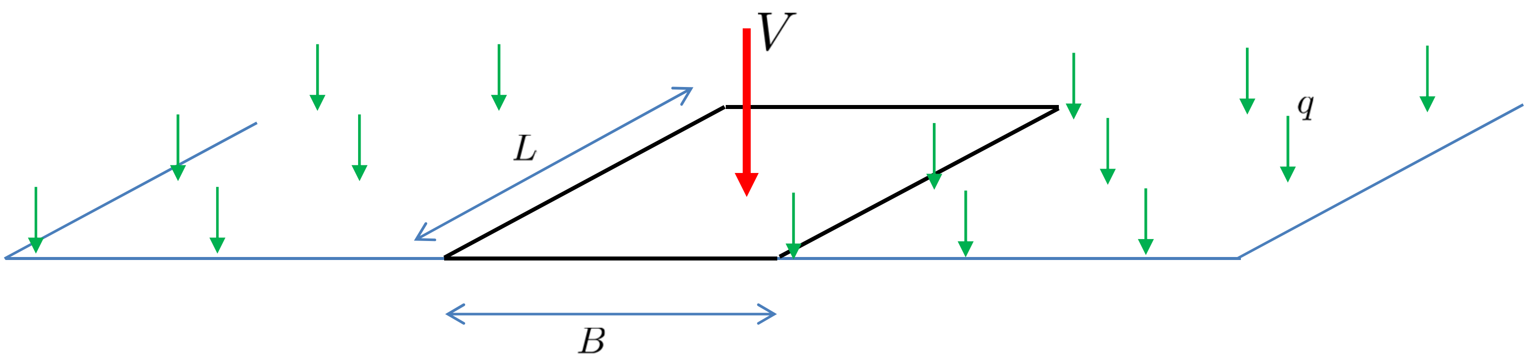
Two families of shape factors:

EC7: $s_\gamma = 1 - 0.3\dfrac{B'}{L'}$: independent of ϕ and always ≤ 1

Meyerhof: $s_\gamma = 1 + 0.1\tan^2(45 + \frac{1}{2}\phi)$: dependent on ϕ and always ≥ 1

Optum^{CE}

Shape



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Who is right? –Only one way to find out: OPTUM G2

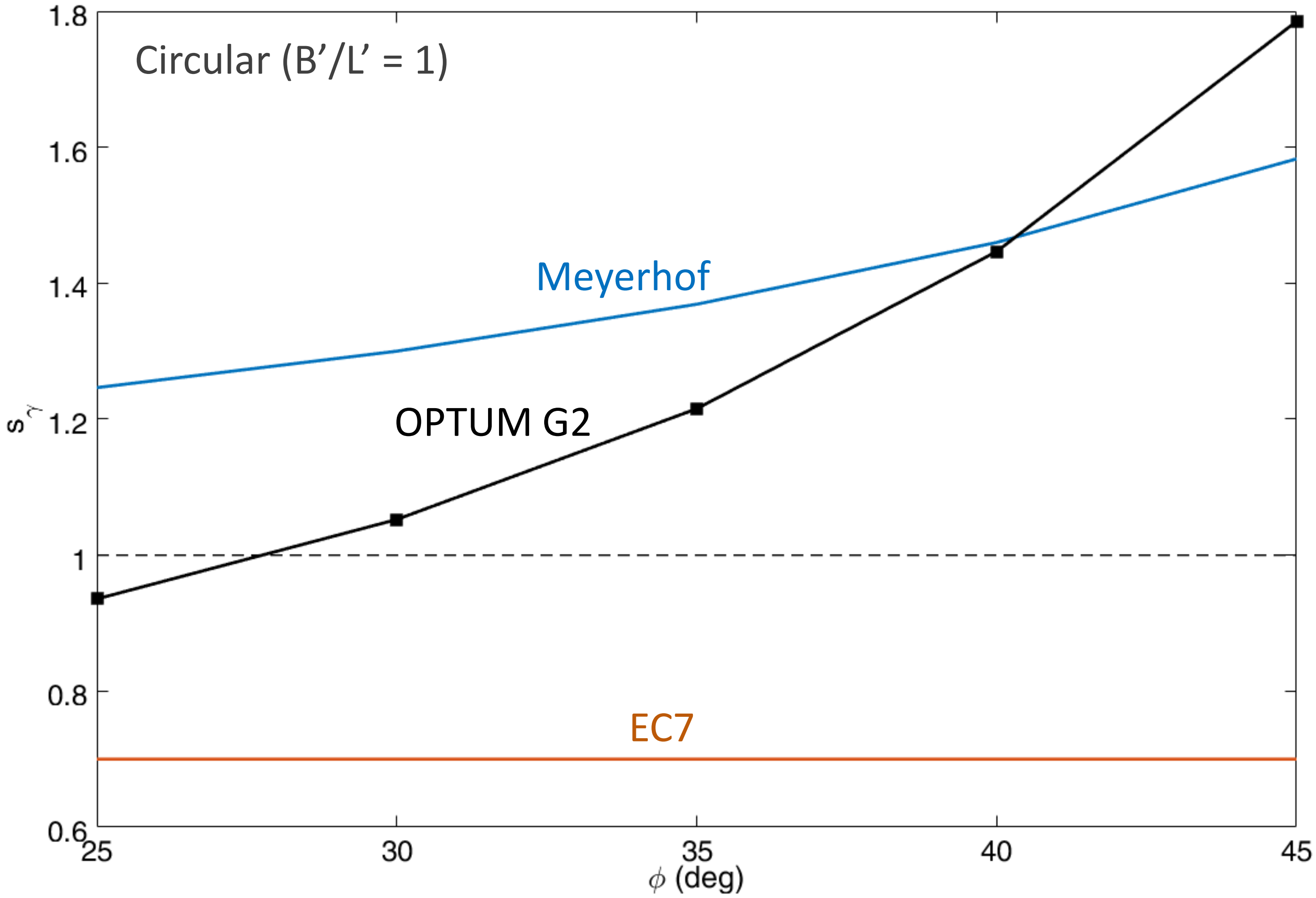
$q_u = \frac{1}{2}\gamma B N_\gamma i_\gamma s_\gamma$

Strip: plane strain (s_γ = 1)
Circular: axisymmetric

$\rightarrow \quad s_\gamma = \dfrac{q_{u,\text{circular}}}{q_{u,\text{strip}}}$

OptumCE

Shape



BEARING CAPACITY EQUATION

Shape

Hold on: N_γ should be calculated on the basis of the *plane strain angle*

That is why $s_\gamma = 0.7$ – both shape *and* stress states not corresponding to plane strain

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Assume:

$$\phi_{ps} = 1.12\phi_{tr}$$

and

$$q_{u,\mathrm{strip}} = \frac{1}{2}\gamma B N_\gamma(\phi_{ps})$$

$q_{u,\mathrm{circular}} =$ calculate on the basis of ϕ_{tr}

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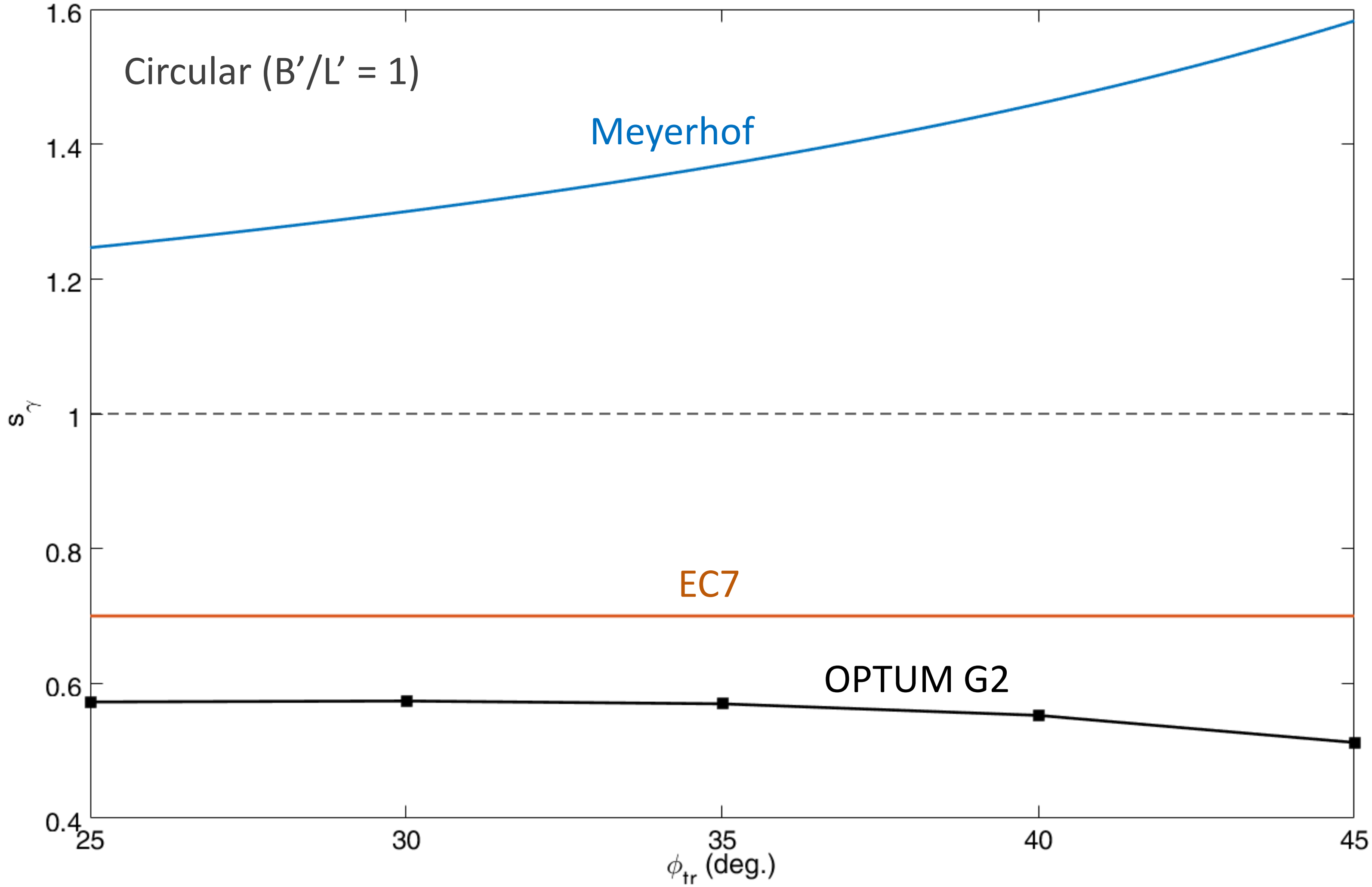

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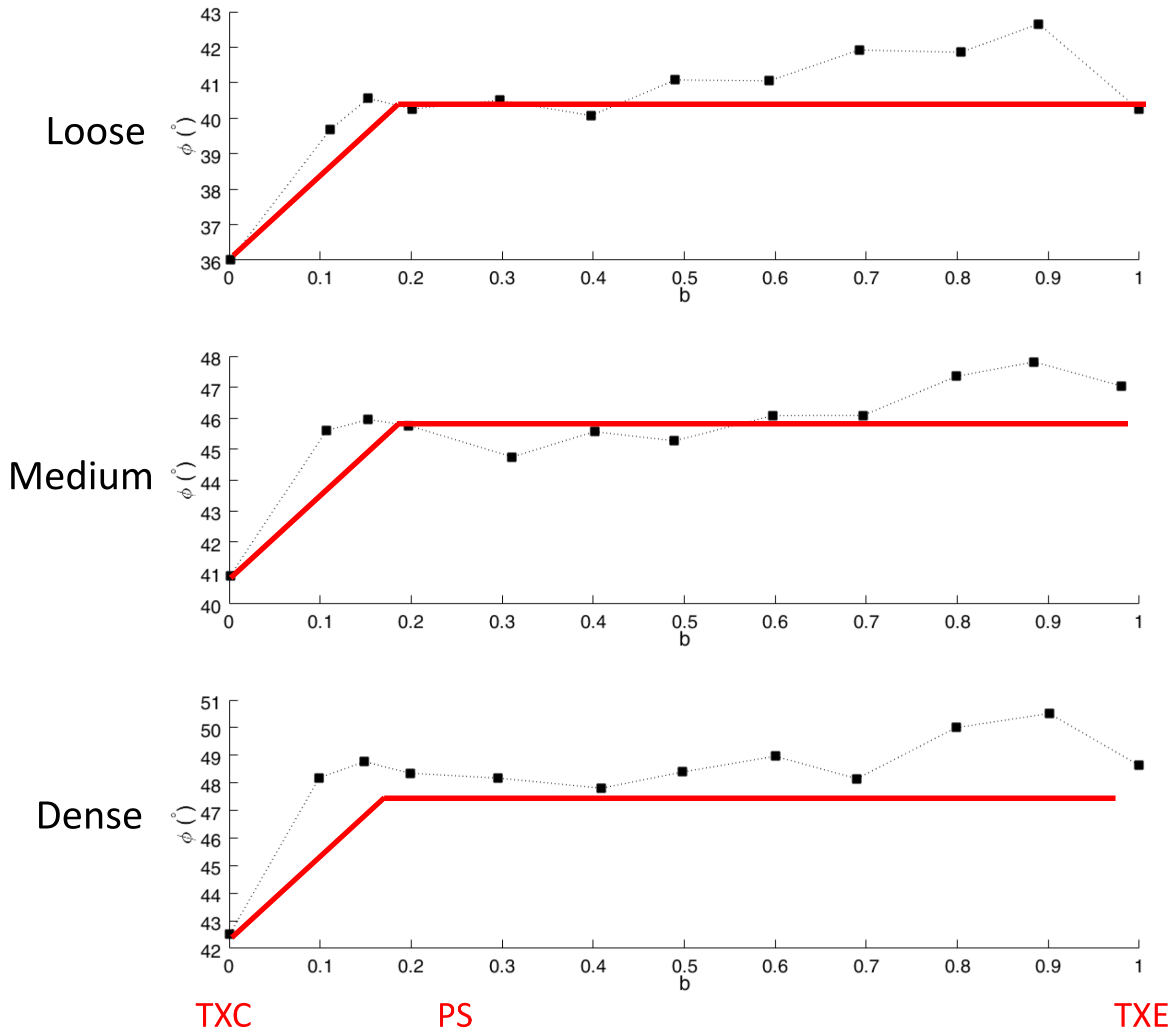
However: not in triaxial compression everywhere for a circular foundation

BEARING CAPACITY EQUATION

FASD



Matched in TXC and:

$$\phi_{ps} = 1.12\phi_{tc}$$

(Kulhawy & Mayne 1990)

Old Danish:

$$\phi_{ps} = 1.1\phi_{tc}$$

New Danish:

$$\phi_{ps} = (1 + 0.1 I_D)\phi_{tc}$$

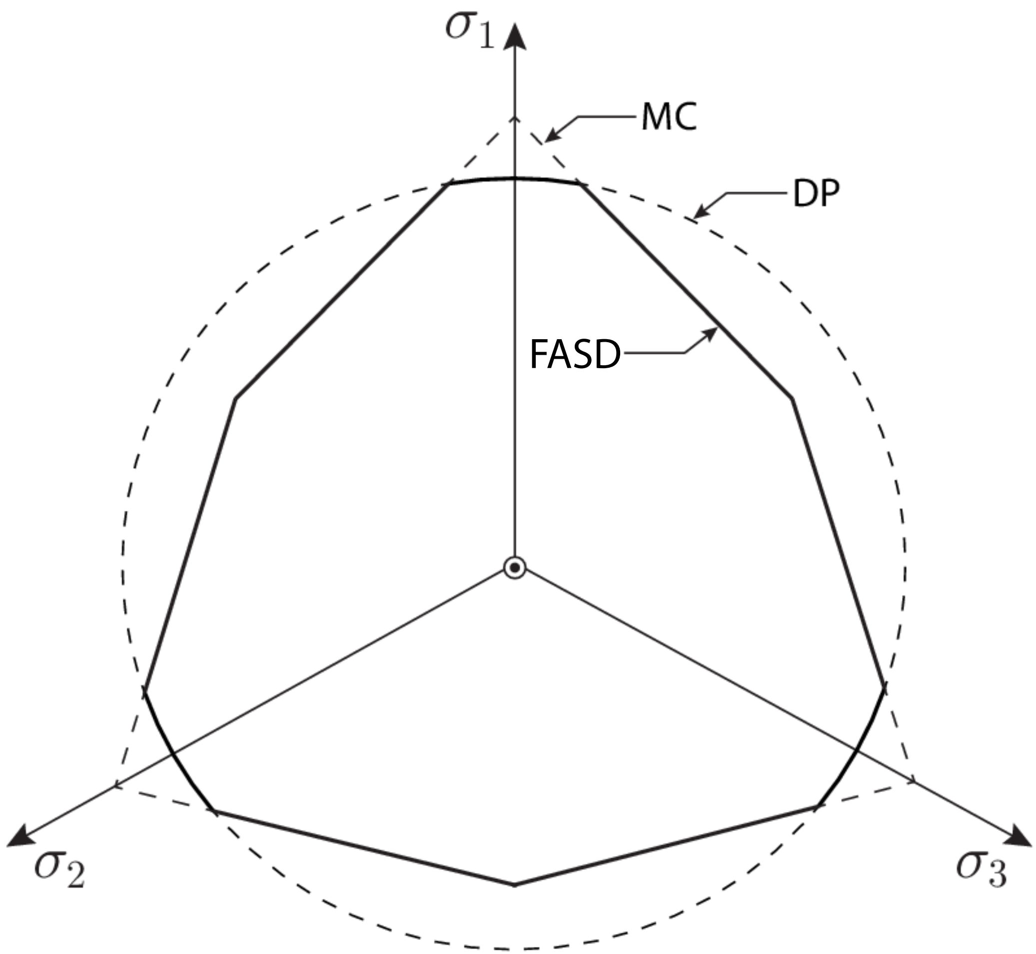
Stakemann (1976):

$$\phi_{ps} = (1 + 0.163 I_D)\phi_{tc}$$

Data: Lade & Wang (2001)

BEARING CAPACITY EQUATION

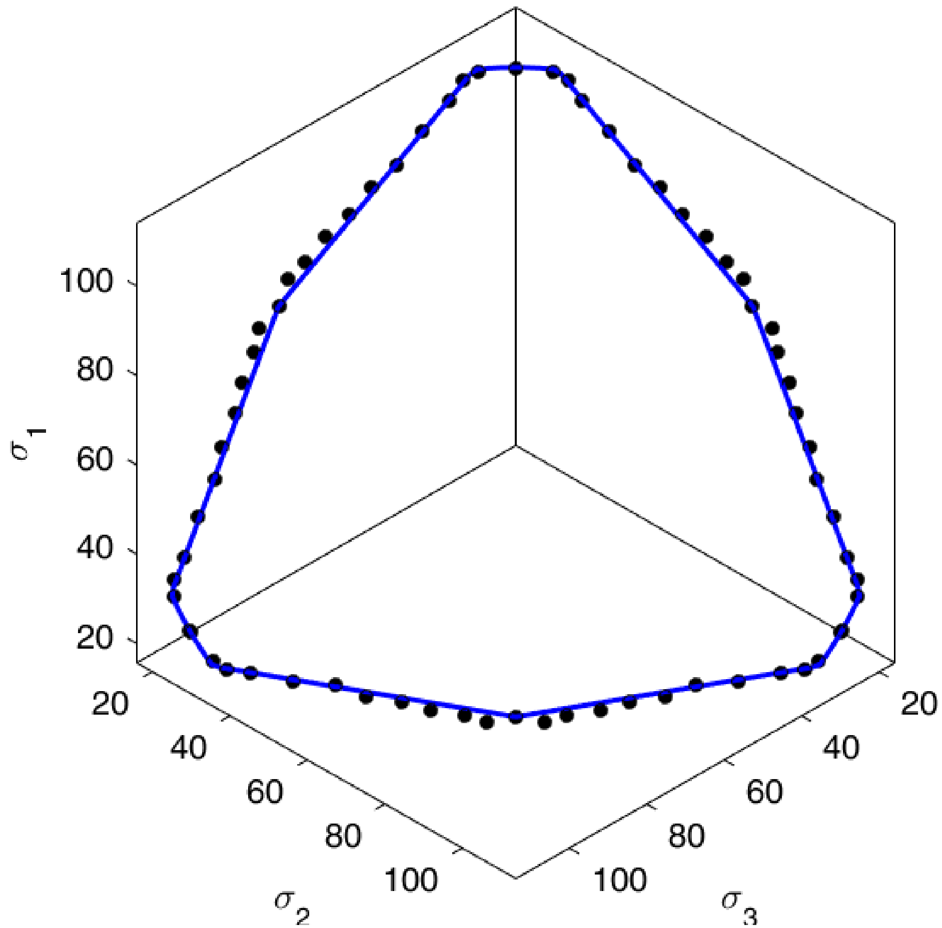
FASD Sand



FASD

$$\phi_{ps} = 1.12\phi_{tc}$$

Loose



BEARING CAPACITY EQUATION

FASD

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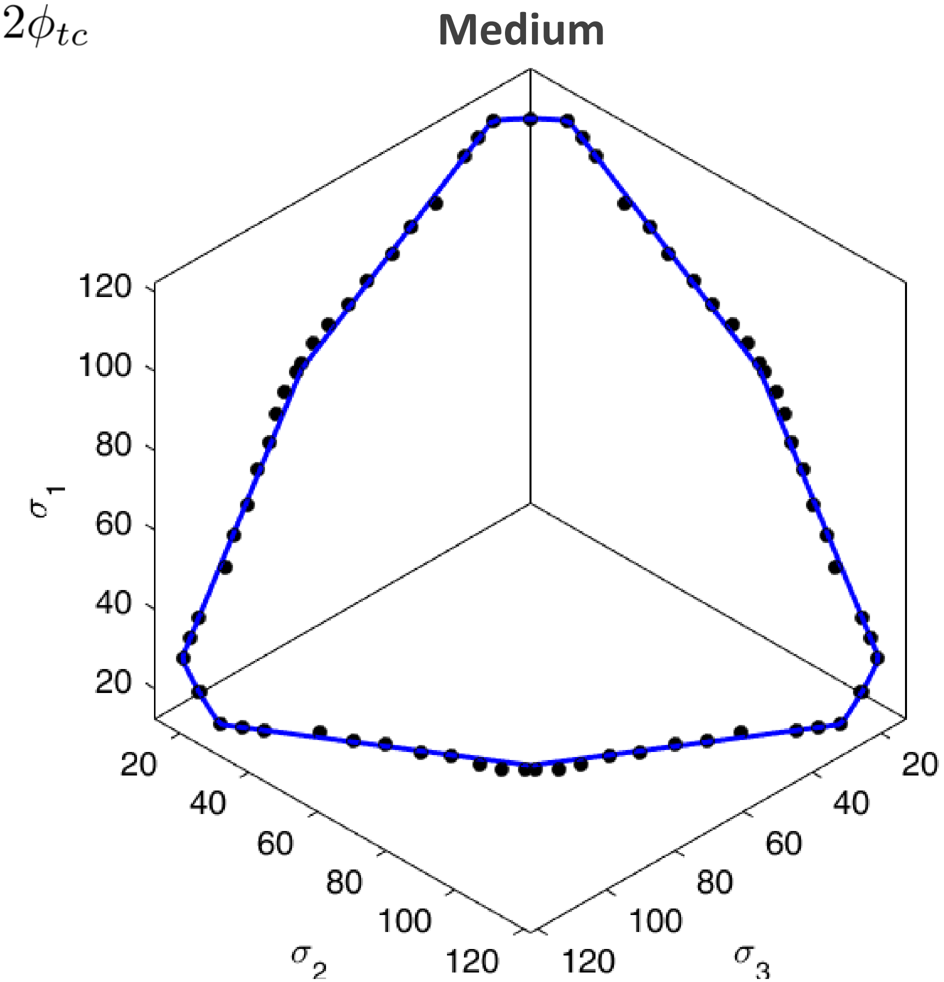

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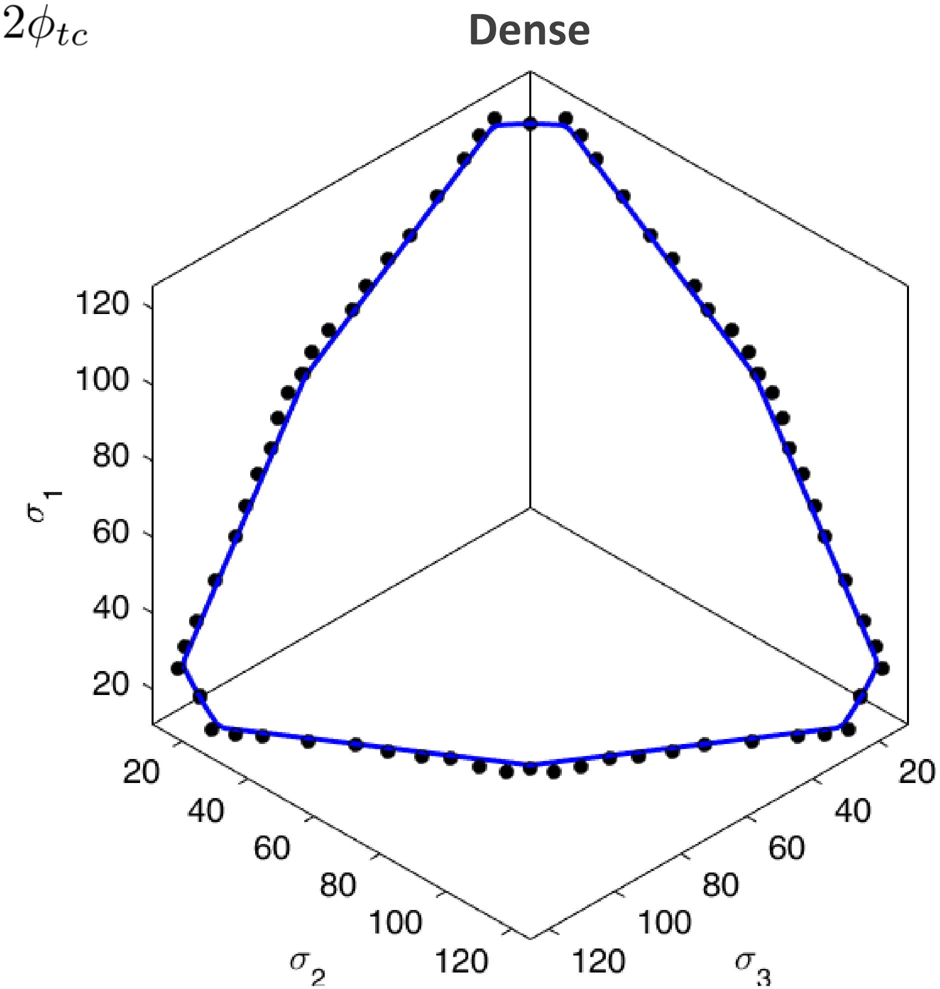

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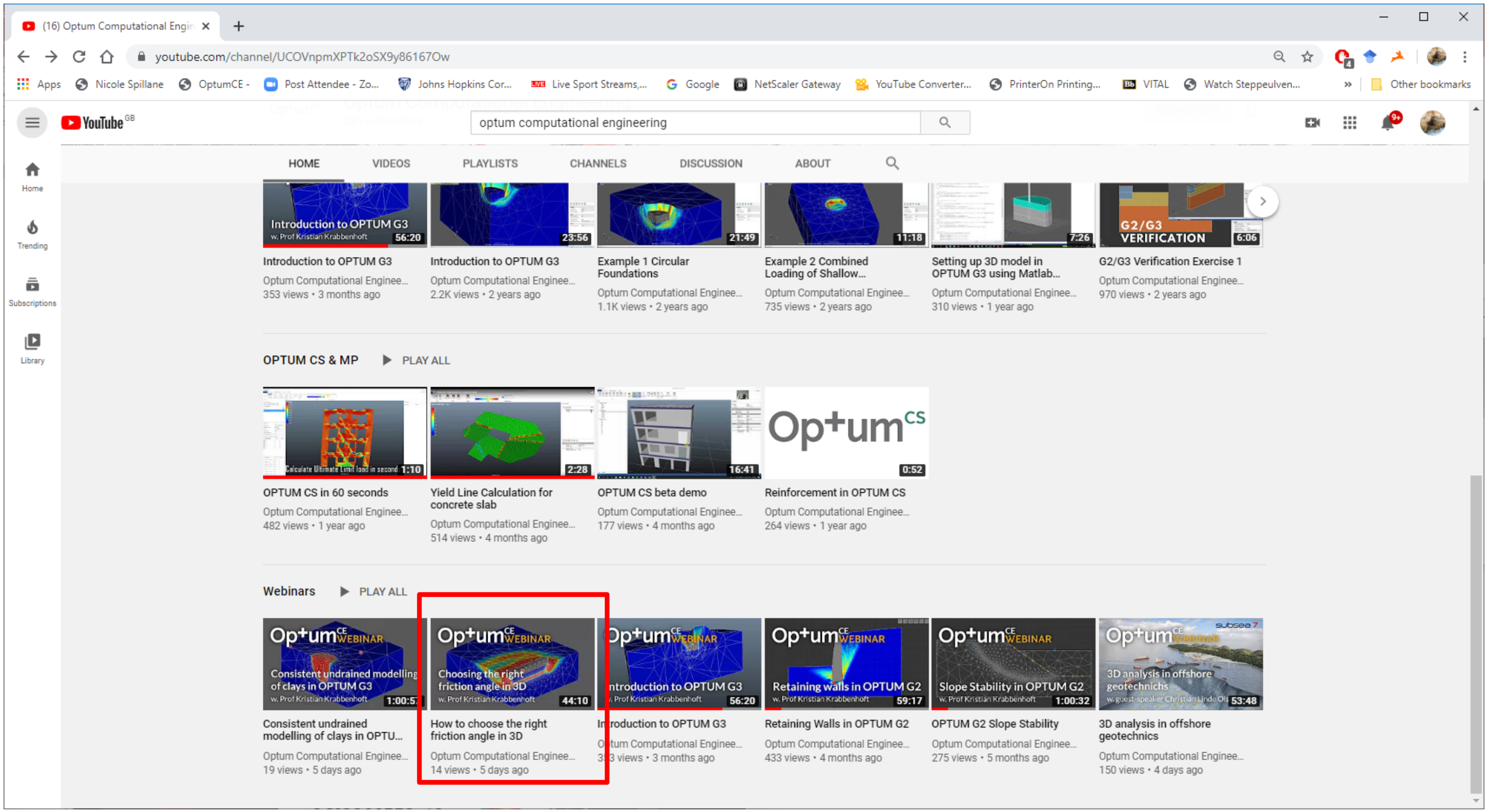
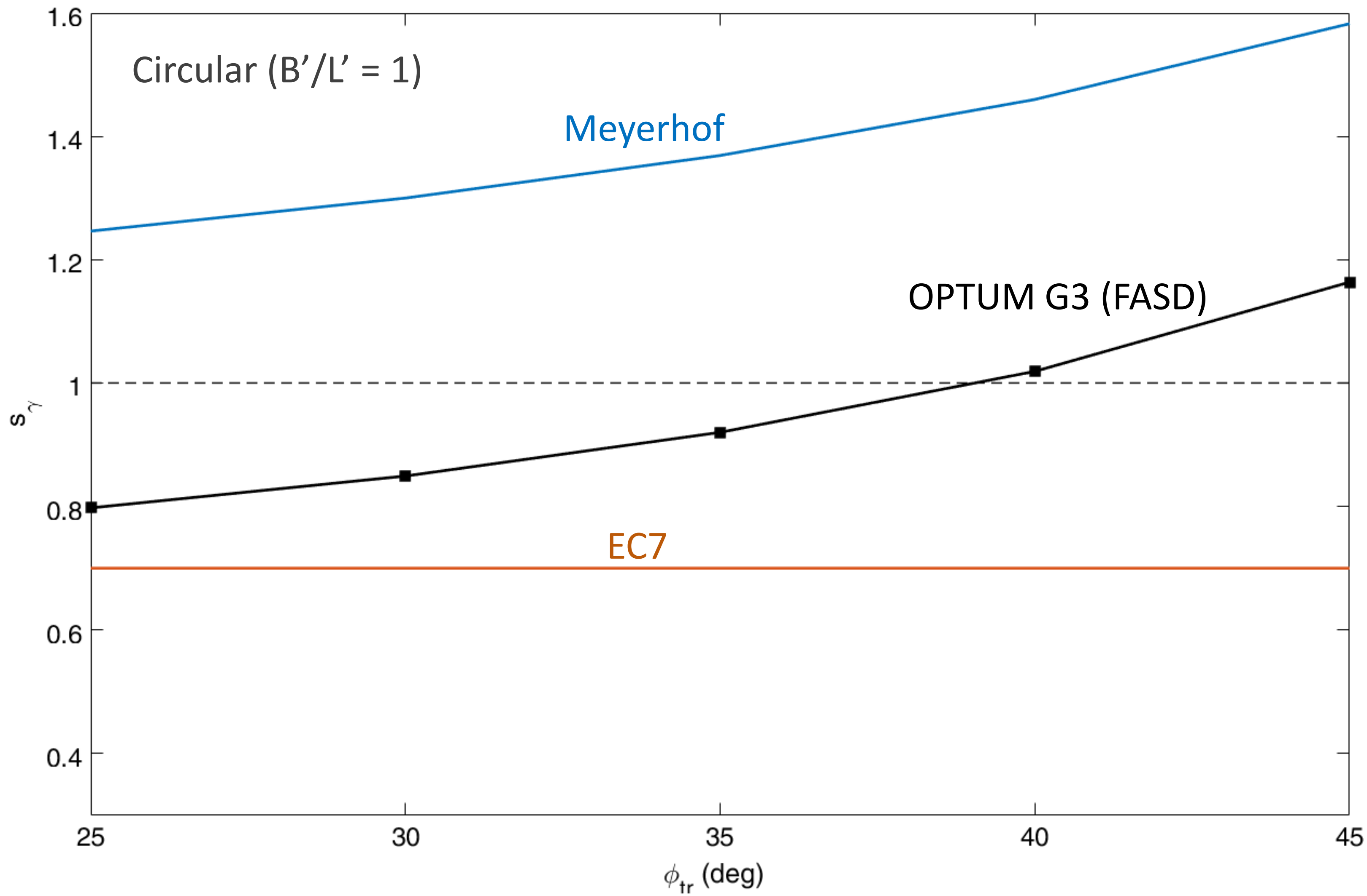


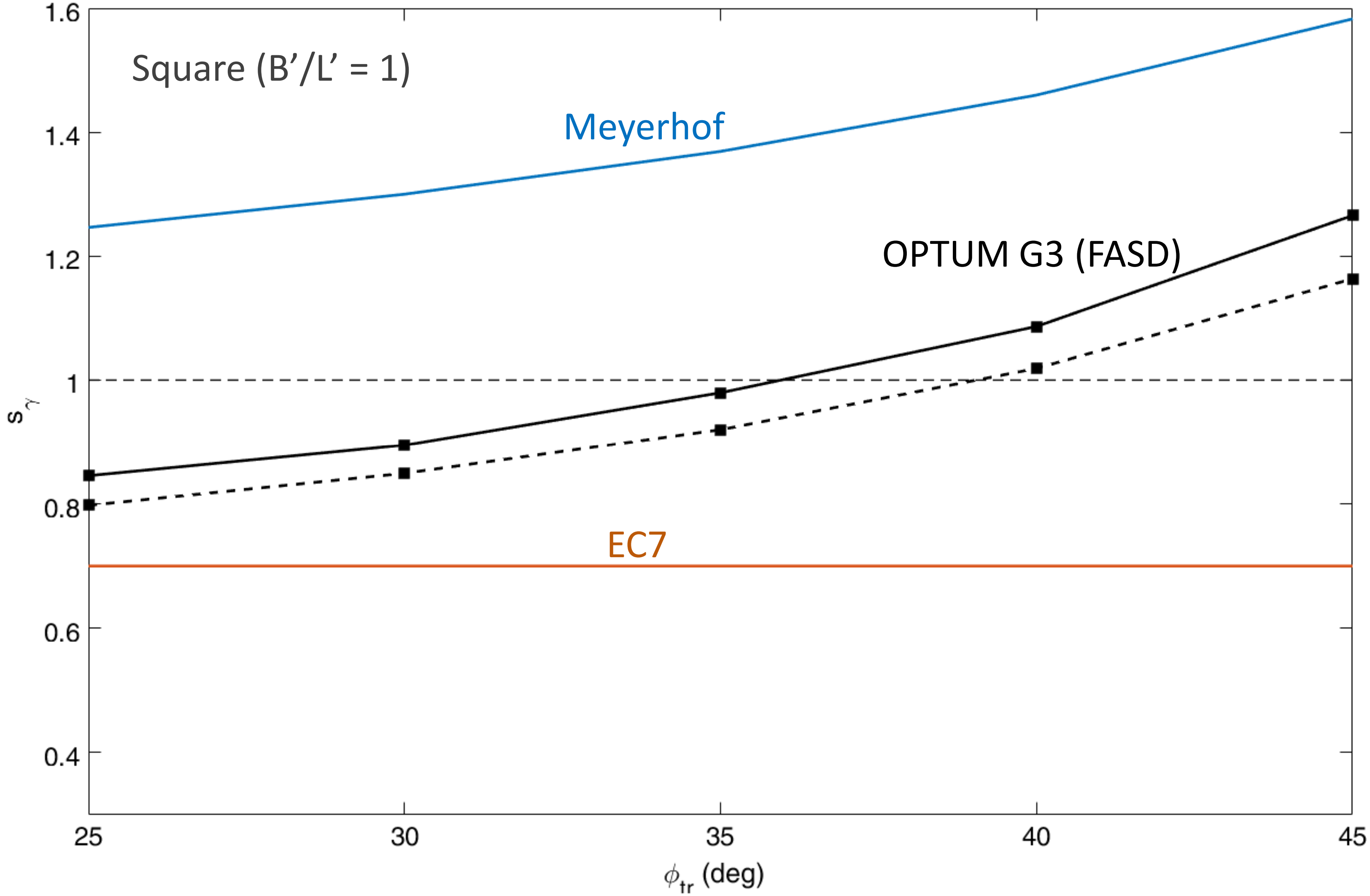
BEARING CAPACITY EQUATION

Shape



BEARING CAPACITY EQUATION

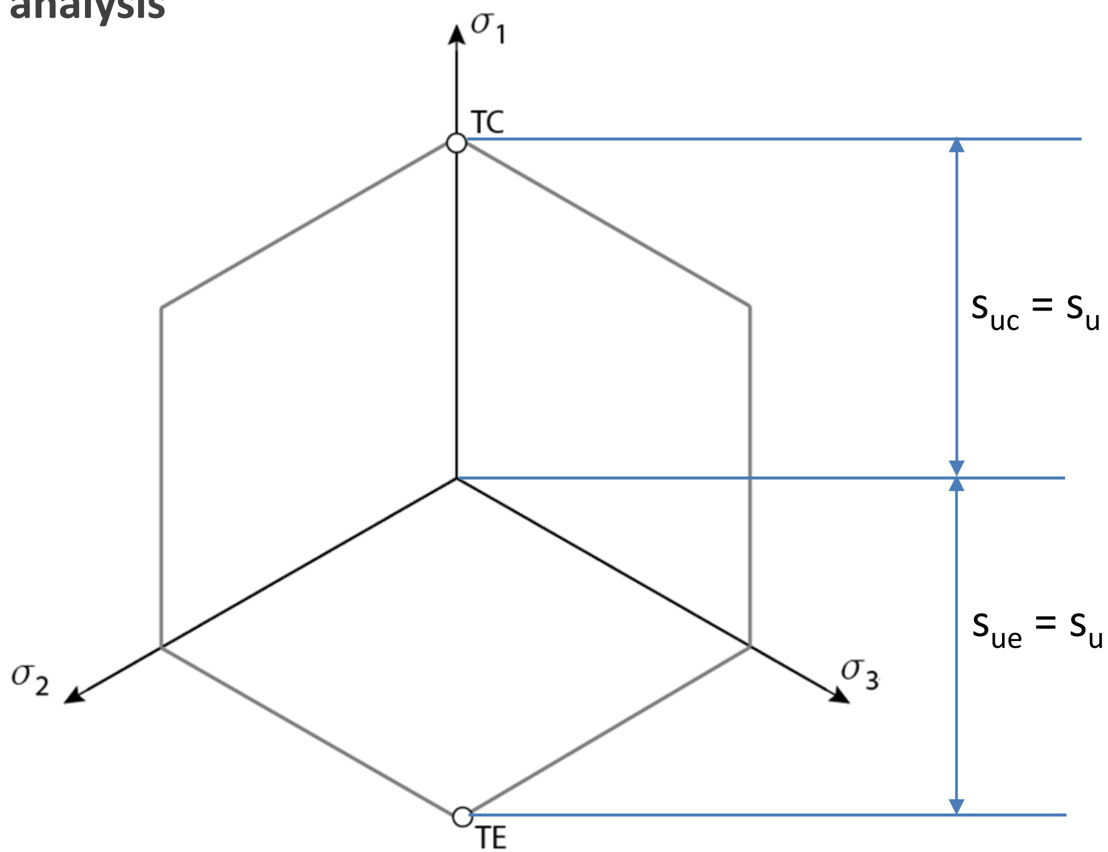
Shape



OptumCE

Shape

Undrained analysis

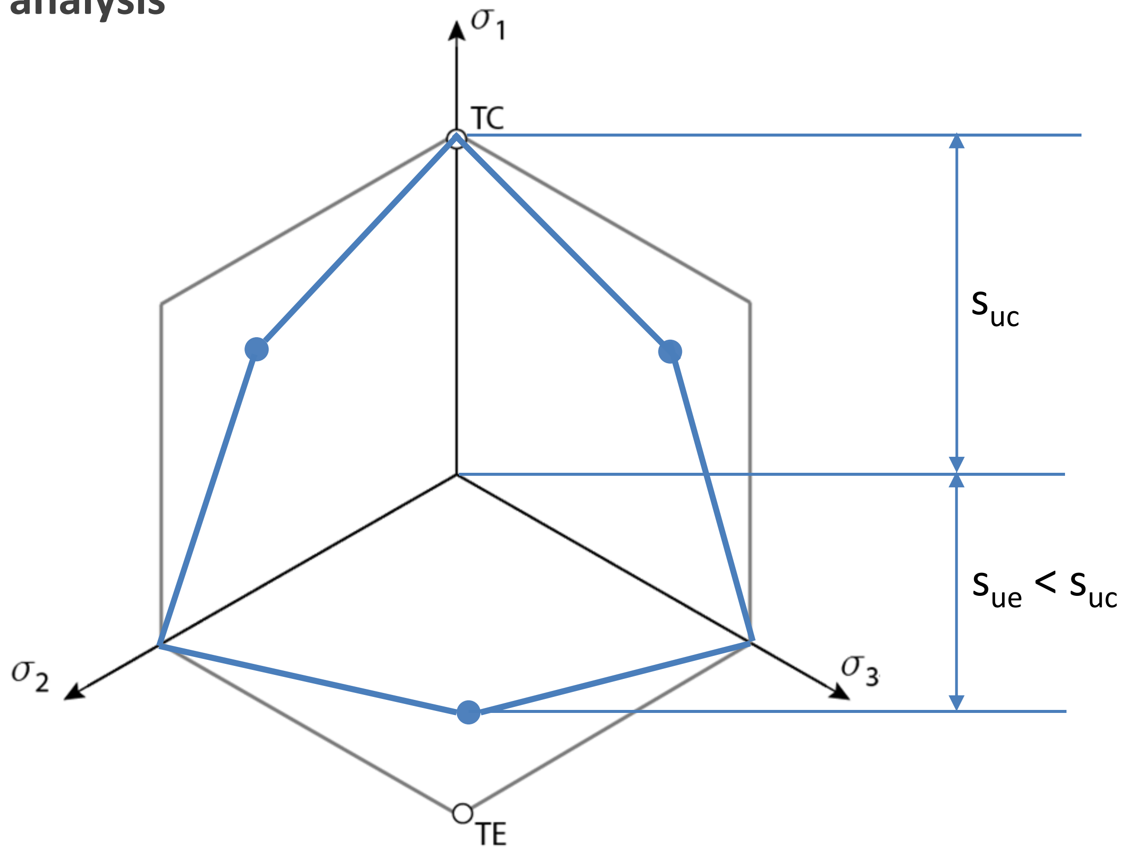


Tresca

OptumCE

Shape

Undrained analysis

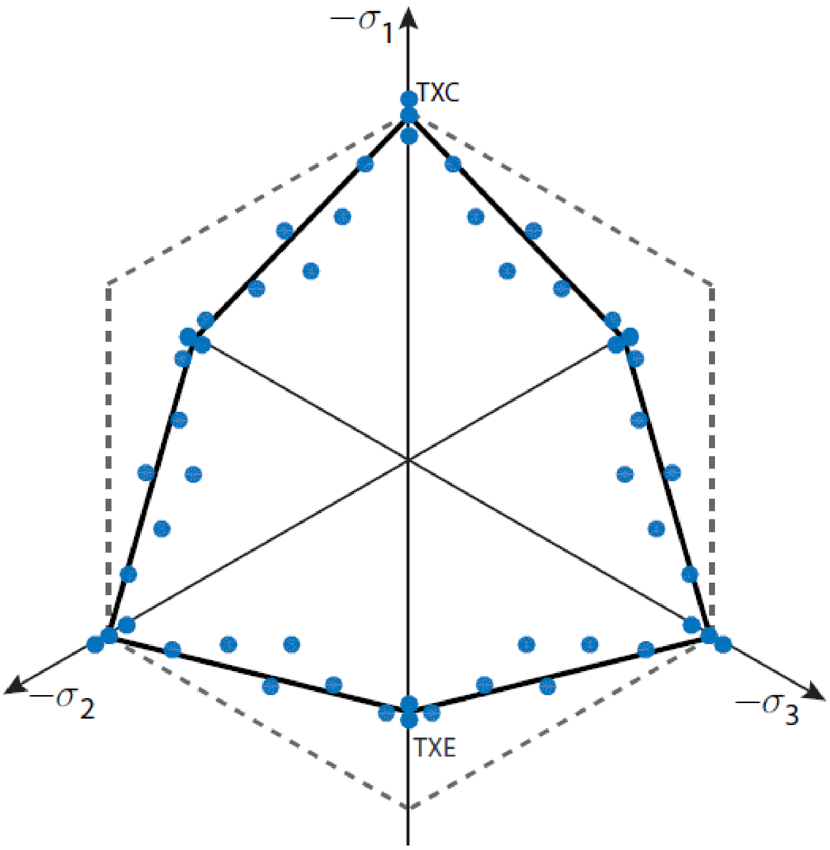


Generalized Tresca (s_{uc}, s_{ue})

Optum^CE

Yield surface

Undrained true triaxial tests on NC Edgar Plastic Kaolinite (Lade 1990)

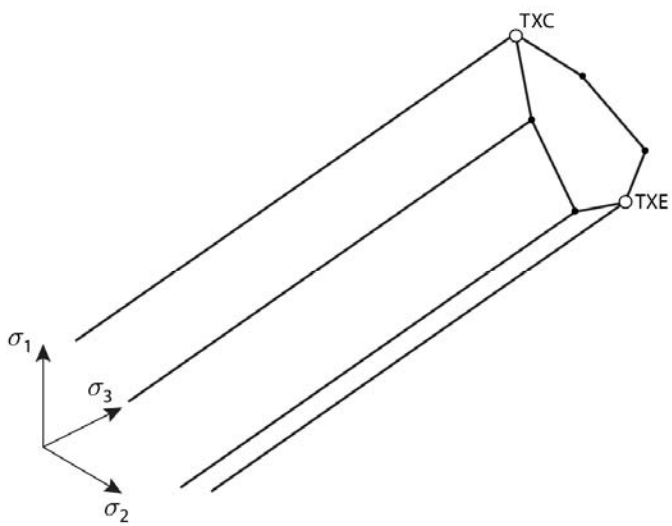


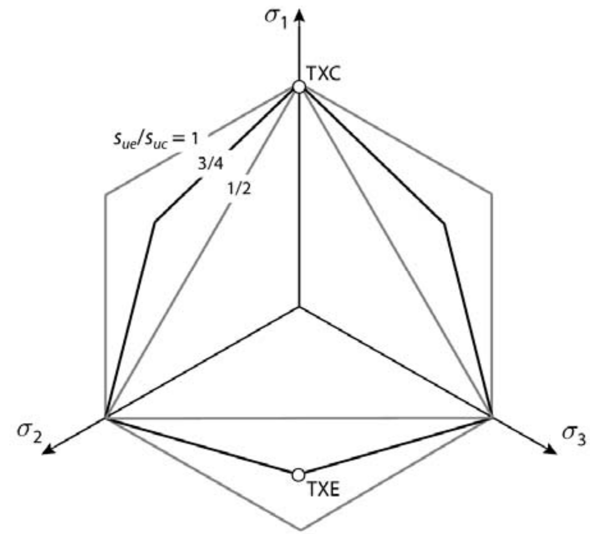
$s_{ue}/s_{uc} = 0.7$

BEARING CAPACITY EQUATION

Generalized Tresca

$$F_u = \sigma_1 - \sigma_3 + (s_{ue}/s_{uc} - 1)(\sigma_1 - \sigma_2) - 2s_{uc}$$





Material	
Name	Tresca Basic
Material Model	Tresca
Color	click to change
Reducible Strength	Yes

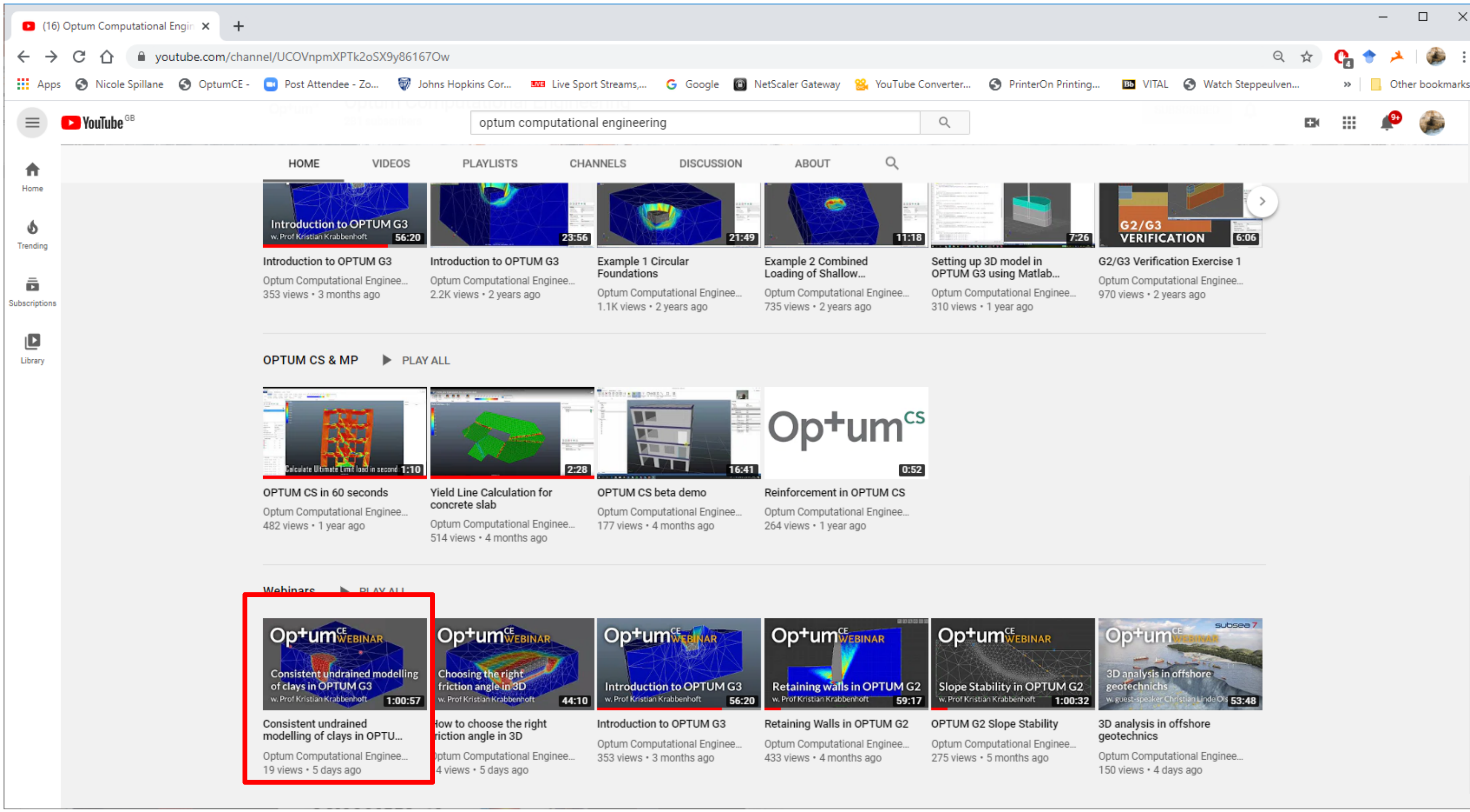
Strength	
Option	Standard
s_u (kPa)	100

Material	
Name	Tresca Basic
Material Model	Tresca
Color	click to change
Reducible Strength	Yes

Strength	
Option	Generalized
s_{uc} (kPa)	30
s_{ue} (kPa)	20

BEARING CAPACITY EQUATION

Generalized Tresca + AUS



BEARING CAPACITY EQUATION

Conclusions

- The bearing capacity equation is pretty good
- However issues with:
  - N_γ (exact solution has been available for the last 15 years)
  - Superposition (well known + conservative)
  - Inclined loading (sliding)
  - Eccentricity – requires some attention to detail (upcoming webinar)
  - Plane strain → 3D: reconsideration of soil model (MC → FASD, Tresca → GT/AUS)
  - Proper modelling of embedment rather than via surcharge
  - All the other well-known limitations – layered soils, increase of strength with depth, effects of torsion (upcoming webinar), etc

BEARING CAPACITY EQUATION

Optum^{CE}

Exact N_c and N_q are universally accepted, so why not exact N_γ?

- Martin's solution is not exact
- Numerical solutions don't qualify as exact
- The concept of exactness is meaningless
- It's complicated, there are so many approximations
- What about the flow rule, large deformations, softening, etc?
- There are more important problems (e.g. cure for cancer)
- The drained case is never critical, hence N_γ is not relevant
- *q*-term always dominates, hence N_γ is not important
- Our N_γ is based on experiments (carried out in the 1950s or 60s)
- Long standing practice will be disrupted

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There now remains the much more challenging problem of purging spurious N_γ values from the geotechnical literature

Ukritchon, B., Whittle, A. J. & Klangvijit, C. (2004). Response to discussion of 'Calculations of bearing capacity factor N_γ using numerical limit analyses' by Martin, C. M. (2004). J. Geotech. Geoenviron. Engng. Div., ASCE, 130, No. 10, 1107–1108.